Lecture presented at the Intern. School of Physics 'Enrico Fermi' - "From Nuclei to Particles" - Varenna, June 1980.

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

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F. Palumbo: SPIN-ISOSPIN ORDERED PHASES IN NUCLEAR MATTER AND IN NUCLEI.

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SUMMARY.

Spin-isospin ordered phases (SIOP) in nuclear matter are described and related to pion condensation. Their properties w.r. to parity and isospin are analyzed in connection with their possible occurrence in nuclei.

A model of SIOP in nuclei and an experiment to detect SIOP by elastic neutrino scattering are proposed.

1. - INTRODUCTION.

In my first lecture I will talk about spin-isospin ordered phases (SIOP) in nuclear matter. The occurrence of such phases has been suggested(1) by the form of the OPE potential. I will discuss in some detail the first model of nuclear binding by the OPE and I will show its connection with pion condensation(2). In this model nuclear matter is fluid in two dimensions and solid in one dimension. It turns out that along this dimension nucleons have a sharp localization(1, 3). This makes very important the role of the core of the N-N interaction, which has been studied for some one-boson-exchange potentials(4).

I will conclude the first talk by discussing the properties of SIOP and pion condensation with respect to parity and isospin(5). Such properties are especially relevant to the actual occurrence of SIOP and pion condensation in nuclei.

SIOP in nuclei will be the subject of my second lecture. I will first talk about an experiment to establish whether SIOP and pion condensation are actually realized(6). This experiment is based on the coherent effects of a neutral weak axial current.

I will finally present a model(7) of SIOP in nuclei without space order and pion condensation. This kind of SIOP cannot be investigated by means of the above experiment, but is characterized by enhanced M2 transition amplitudes.

2. - OPE POTENTIAL.

The OPE is strong and long range, but it does not contribute to the binding energy in Hartree approximation unless the nuclear system is in a SIOP. This is due to its spin-isospin dependence

$$V_{OPE} = \frac{1}{3} f^2 \mu \vec{\tau}_1 \cdot \vec{\tau}_2 \left\{ -\frac{4\pi}{3} \delta(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 + (1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}) S_T \right] \frac{e^{-\mu r}}{\mu r} \right\} . \quad (1)$$

In fact assuming the spins quantized along the z-axis

$$\langle V_{OPE} \rangle_{direct} = \frac{1}{2} \sum_{\tau_3(1)\sigma_3(1)} \sum_{\tau_3(2)\sigma_3(2)} \int d\vec{r}_1 \int d\vec{r}_2 \langle \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) \cdot$$

$$\left| V_{OPE} \right| \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) \rangle \; \varrho_{\tau_3(1)\sigma_3(1)}(\vec{r}_1) \; \varrho_{\tau_3(2)\sigma_3(2)}(\vec{r}_2) \; , \quad (2)$$

where $\varrho_{\tau_3\sigma_3}$ is the one-body density matrix of nucleons of isospin τ_3 and spin σ_3. The spin-isospin matrix element is

$$\langle \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) \left| V_{OPE} \right| \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) \rangle = \frac{1}{3} f^2 \mu \left\{ -\frac{4\pi}{3} \delta(\vec{r}) + \right.$$

$$\left. + \left[1 + (1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2})(3\frac{z^2}{r^2} - 1) \right] \frac{e^{-\mu r}}{\mu r} \right\} \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) \; . \quad (3)$$

It is convenient to introduce the spin-isospin density operator

$$S_{ik} = \bar{\psi} \tau_i \sigma_k \psi \; . \quad (4)$$

Eq. (2) can be rewritten in terms of the average value of S_{33}

$$\langle S_{33} \rangle = \sum_{\tau_3\sigma_3} \varrho_{\tau_3\sigma_3} \tau_3 \sigma_3 \; , \quad (5)$$

$$\langle V_{OPE} \rangle_{direct} = \frac{1}{2} \int d\vec{r}_1 \int d\vec{r}_2 \langle S_{33}(\vec{r}_1) \rangle \langle S_{33}(\vec{r}_2) \rangle \frac{1}{3} f^2 \mu \left\{ -\frac{4\pi}{3} \delta(\vec{r}) + \right.$$

$$\left. + \left[1 + (1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2})(3\frac{z^2}{r^2} - 1) \right] \frac{e^{-\mu r}}{\mu r} \right\} . \quad (6)$$

Eqs. (5) and (6) show that if $\varrho_{\tau_3\sigma_3}$ is independent of $\tau_3\sigma_3$, $\langle V_{OPE} \rangle_{direct} = 0$. It is therefore natural to think that a SIOP should be favored at sufficiently high density. At high density in fact, the exchange potential energy per particle grows like the density ϱ, the kinetic energy per particle like $\varrho^{5/3}$ and the direct potential energy per particle like ϱ^2. This last term is therefore dominating[8], and if the OPE were the whole N-N interaction, a SIOP would necessarily be established which would eventually lead to nuclear collapse[1]. The problem is therefore to investigate whether and to which extent other components of the N-N interaction will favor or handicap possible SIOP.

Let us remind for later reference that the contact term in eq. (1) is not accettable in the nuclear Hamiltonian, unless an infinite repulsive core is present. Such a term in fact, being not definitive in sign, would make the Hamiltonian unbounded from below.

The OPE potential must therefore be regularized. An example of regularization is given by the one-boson-exchange potentials(9) that we will use in Section 6. Another crude form of regularization that will be adopted in the next Section is to simply cut-off the potential at short distance.

3. - NUCLEAR BINDING BY THE OPE POTENTIAL.

Spin-isospin order in nuclear matter can be realized only localizing nucleons to some extent. Now unless nuclear matter is anisotropic, the tensor term does not contribute in eq. (6), because the factor $3z^2-r^2$ averages to zero. SIOP will therefore be anisotropic, with maximum asymmetry between the z-axis and the x-y-plane. It seems therefore convenient to localize nucleons either along the z-axis or in the x-y-plane. We assume localization to take place along the z-axis and use Bloch s. p. w. f. (1)

$$\psi_{\vec{K}\lambda}(\vec{r}) = \frac{1}{L} \exp (i\vec{K}_\perp \cdot \vec{r}_\perp)\, \chi_{K_z\lambda}(z)\, \xi_\lambda \tag{7}$$

where L is the length of the side of the quantization box, ξ_λ the spin-isospin w. f. and λ the spin-isospin quantum number with the following correspondence: $\lambda = 1$, spin-up protons; $\lambda = 2$, spin-down protons; $\lambda = 3$, spin-up neutrons; $\lambda = 4$, spin-down neutrons. Moreover we take

$$\chi_{K_z\lambda}(z) = \chi_{K_z}(z + \frac{1}{2}\varepsilon_\lambda d) \ , \tag{8}$$

$$\chi_{K_z}(z) = \frac{1}{L^{1/2}} \exp\left\{iK_z\left[z + \eta\frac{d}{\pi}\sin(\frac{\pi}{d}z)\right]\right\}\left[1+\eta\cos\frac{\pi}{d}z\right]^{1/2} \quad |\eta| \leq 1 \ . \tag{9}$$

The above w. f. are orthonormal. The factor $\exp\left[iK_z\eta\frac{d}{\pi}\sin(\frac{\pi}{d}z)\right]$, whose presence is necessary in order to guarantee orthogonality, can be omitted if we impose that only states for which

$$|K_z| \leq \frac{\pi}{2d} \ , \tag{10}$$

can be occupied.

In this case, however, we would not be able to explore the possibility of long wave-length for the density oscillations. We will therefore use w. f. (9).

We will also assume the OPE to be the whole N-N interaction. This oversimplification will enable us to get an idea of the critical density ϱ_c for the onset of SIOP. Since we neglect the other components of the N-N interaction, the precise form of regularization of the OPE is not significant, and we simply cut-off the OPE at short distance.

Looking at eq. (5) we find convenient to assume the same phase for spin-up protons and spin-down neutrons, opposite to the phase of spin-down protons and spin-up neutrons

$$\varepsilon_1 = \varepsilon_4 = -\varepsilon_2 = -\varepsilon_3 = 1 \ . \tag{11}$$

This choice which does not take into account $\langle V_{OPE}\rangle_{exchange}$ minimizes $\langle V_{OPE}\rangle_{direct}$. It is justified by the small value of the η-dependent part of $\langle V_{OPE}\rangle_{exchange}$.

By performing a variational calculation of the energy per particle of nuclear matter w. r. to the parameters η and d for given value of the average density $\bar{\varrho}$, we obtain

$$\eta = \begin{cases} 0 & \bar{\varrho} < 4.7\ \varrho_o \\ 1 & \bar{\varrho} > 4.7\ \varrho_o \end{cases} \tag{12}$$

$$2d = \begin{cases} \text{undetermined} & \bar{\varrho} < 4.7\ \varrho_o \\ 4\ \text{fm} & \bar{\varrho} > 4.7\ \varrho_o\ , \end{cases} \tag{13}$$

ϱ_o being normal nuclear density. The critical density $\varrho_c = 4.7\ \varrho_o$ turns out to be very large.

4. - CONNECTION WITH PION CONDENSATION.

At the same time in which SIOP have been proposed, the possibility of pion condensation has been considered[2]. The starting point here is that if barionic matter is dense enough the vacuum becomes unstable against creation of pions.

The average value of the pion field obeys the equation

$$(\Box + m^2) \langle \varphi_i \rangle = \frac{f}{\mu}\ \partial_k \langle S_{ik} \rangle\ . \tag{14}$$

If $\langle \varphi_i \rangle \neq 0$, also $\langle S_{ik} \rangle \neq 0$ and barions are in SIOP. The converse is obviously true[10].

Condensation of charged pions is related to superconductivity in layers[3] and will not be discussed here.

Due to the one-to-one correspondence between SIOP and pion condensation in nuclear matter[11] we will refer in the following to one or the other indifferently.

5. - ONE-DIMENSIONAL LOCALIZATION.

The parameter η has a discontinuity jump across the critical density from $\eta = 0$ to $\eta = 1$, qualifying the phase transition as a first order one.

For $\eta = 1$ we have complete localization along the z-direction, because $\chi_{K_z}(z)$ vanishes at

$$z = (2n + 1)\, d\ . \tag{15}$$

We can therefore replace the χ_{K_z} by Heitler-London w.f.[3]

$$\chi_n(z) = \left[1 + \cos\left(\frac{\pi}{d}\, z + 2n\pi\right)\right]^{1/2} \theta(d - z - 2nd)\, \theta(d + z + 2nd) \tag{16}$$

$\theta(z)$ being the step function.

By performing a Wannier transformation on the $\chi_n(z)$ we obtain

$$\chi'_{K_z}(z) = \frac{1}{L^{1/2}} \sum_{n=-L/4d}^{L/4d} \exp(i\, 2nd K_z)\ \chi_n(z) \tag{17}$$

and the s.p.w.f.

$$\psi'_{\vec{K}_z \lambda}(\vec{r}) = \frac{1}{L}\ \exp(i \vec{K}_\perp \cdot \vec{r}_\perp)\ \chi'_{K_z \lambda}(z)\ \xi_\lambda\ . \tag{18}$$

The χ'_{K_z} are othogonal only for K_z satisfying condition (10). This condition is not to stringent at normal density in view of the value of d we have obtained in eq. (13).

It is very easy to check that the $\psi'_{\vec{K}\lambda}$ give rise to the same s.p. diagonal density as the $\psi_{\vec{K}\lambda}$, so that $\langle V_{OPE} \rangle_{direct}$ remains the same. The nondiagonal s.p. den sity, however, is different and gives rise to a much lower kinetic energy. As a result the variational calculation with the w.f. (18) gives a much lower critical density

$$\varrho_c = 1.5\ \varrho_o\ . \qquad (19)$$

6. - ROLE OF THE CORE OF N-N POTENTIAL.

We have seen that the critical density is not too higher than normal density if we take into account only the OPE potential tail. It is therefore essential to know whether other components of the N-N interaction will favor or handicap the onset of SIOP. Particularly important can be the role of the core of the N-N potential in view of the complete localization along the z-axis.

A calculation taking into account the short range correlations induced by the core is however very difficult. Correlations must in fact be weaker between, say, spin-up protons and spin-down neutrons (which are already kept apart to some extent with w.f. (18), than between spin-up protons and spin-down neutrons. This requires Brueckner effective interactions or Jastrow correlations functions spin-isospin dependent and anisotropic.

It has been argued[12] that one should omit the contact term of the OPE, on the ground that the w.f. (evaluated in the normal phase), are very small at the origin. This would be appropriate if this term could be treated in perturbation theory, which is not the case in the presence of a first order phase transition. Actually, if such a term were present in the absence of a hard core, the w.f. would increase without bound at $r = 0$. The real problem is therefore to evaluate its contribution after regularization.

Due to the difficulty of properly treating short range correlations in SIOP, a preliminary investigation of the role of the core has been done[4] using one-boson-exchange potentials[9] with w.f. which contain only spin-isospin correlations. Since the short range terms of the potentials have different spin-isospin dependence, choices for the ε_λ other than (11) can be advantageous. A number of such choices in addition to (11) have been studied leading to different SIOP.

The calculation show that the core favors SIOP w.r. to the normal phase already at normal density. One obtains once again sharp localization along the z-axis, which requires the calculation to be redone with Heitler-London w.f.. Preliminary results[13] show a considerable improvement w.r. to Bloch w.f..

7. - ISOSPIN AND PARITY IN SIOP.

Parity is broken in SIOP as a consequence of the breaking of translational invariance. It has therefore been argued that the signature for such order in finite nuclei should be the existence of parity doublets[12]. The reason is that from a state Φ of undefinite parity we can construct the degenerate state $P\Phi$ and therefore the degenerate parity doublets $(1 \pm P)\Phi$.

This is not necessarily true, however[5]. Parity is broken as a symmetry w.r. to arbitrary points in nuclear matter, but can be a good symmetry w.r. to selected points. In the case of eq. (11), for instance, parity is even w.r. to the points $z = (n/2)d$, $n = 0, \pm 1, \ldots$. This is all we need in order to construct states of definite parity in nuclei, where parity is defined only w.r. to the c.m..

Degenerate states (of the same parity) must however be present due to the breaking of isospin.

That isospin is also broken in the presence of SIOP can be shown in the following way. The operators S_{ik} are isovector for fixed k. Therefore applying the Wigner-Eckart theorem we have

$$< T T_z | S_{3k} | T T_z > \sim \frac{T_z}{T(T+1)} \quad . \qquad (20)$$

For symmetric nuclear matter $T_z = 0$, and S_{3k} must vanish for a state of definite T.

8. - COHERENT EFFECTS OF A WEAK AXIAL NEUTRAL CURRENT.

Due to the existence of weak neutral currents, the elastic process $\nu + A \rightarrow \nu + A$ should have a sharp coherent forward peak, just as $e + A \rightarrow e + A$ does(14).

To see this let us consider the current-current effective Lagrangian derived from the Weinberg-Salam model

$$\mathscr{L}_{eff} = \frac{1}{\sqrt{2}} G l^{\mu} J_{\mu} \, , \qquad (21)$$

where l_{μ} is the lepton current and J_{μ} the hadronic current

$$J_{\mu} = \bar{\psi} \left[-\frac{1}{2} \sin^2\theta_W \gamma_{\mu} + \frac{1}{4} \tau_3 \gamma_{\mu} (1 - 2\sin^2\theta_W) + \frac{1}{4} \tau_3 \gamma_{\mu} \gamma_5 \right] \psi \qquad (22)$$

omitting strangeness and charm.

For a normal nucleus with N = Z only the first term of J_{μ} contributes to elastic scattering and the differential cross-section is

$$\frac{d\sigma}{dQ^2} = \frac{G^2}{2\pi} |F(Q)|^2 (1 + Q^2 \frac{Am + 2E}{4AmE^2}) \quad , \qquad (23)$$

where Q is the momentum transfer, E the neutrino laboratory energy and F(Q) the nuclear form factor. The integrated cross section for $100 < Q < 300$ MeV/c is of the order of 10^{-38} cm^2 on ^{12}C for $0.2 < E < 1$ GeV. Experimental difficulties lie in the detection of recoil nuclei of small momentum. On the other hand increasing the momentum makes the cross section smaller due to the damping by the nuclear form factor.

These experimental difficulties appear less serious for nuclei in SIOP. In this case the last term of (22) also contributes to elastic scattering. Its nonrelativistic approximation

$$\bar{\psi} \frac{1}{4} \tau_3 \gamma_{\mu} \psi \rightarrow \frac{i}{4} \bar{\psi} \tau_3 \left[\frac{1}{mc} \vec{\sigma} \cdot \vec{p} \, , \, -\vec{\sigma} \right] \psi \qquad (24)$$

exhibits in fact the spin-isospin density operator S_{3k} whose average value is nonvanishing in SIOP.

The cross-section is essentially the same as (23) with(6)

$$|F(Q)|^2 = \eta^2 A^2 \exp \left[- \frac{(2d_o)^2 Q^2}{2} \right] , \qquad (25)$$

to be compared with

$$|F(Q)|^2 = A^2 \exp(-2b^2 Q^2) \qquad (26)$$

for the normal case. In the above equations η is the amplitude of density fluctuations, $2d_o$ is the fluctuation width and $b^2 = 1/6$ (r.m.s.r.)^2. Due to the fact that $2d_o \ll$ (r.m.s.r.), one can explore values of Q where (26) is negligeable. This allows to establish an upper bound on η.

9. - SIOP WITHOUT SPATIAL ORDER.

Being nuclei finite, it is not necessary to localize nucleons to have SIOP, but it is sufficient to correlate nucleons of different spin-isospin in a proper way.

Here we consider the possibility of realizing spin-isospin order in the zero-point motion, by allowing spin-up protons and spin-down neutrons to oscilate w.r. to spin-down protons and spin-up neutrons. The correlation is not static, and on the average there is neither space order nor pion condensation.

Oscillations can be allowed in one, two or three dimensions. The case of one dimensional oscillations, which is closer to the case of SIOP with spatial order considered so far, turns out to be energetically favoured. This model has been studied by considering only a OPE potential tail, like in Section 3. Its actual realization is therefore far from established, the mentioned difficulties with a proper treatment of short range correlations being obviously present also here. The model is however characterized by specific predictions: First this kind of SIOP can exist only in light oblate or (approximately) spherical nuclei; second isospin is broken with $\langle T^2 \rangle \sim 1/3$; third there is an enhancement of B(M2, $K = 0 \rightarrow K = 0$) transition amplitudes by a factor 20.

FOOTNOTES AND REFERENCES.

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