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ABSTRACT

A semimicroscopic model of the nucleus is presented in which spin-isospin order is realized as zero-point one-dimensional oscillations of spin-up protons and spin-down neutrons against spin-down protons and spin-up neutrons. This phase is favored by the OPE potential in not too heavy spherical and oblate nuclei. The model is characterized by isospin admixtures in the ground state and enhancement of the B(M2,K=0 $\rightarrow$ K=0) by a factor 20.

It has been suggested[(1)] that the OPE potential should give rise to a spin-isospin ordered phase. In infinite nuclear matter such an order, however, requires some localization of nucleons. In a number of specific models[(2)] localization takes place only along one direction, the direction of spin quantization, giving rise to a laminated structure where spin and/or iso-spin alternate their orientation going from one layer to the next one.

The actual occurrence of this phase depends on the balance between the kinetic energy increase necessary to localize nucleons, the attraction coming from the OPE potential and the change in the short range interaction energy. This last effect is most difficult to evaluate and makes uncertain the determination of the critical density. It is therefore hard to establish theoretically whether this ordered phase is actually present in nuclei.

We want to explore the possibility of a realization of spin-isospin order in nuclei which does not require localization of nucleons in layers. Being nuclei finite it is in fact sufficient to correlate nucleons with different spin-isospin in appropriate way in order to get a nonvanishing contribution from the OPE potential. Such a

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correlation can be realized by allowing protons with spin-up and neutrons with spin-down to oscillate with respect to protons with spin-down and neutrons with spin-up. Spin-isospin order is thus obtained in the zero-point motion.

Oscillations can occur in one, two or three dimensions. We consider here the case of one-dimensional oscillations, which is closer to the model developed in nuclear matter.

In analaogy with the procedure adopted in refs. (1,2) we introduce displaced creation operators

$$(C'_{a\sigma_3\tau_3})^+ = \exp(-\frac{i}{2\hbar}\sigma_3\tau_3 d\, p_z) C^+_{a\sigma_3\tau_3} \exp(\frac{i}{2\hbar}\sigma_3\tau_3 d\, p_z) \tag{1}$$

where we label by a the spatial quantum numbers. Out of the operators (1) we construct a Slater determinat

$$\Lambda = \prod_{h\sigma_3\tau_3} (C'_{h\sigma_3\tau_3})^+ |0\rangle , \tag{2}$$

where h denotes occupied states.

Consistently with the traditional approach to the macroscopic description of collective vibrations[3] we redefine d as a collective variable and assume a total w.f. of the form

$$\Psi_n = \Phi_n (d)\, \Lambda (d). \tag{3}$$

The collective Hamiltonian in the harmonic approximation is

$$H = \frac{p_d^2}{2M} + \frac{1}{2}(C + K_{OPE}) d^2, \tag{4}$$

where M is the reduced mass, K_{OPE} the contribution to the restoring constant coming from the OPE and C the contribution coming from the other components of the N-N potential.

In order to establish whether spin-isospin order in the zero-point motion is favored, we should compare the energy of this phase with the one of other possible ordered phases, like for instance pure isospin order, and of the disordered phase. A proper treatment of short range correlations however is so difficult, that this is practically out of the present possibilities.

We therefore confine ourselves to the study of the characteristic predictions of the model which can be tested experimentally.

Before doing this we evaluate K_{OPE}. A necessary condition for spin-isospin order to be favored in the zero-point oscillation is in fact that K_{OPE} be negative.

We assume spin quantization along the Z-axis. The direct part of the expectation value of V_{OPE} is

$$\langle\Lambda|V_{OPE}|\Lambda\rangle_D = \frac{1}{2}\int d\vec{r}_1 \int d\vec{r}_2 \langle\Lambda|S_{33}(\vec{r}_1)|\Lambda\rangle\langle\Lambda|S_{33}(\vec{r}_2)|\Lambda\rangle \cdot \left[V_C(r_{12}) + 3(\frac{z_{12}^2}{r_{12}^2} - 1) V_T(r_{12})\right] , \tag{5}$$

where

$$S_{ik} = \bar{\Psi}\, \tau_i \sigma_k \Psi \tag{6}$$

is the spin-isospin density operator and V_C and V_T are the central and tensor terms of the OPE. The contact term of V_C will be omitted. For a discussion of this point see ref. (4).

The average value of S_{33} is

$$\langle\Lambda|S_{33}|\Lambda\rangle = \sum_{\tau_3\sigma_3} \varrho_{\tau_3\sigma_3}\, \tau_3\sigma_3 \tag{7}$$

where

$$\varrho_{\tau_3\sigma_3}(\vec{r},d) = \sum_h \varphi^*_{h\sigma_3\tau_3}(\vec{r},d)\,\varphi_{h\sigma_3\tau_3}(\vec{r},d) \tag{8}$$

$$\varphi_{h\sigma_3\tau_3}(\vec{r},d) = (e^{\frac{-i}{\hbar}\tau_3\sigma_3 p_z d}\,\varphi_{h\tau_3\sigma_3})(\vec{r}) \tag{9}$$

We disreguard the contribution to K_{OPE} coming from the exchange term according to the estimates of refs. (1,2) in nuclear matter.

In order to get a workable expression for the mean value (5) we approximate the one-body density by a gaussian of the form

$$\varrho_{\tau_3\sigma_3}(\vec{r},d) = \varrho_o \exp\left[-\frac{r_\perp^2}{R_1^2} - \frac{(Z-\frac{1}{2}\sigma_3\tau_3 d)^2}{R_3^2}\right] \tag{10}$$

which allows a unified description of sphericl as well as deformed nuclei. We consider for simplicity only N=Z nuclei. R_1 and R_3 are related to the deformation parameter δ and the nuclear radius R through

$$R_1 = \left[\frac{2}{5(1+\delta)}\right]^{1/2} R \qquad ; \quad R_3 = \left[\frac{2}{5(1-\delta)}\right]^{1/2} R \tag{11}$$

We can now expand $\langle V_{OPE}\rangle_D$ to first order in δ and second order in d

$$\langle V_{OPE}\rangle_D \simeq \frac{1}{2}K_{OPE}\, d^2 = \frac{1}{2}(K_o + K_1\,\delta)\, d^2 \tag{12}$$

where

$$K_i = \frac{1}{4\sqrt{5\pi}}\,\frac{v_o}{\mu}\,\frac{A^2}{R^3}\int_o^\infty dt\, \exp(-\frac{2}{\sqrt{5}}\,\mu R t - t^2)\, t\, p_i(t)$$

$$p_o = 1 - \frac{2}{(2\mu R)^2} - \frac{8}{\sqrt{5}(2\mu R)}\, t - \frac{6}{5}t^2$$

$$p_1 = \frac{18}{(2\mu R)^2} - \frac{1}{4} + \frac{18}{\sqrt{5}(2\mu R)}\, t + (1.7 - \frac{40}{7(2\mu R)^2})\, t^2 + \tag{13}$$

$$- \frac{8}{7}\,\frac{\sqrt{5}}{2\mu R}\, t^3 - \frac{18}{35}t^4$$

$$t = \frac{5}{2}\,\frac{r_{12}}{R} \qquad ; \quad \mu = \text{pion mass}$$

Evaluation of the above formulae shows that the most attractive contribution is obtained for not too heavy deformed oblate nuclei. For ligth spherical nuclei small negative values of K_{OPE} are still obtained, while K_{OPE} is positive in all the other cases. Typical values of K_{OPE} are

For A = 12, δ = -0.4 ,	K_{OPE} = -13.6 MeV fm^{-2}
For A = 16, δ = 0 ,	K_{OPE} = - 4.7 MeV fm^{-2}
For A = 28, δ = -0.4 ,	K_{OPE} = -10.3 MeV fm^{-2}

These values K_{OPE} must be compared with C, eq. (4). An rough estimate of the latter quantity can be based on the energy of the electric dipole giant resonance. The phenomenological estimate reported by Bohr and Mottelson[5] can be adapted to our model by rescaling C by a factor $(\frac{A}{2})^2$ and M by $(\frac{2}{A})^2$. This gives $C \sim 10.2\ A^{1/3}$ MeV fm^{-2}.

The excitation energy is therefore

$$\hbar\omega = 2\hbar\sqrt{\frac{C+K_{OPE}}{Am}} \simeq \begin{array}{ll} 11.8 & \text{for } A=12 \\ 14.4 & \text{for } A=16 \\ 10.9 & \text{for } A=28 \end{array}$$

which results considerably lower than the energy of the electric dipole resonance.

The corresponding excited state is characterized by enhanced M2 transition probabilities. In fact

$$B(\lambda, I=K=0 \rightarrow I,K) = \frac{2}{1+\delta_{K0}} \left| \langle \Phi_{1K} \left| \mathcal{M}(\lambda=I, \nu=K) \right| \Phi_0 \rangle \right|^2 \tag{14}$$

where

$$\mathcal{M}(I,K) = \langle \Lambda \left| M(I,K) \Lambda \right| \rangle \sim \frac{i}{2\hbar} d \sum_{h\sigma_3\tau_3} \sigma_3\tau_3 \langle \varphi_{h\sigma_3\tau_3} \left| \left[p_3, M(I,K) \right] \right| \varphi_{h\sigma_3\tau_3} \rangle . \tag{15}$$

It is easy to show from this equation that the lowest nonvanishing multiple operator is the magnetic quadrupole one, whose expression is

$$\mathcal{M}(M2,K) \simeq \frac{1}{8}\sqrt{\frac{15}{2}} \langle 1 0 1 0 | 2 0 \rangle (g_p - g_n)\, d \frac{e\hbar}{2mc}\, \delta_{K0} \tag{16}$$

Note that only the K=0 component is different from zero. The corresponding B(M2) is of the order of 20 w.u.

Enhanced transition probabilities of different multipolarity to other states are also allowed. These can occur either because the excited states have a different order or because a different order coexists in the ground state[6].

A further characterization of the model is the breaking of isospin, whose average value in the ground state is $\langle T^2 \rangle \sim 1$.

Needless to say our numerical estimates can only be indicative. However the predictions concerning the oblate character of the deformation, the lowering of excitation energy, the enhancement of the B(M2,K=0 → K=0) and the presence of isospin mixing can be taken in our view as qualitatively significant.

The model can be extended in several ways. Oscillations along the x and y axis can also be allowed associated with the same or different spin-isospin order (for instance only isospin order as in the Goldhaber-Teller model[7]). Furthermore the spin can be quantized along an axis perpendicular to the symmetry axis. The resulting correlation migth be consistent with prolate deformations. A thorough analysis of these extensions will be reported in a separate paper[6].

We conclude by recalling that spin-isospin order in infinite nuclear matter is equivalent to neutral pion condensation[8]. In the present model however, the average value of the pion field vanishes due to the nonstatic character of the spatial correlation.

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