

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-80/38

M. Greco, Y. Srivastava and G. Penso :
QUANTUM CHROMODYNAMICS AND DUALITY IN
 e^+e^- ANNIHILATION

Estratto da :

Phys. Rev. D21, 2520 (1980)

Quantum chromodynamics and duality in e^+e^- annihilation

M. Greco and Y. Srivastava*

Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, Frascati, Italy

G. Penso

*Istituto di Fisica dell'Università di Roma, Roma, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Roma, Italy*

(Received 1 December 1978)

We present a method based on duality for smoothing the ratio $R \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. This method allows us to extract from all available data the contribution of the three light quarks and separately that of the charm quark. These contributions are successfully compared with quantum-chromodynamics predictions. We find $m_c = 1.45 \pm 0.05$ GeV and $\Lambda^2 = 0.5-0.6$ GeV². The method itself provides a self-consistency check of semilocal duality.

Recent experiments in e^+e^- annihilation into hadrons have provided an accurate determination of

$$R(s) \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

in the low- and medium-energy region. Thus, R is reliably known from threshold up to 8 GeV.¹ Such high-quality data ought naturally to provide for precise tests of proposed models and theories. However, a direct confrontation with theoretical predictions has been hampered by the local fluctuations in R due to resonances as well as the presence of multihadron thresholds. To this end, in this paper we present a method based on duality which helps smooth out R . Our procedure simultaneously allows for a check of the concept of duality itself in e^+e^- annihilation.²

Earlier, to overcome the above-mentioned difficulty, three different methods have been suggested. In the first one,^{3,4} the experimental data are extrapolated from the timelike to the spacelike region via dispersion relations, and then compared with the theoretical quantum-chromodynamics (QCD) predictions for the function $D(Q^2)$ defined as

$$D(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R(s)ds}{(Q^2+s)^2} = \frac{3\pi}{\alpha} Q^2 \left. \frac{d\Pi}{ds} \right|_{s=-Q^2}, \quad (1)$$

where $\Pi(s)$ is the hadronic contribution to the vacuum-polarization tensor. Ignorance of R beyond the energy region accessible experimentally directly introduces an indeterminacy in $D(Q^2)$, depending upon the assumptions regarding the number and mass of heavy flavors made for the extrapolation of the data.

In the second approach⁵ a smearing procedure has been suggested which is applied directly in the

timelike region. To be precise, the quantity

$$\bar{R}(s, \Delta) = \left(\frac{\Delta}{\pi}\right) \int_{4m_\pi^2}^{\infty} \frac{R(s')ds'}{(s'-s)^2 + \Delta^2} \quad (2)$$

is compared with the theoretical prediction for $R(s)$ given by QCD perturbation theory. The price one has to pay here is the introduction of an arbitrary parameter Δ .

Yet another method, similar in spirit to ours, though technically somewhat different, has been proposed.^{6,7} We critically review these works at the end of the paper.

As stated before, our smoothing procedure is based on duality, which has been successfully applied earlier for this process.² Given the experimental value, $R_{\text{exp}}(s)$, we construct the zeroth moment

$$M(\bar{s}) = \int_{4m_\pi^2}^{\bar{s}} R_{\text{exp}}(s)ds. \quad (3)$$

Figure 1(a) shows the values of $R_{\text{exp}}(s)$ used by us. The data have been organized as follows. For $W (\equiv \sqrt{s})$ from 0.1 to 0.9 GeV, the dominant contribution is due to the ρ resonance for which the Gounaris-Sakurai form⁸ has been assumed; ω and ϕ resonances are introduced as δ functions. From 0.9 to 3.45 GeV, all available data¹ from ACO, VEPP-2M, DC1-M3N, ADONE- $\gamma\gamma$ 2, ADONE-MEA, and SPEAR-MARK I have been averaged over 100-200 MeV intervals as shown in Fig. 1(a). In the interval 3.45 to 7.8 GeV, we have utilized data¹ from DELCO, DASP, PLUTO, and SPEAR-MARK I whenever available. The heavy-lepton contribution has been subtracted. Also, radiative corrections have been applied⁹. The band shown in Fig. 1(a) is mainly due to systematic discrepancies between experiments in the interval 15-25 GeV² and to statistical errors above 25 GeV². Using the above inputs, we plot $M(\bar{s})$ in Fig. 1(b).

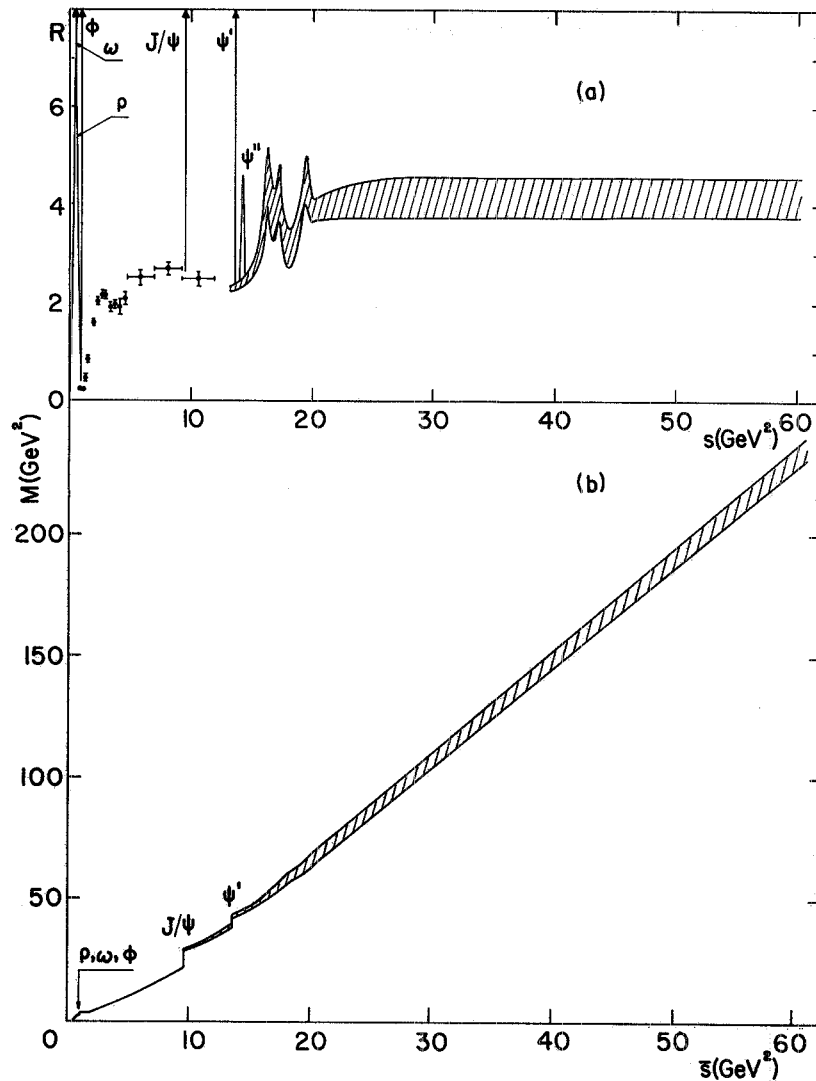


FIG. 1. (a) $R_{\text{exp}}(s)$ vs s as explained in the text. (b) $M(\bar{s})$ vs \bar{s} .

Here the band reflects only the systematic discrepancies between various experiments, the statistical error on the integral being negligible.

Clearly, the local structures present in $R_{\text{exp}}(s)$ have almost disappeared in $M(\bar{s})$. This, therefore, allows a much cleaner separation of various quark contributions. In fact, these data naturally divide into three regions with effective slopes: 2 ($s \approx 0-5 \text{ GeV}^2$), 2.5 ($5-9 \text{ GeV}^2$), and 4.2 ($9-60 \text{ GeV}^2$).

For clarity, we present separately in Fig. 2, the low-energy data alone on a bigger scale. This figure confirms that the average value of R between $0-9 \text{ GeV}^2$ agrees with the values of R averaged over the ρ, ω, ϕ resonances. This gives us confidence in the idea of semilocal duality.

A more accurate test of duality in this region cannot be made at present because of the lack of

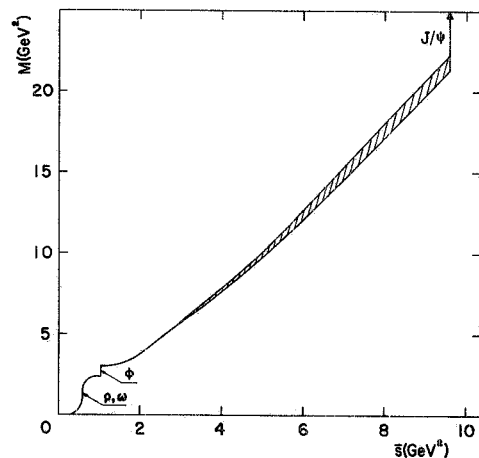


FIG. 2. $M(\bar{s})$ vs \bar{s} for the energy region $\bar{s} \leq 10 \text{ GeV}^2$.

separation of s -quark contribution from those of u and d quarks. This also shows up in the ϕ continuum which apparently sets in later (around $s \geq 5 \text{ GeV}^2$) and which may be responsible for the change in the slope from 2 to 2.5. The above conclusions rely on the experimental assumption that at low energies ($s \approx 2-6 \text{ GeV}^2$) all produced particles are pions. Indeed, the number of kaons has been observed¹ to increase with energy and due to the smaller detection efficiency for kaons (relative to pions) at low energies, the true $R_{\text{exp}}(s)$ may be higher than the value shown in Fig. 1(a). In turn, this might further reduce the difference between the two slopes (i.e., 2 and 2.5).

At this point we can make a simple consistency check with the first-order correction expected from QCD:

$$R(s) \approx 3 \sum_{i=u,d,s} Q_i^2 \left(1 + \frac{\alpha(s)}{\pi} \right). \quad (4)$$

Taking three massless quarks only, we have

$$\alpha(s) \approx \frac{4\pi}{9 \ln(s/\Lambda^2)}.$$

Thus, for a value of $\Lambda^2 \approx 0.5 \text{ GeV}^2$ we find $R(\sim 9 \text{ GeV}^2) \approx 2.30$. (The inclusion of the c quark changes this result by $\leq 1\%$.) We compare with a smoothed value, R_{av} , defined as

$$R_{\text{av}}(\bar{s}) = \frac{1}{\bar{s}} \int_{4m_c^2}^{\bar{s}} R_{\text{exp}}(s) ds. \quad (5)$$

From Fig. 2, we find $R_{\text{av}}(\sim 9 \text{ GeV}^2) = 2.27 \pm 0.05$ (statistical error only) in good agreement with the

above value. Notice that due to the summation over R_{exp} , the statistical error on R_{av} is considerably reduced. Thus, the low-energy region ($\sqrt{s} \leq 3 \text{ GeV}$) can be accounted for by a constant α_{eff} equal to its asymptotic-freedom estimate $\alpha(s \approx 9 \text{ GeV}^2) \approx 0.48$.¹⁰

Let us now discuss the contribution to R of the charm quark alone. Owing to the distance of this threshold from those of all other quarks, it is possible to isolate this quark's contribution $M_c(\bar{s})$. This is achieved by subtracting from $M(\bar{s})$ for $\bar{s} \geq 9 \text{ GeV}^2$ the contribution of light quarks evaluated using the mean value $R_{\text{light}}(s) \approx 2.4 \pm 0.1$. The result is plotted in Fig. 3. This figure provides a precise test of duality since the straight-line behavior of $M_c(\bar{s})$ when extrapolated down to $8-9 \text{ GeV}^2$ averages quite accurately over the low-lying resonances J/ψ and ψ' . This aspect of duality has been noticed previously.²

We evaluate the effective charm threshold and deduce therefrom the c -quark mass using two different parametrizations. First, we use the naive formula

$$R_c^{(1)}(s) = \bar{R}_c \theta(s - s_c) \quad (6)$$

and the QCD prediction⁵

$$R_c^{(2)}(s) = (3Q_c^2)^{\frac{1}{2}} \beta (3 - \beta^2) \left[1 + \frac{4}{3} f(\beta) \alpha(s) \right], \quad (7)$$

where $Q_c = \frac{2}{3}$ is the charge of the c quark, $\beta = (1 - 4m_c^2/s)^{1/2}$,

$$f(\beta) = \frac{\pi}{2\beta} - \frac{(\beta+3)}{4} \left(\frac{\pi}{2} - \frac{3}{4\pi} \right) \quad (8)$$

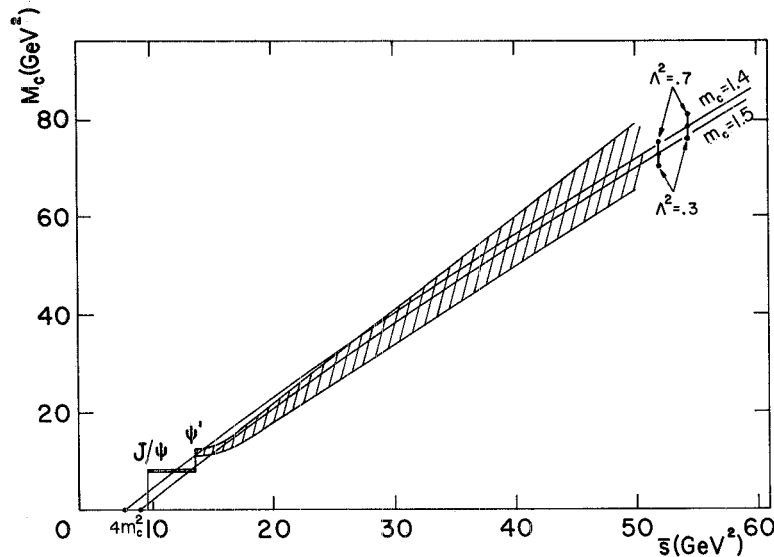


FIG. 3. $M_c(\bar{s})$ vs \bar{s} . The band in the figure takes into account both the experimental error associated with $R_{\text{exp}}(s)$ as well as the error in the subtraction. The theoretical curves are for $m_c = 1.4$ (1.5) GeV and $\Lambda^2 = 0.5 \text{ GeV}^2$. For each mass value, a representative point for $\Lambda^2 = 0.3$ and 0.7 GeV^2 is also shown.

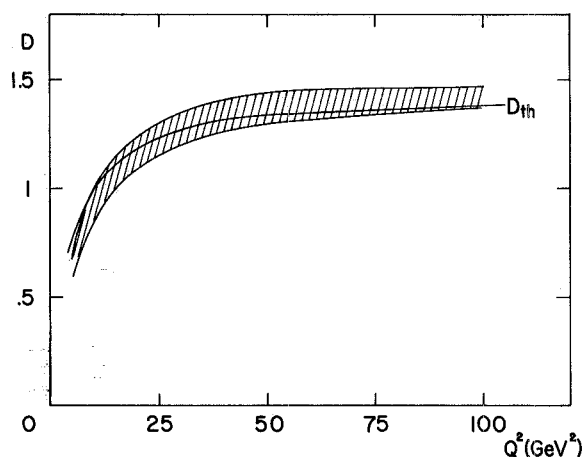


FIG. 4. $D^c(Q^2)$ vs Q^2 . The solid line is $D_{th}^c(Q^2)$ for $m_c = 1.45$ GeV and $\Lambda^2 = 0.5$ GeV². The experimental band for $D_{exp}^c(Q^2)$ is obtained as in Fig. 3.

and¹¹

$$\alpha(s) \approx \frac{12\pi}{27 \ln(s/\Lambda^2) - 2 \ln\left(\frac{s + 5m_c^2}{\Lambda^2 + 5m_c^2}\right)}. \quad (9)$$

Using Eq. (6) and the data, we find $\frac{1}{2}\sqrt{s_c} = \bar{m}_c \approx (1.3-1.4$ GeV) and $\bar{R}_c \approx (1.6-1.8)$. If we interpret \bar{R}_c to be $(3Q_c^2)$ then the above value for the charge Q_c is too high. As we show now, the QCD correction ameliorates this situation. In Fig. 3, the QCD result as given by Eq. (7) is plotted for different values of $m_c = 1.4$ GeV and 1.5 GeV and $\Lambda^2 = 0.3, 0.5,$ and 0.7 GeV². It is clearly seen from the figure that the QCD expression with the correct value of the charge, a mass value (1.45 ± 0.05) GeV and $\Lambda^2 = 0.5-0.6$ GeV² reproduces the data quite well.

We consider the above as a success of QCD and our smoothing procedure. We stress again that our method uses *all* data, including narrow resonances such as J/ψ and ψ' . In fact, the amount of resonance contributions allows us to verify the

basic hypothesis of semilocal duality. On the contrary, the smearing procedure of Ref. 5 discards such narrow structures.

Using the result of the above analysis we can also discuss the charm contribution to the function $D(Q^2)$ in the spacelike region. From Eqs. (1) and (7) and the obtained values of $m_c = 1.45$ GeV and $\Lambda^2 = 0.5$ GeV², we plot in Fig. 4 $D_{th}^c(Q^2)$. For comparison with experimental data, we also plot $D_{exp}^c(Q^2)$ constructed in the following way. The contribution of the $u, d,$ and s quarks to $R_{exp}(s)$ is subtracted as before, in the range $4m_c^2 \leq s \leq 60$ GeV². For $s \geq 60$ GeV², $R_c(s)$ has been extrapolated using the QCD formula (7). Once again the agreement is quite reasonable.

In Ref. 6 the same integral as given in Eq. (3) is considered. However, in that analysis it is assumed that all strangeness contribution is given by the ϕ resonance alone disregarding the ϕ continuum which is rather important. Owing to better input data we are able to isolate the charm contribution, give an accurate value of the charm quark mass, and also provide a check of duality. A similar analysis⁷ wrongly concludes that to explain data below 7.8 GeV, five flavors are required with the fifth quark mass 2.3 GeV.

In conclusion, we have presented a duality-based smoothing procedure for $R(s)$, which led us to make a rather direct comparison with the QCD predictions. The method allows us to use all the experimental data down to threshold and gives excellent support for semilocal duality. Isolating the charm contribution, we find $m_c = 1.45 \pm 0.05$ GeV. All our analysis in the spacelike and timelike region is consistent with the QCD predictions for a value of $\Lambda^2 = 0.5-0.6$ GeV². Obviously, a similar analysis for b and higher-mass quarks can be attempted when higher-energy data become available.

It is a pleasure to thank G. Parisi for helpful comments. The work of Y.S. was supported in part by the National Science Foundation.

*Permanent address: Northeastern University, Boston, Mass.

¹J.-E. Augustin *et al.*, Phys. Rev. Lett. **34**, 764 (1975); V. A. Sidorov, in *Proceedings of the XVIII International Conference on High Energy Physics, Tbilisi, 1976*, edited by N. N. Bogolubov *et al.* (JINR, Dubna, 1977), Vol. II, p. B13; C. J. Feldman, in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979), p. 777; G. P. Murtas, *ibid.*, p. 269; J. P. Perez-Y-Jorba, *ibid.*, p. 277; R. Baldini-Celio *et al.*, Lett. Nuovo

Cimento **24**, 324 (1979); C. Bacci *et al.*, Phys. Lett. **86B**, 234 (1979); B. Esposito *et al.*, Lett. Nuovo Cimento **19**, 21 (1977); B. H. Wiik and G. Wolf, DESY Report No. 78/23, 1978 (unpublished), and references therein.

²For earlier applications of duality in annihilation see for instance A. Bramon, E. Etim, and M. Greco, Phys. Lett. **41B**, 609 (1972); M. Greco, Nucl. Phys. **63B**, 398 (1973); J. J. Sakurai, Phys. Lett. **46B**, 207 (1973); E. Etim and M. Greco, Lett. Nuovo Cimento **12**, 91 (1975); M. Greco, Phys. Lett. **77B**, 84 (1978).

³S. Adler, Phys. Rev. D **10**, 3714 (1974).

- ⁴A. De Rújula and H. Georgi, *Phys. Rev. D* **13**, 1296 (1976).
- ⁵E. Poggio, H. Quinn, and S. Weinberg, *Phys. Rev. D* **13**, 1958 (1976).
- ⁶R. Shankar, *Phys. Rev. D* **15**, 755 (1977).
- ⁷R. Moorehouse, M. Pennington, and G. Ross, *Nucl. Phys.* **B124**, 285 (1977).
- ⁸G. Gounaris and J. Sakurai, *Phys. Rev. Lett.* **21**, 244 (1968).
- ⁹M. Greco, G. Pancheri-Srivastava, and Y. Srivastava, *Nucl. Phys.* **B101**, 234 (1975); F. Yndurain, *Nucl. Phys.* **B136**, 533 (1978).
- ¹⁰That the comparison between Eqs. (4) and (5), which is justified by duality, leads to $\Lambda^2 \approx 0.5 \text{ GeV}^2$ is very

satisfactory since the same Λ^2 is reproduced by our subsequent analysis of higher-energy data 9–50 GeV^2 as well as various analyses of the deep-inelastic lepton scattering at the first-order QCD level. It should be pointed out that the value of Λ^2 does change substantially if second-order effects are included.

- ¹¹A slightly different expression is obtained by O. Nachtmann and W. Wetzel, Heidelberg Report No. HD-THEP-78, 1978 (unpublished) and verified by R. Barbieri and G. Curci. However, the numerical change is very little for our purposes. Thus, we continue to use formula (7). We thank G. Curci for communicating to us these results.