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F. Palumbo: COHERENT EFFECTS OF A WEAK NEUTRAL AXIAL CURRENT ON NUCLEI WITH SPIN-ISOSPIN ORDER.

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F. Palumbo: COHERENT EFFECTS OF A WEAK NEUTRAL AXIAL CURRENT ON NUCLEI WITH SPIN-ISOSPIN ORDER.

The weak neutral axial current gives rise to coherent eleastic scattering of neutrinos by huclei with static spin-isospin order or pion condensation. The cross-section is comparable to that for normal nuclei at much lower momentum transfer. This allows to perform the experiment at values of the momentum transfer high enough for the recoil nucleus to be detectable and the contribution to the cross-section from the vector current to be negligeable.

The possible existence of spin-isospin order[1] or pion condensation[2] has been first proposed and investigated in the framework of infinite nuclear matter. Such order is characterized by localization of the nucleons which in the simplest models takes place along one direction only, the direction of spin quantization, giving rise to a laminated structure where spin-isospin alternate their orientation going from one layer to the next one.

It is generally assumed that if spin-isospin order or pion condensation are at all realized in nuclei, these will have a structure similar to that of nuclear matter, although other possibilities are not excluded (for instance non static spin-isospin order[3]). The experimental research of these ordered states is therefore based on the specific effects of this kind of structure. The purpose of this paper is to point out that one such effect is the coherent elastic scattering of neutrinos at values of the momentum transfer where the coherent elastic scattering by a normal nucleus[4] is negligeable. The coherent scattering of neutrinos by ordered nuclei is due to the weak neutral axial current. In the Weinberg-Salam model this current with the normalization of ref. 4 is

$$J_\mu = \frac{1}{2} \bar{\Psi} \gamma_\mu \gamma_5 \tau_3 \Psi \quad , \tag{1}$$

and its average value in a nuclear state with static spin-isospin order is different from zero. In non relativistic approximation in fact

$$J_k = -i \bar{\Psi} \sigma_k \tau_3 \Psi \tag{2}$$

and J_o is of order $1/c$. The average value of iJ_k is just the spin-isospin order parameter.

Let us therefore evaluate the neutrino-nucleus elastic cross section keeping the contribution from the axial current only. This point will be justified later. We suppose the nucleus to have a laminated structure with axial symmetry, with the layers ortogonal to the symmetry axis whose versor we denote by v. Let the initial state be

$$|\vec{P}\rangle = \frac{1}{(2\pi)^{3/2}} e^{i\vec{P}\cdot\vec{R}} \frac{1}{\sqrt{4\pi}} \int d\vec{v}\, \Phi(\vec{v}) \, , \tag{3}$$

where $\Phi(\vec{v})$ is the intrinsic w.f. describing the nucleus in its principal frame of inertia. We will make the usual assumption

$$\langle \varphi(\vec{v}) | 0 | \varphi(\vec{v}') \rangle = \delta(\vec{v} - \vec{v}') \langle \varphi(\vec{v}) | 0 | \varphi(\vec{v}) \rangle , \tag{4}$$

where 0 is any single particle operator.

Let the final state be

$$| \vec{P}' \vec{v} \rangle = \frac{1}{(2\pi)^{3/2}} e^{i\vec{P}' \cdot \vec{R}} \Phi(\vec{v}) . \tag{5}$$

We then have

$$\begin{aligned} \langle \vec{P} | J_k | \vec{P}', \vec{v} \rangle &= -i\delta(\vec{P}' - \vec{P} - \vec{Q}) \frac{1}{4\sqrt{4\pi}} \cdot \\ &\cdot \langle \Phi(\vec{v}) | \sum_i \sigma_k(i) \tau_3(i) e^{-i\vec{Q}(\vec{r}_i - \vec{R})} | \Phi(\vec{v}) \rangle = \\ &= -i\delta(\vec{P}' - \vec{P} - \vec{Q}) \frac{1}{2\sqrt{4\pi}} v_k S_{33}(\vec{Q}, \vec{v}) , \end{aligned} \tag{6}$$

where

$$\begin{aligned} S_{33}(\vec{Q}, \vec{v}) &= \int d\vec{r} \, e^{-i\vec{Q} \cdot \vec{r}} \langle \Phi(\vec{v}) | \overline{\psi} \tau_3 \sigma_3 \psi | \Phi(\vec{v}) \rangle = \\ &= \int d\vec{r} \, e^{-i\vec{Q} \cdot \vec{r}} \sum_{\sigma_3 \tau_3} \varrho_{\sigma_3 \tau_3} \sigma_3 \tau_3 , \end{aligned} \tag{7}$$

$\varrho_{\sigma_3 \tau_3}$ being the density of nucleons of spin σ_3 and isospin τ_3 .

The resulting differential cross-section is

$$\begin{aligned} \frac{d^3\sigma}{dQ^2 \, d\Omega_Q \, d\Omega_v} &= \frac{G^2}{2\pi} \frac{1}{4(4\pi)^2} \frac{1}{p^2} \left[(\vec{v} \cdot \vec{p})^2 - (\vec{v} \cdot \vec{p})(\vec{v} \cdot \vec{Q}) + \frac{Q^2}{4} \right] \cdot \\ &\cdot \left| S_{33}(\vec{Q}, \vec{v}) \right|^2 , \end{aligned} \tag{8}$$

where $\vec{p}$ is the momentum of the incoming neutrinos in the laboratory frame.

In order to get an estimate of this cross-section we parametrize $\varrho_{\sigma_3\tau_3}$ as follows

$$\varrho_{\sigma_3\tau_3} = C_{\sigma_3\tau_3} \frac{A}{\pi^{3/2} R_1^2 R_3^2} \exp\left(-\frac{r_\perp^2}{R_1^2}\right) \left[\exp\left(-\frac{z^2}{R_3^2}\right) + f_{\sigma_3\tau_3}(z)\right] , \quad (9)$$

with

$$\sum_{\sigma_3\tau_3} C_{\sigma_3\tau_3} = 1 , \qquad \int dz\, f_{\sigma_3\tau_3}(z) = 0 . \quad (10)$$

$f_{\sigma_3\tau_3}$ describes the spin-isospin density oscillations. For a normal nucleus $f_{\sigma_3\tau_3} = 0$. Only the term involving $f_{\sigma_3\tau_3}$ obviously contributes to $S_{33}(\vec{Q}, \vec{v})$

$$S_{33}(\vec{Q}, \vec{v}) = A \exp\left\{-\frac{1}{4}\left[Q^2 - (\vec{Q}\cdot\vec{v})^2\right] R_1^2\right\} f(\vec{Q}\cdot\vec{v}) , \quad (11)$$

where

$$f(\vec{Q}\cdot\vec{v}) = \frac{1}{\sqrt{\pi}\, R_3} \sum_{\sigma_3\tau_3} C_{\sigma_3\tau_3} \int dz\, e^{-i\vec{Q}\cdot\vec{v}z} f_{\sigma_3\tau_3}(z) . \quad (12)$$

In order to integrate the cross-section over angles we observe that the fall-off of $f(\vec{Q}\cdot\vec{v})$ must be governed by the width λ of spin-isospin density ripples

$$\left| f(\vec{Q}\cdot\vec{v}) \right| \sim \eta\, e^{-\frac{(\vec{Q}\cdot\vec{v})^2 \lambda^2}{4}} , \quad (13)$$

where η mesures the amplitude of oscillations (actually η will be an oscillating function of $\vec{Q}\cdot\vec{v}$). In nuclear matter[5] $\lambda \sim 1$ fm , so that in not too light nuclei $(\frac{\lambda}{R_1})^2 \ll 1$. For $QR_1 \gg 1$ we can therefore approximate $\left|S_{33}(\vec{Q},\vec{v})\right|^2$ by

$$\left|S_{33}(\vec{Q}\cdot\vec{v})\right|^2 \sim A^2 \sqrt{\frac{\pi}{2}} \frac{1}{QR_1} \frac{1}{2}\left[\delta\left(\frac{\vec{Q}\cdot\vec{v}}{Q}+1\right)+\delta\left(\frac{\vec{Q}\cdot\vec{v}}{Q}-1\right)\right]\left|f(\vec{Q}\cdot\vec{v})\right|^2. \tag{14}$$

In this approximation the recoil nucleus will be polarized with $\vec{v}$ parallel or antiparallel to $\vec{Q}$.

Finally

$$\frac{d\sigma}{dQ^2} = \frac{G^2}{2\pi}\frac{1}{24}A^2\left(1+\frac{3}{4}\frac{Q^2}{p^2}\right)\sqrt{\frac{\pi}{2}}\frac{1}{QR_1}\frac{1}{2}\left[\left|f(Q)\right|^2+\left|f(-Q)\right|^2\right] \sim$$

$$\sim \frac{G^2}{2\pi}\frac{1}{24}A^2\left(1+\frac{3}{4}\frac{Q^2}{p^2}\right)\sqrt{\frac{2}{\pi}}\frac{\eta^2}{QR_1}\, e^{-\frac{Q^2\lambda^2}{2}} . \tag{15}$$

Let us compare this cross-section with the cross-section for elastic neutrino scattering by a normal nucleus[4], where the vector current only contributes

$$\frac{d\sigma}{dQ^2} = \frac{G^2}{2\pi}\sin^4\theta_W A^2\left(1+\frac{1}{4}\frac{Q^2}{p^2}\right) e^{-\frac{Q^2R^2}{2}} . \tag{16}$$

Since[6] $\sin^4\theta_W \sim \frac{1}{24}$, the essential difference is the replacement of $e^{-\frac{Q^2R^2}{2}}$ by $\sqrt{\frac{2}{\pi}}\frac{\eta^2}{QR_1}\, e^{-\frac{Q^2\lambda^2}{2}}$.

We see that in not too light nuclei the contribution from the vector current is negligeable w. r. to the contribution from the axial current for $Q \sim \frac{1}{\lambda} \gg \frac{1}{R}$, justifying our calculation of the cross-section for ordered nuclei.

This cross-section appears easier to be mesured than the cross--section for normal nuclei. The main difficulty of the experiment on normal nuclei is in fact the detection of a recoil nucleus of low momentum Q. That Q must be low follows from the gaussian damping $e^{-\frac{Q^2R^2}{2}}$ of the cross-section, a damping which is more effective the heavier the nucleus. It has been estimated[4] that already for C^{12} the maximum value of Q for the cross-section to be not too small is 300 MeV/c. But in the case of ordered nuclei the damping becomes $e^{-\frac{Q^2\lambda^2}{2}}$ (approximately), independent of the mass number, so that the maximum value of Q is $\frac{R}{\lambda}$ times higher.

Bearing this in mind, details of the estimates of the cross-section as well as considerations about the experiment can be taken directly from ref. 4.

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