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M. Ramon-Medrano, G. Pancheri-Srivastava and
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DYNAMICAL GROWTH OF SUPERSYMMETRY^(x)

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ABSTRACT

A self-interacting Majorana spinor theory, even without derivative couplings, can be equivalent to a supersymmetric theory at the quantum level. A particular example is provided which leads to a supersymmetric Yang-Mills Lagrangian.

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In this paper we show that a classical four-fermion interaction of a triplet of massless Majorana fields is equivalent at the quantum level, to a non-Abelian gauge theory with (global) supersymmetry. Thus, while the classical Lagrangian possesses only a global SU(2) symmetry, quantum corrections endow this Lagrangian with a local SU(2) symmetry as well as supersymmetry.

Several years ago, Volkov and Akulov⁽¹⁾ provided a non-linear realization of global supersymmetry. This has been the only known example (at least to us) of supersymmetry in terms of a single Majorana field with derivative coupling (in 4-dimensions, it goes up to 4 derivatives and number of fields up to 8⁽²⁾). This rather complicated Lagrangian has been shown⁽³⁾ to be equivalent (with a constraint) to the free Wess-Zumino model, at the classical level. In this context the interest of our example lies in that it shows how a simple, self-interacting Majorana field is able to generate - at the quantum level - a linear realization of supersymmetry in conjunction with a Yang-Mills theory.

Before discussing our example, we mention some related earlier work. That perturbatively non-renormalizable 4-fermion interactions may indeed be renormalizable has been discussed recently in a variety of papers⁽⁴⁾. Their equivalence, in certain cases, to gauge theories both for Abelian⁽⁵⁾ and non-Abelian⁽⁶⁾ examples has also been shown. The technique which we follow has been extensively used for scalar and Dirac fields in connection with σ -models and gauge theories^(4, 6, 7, 8). A general discussion of 4-fermion theories in D (< 4) dimension has been given by G. Parisi⁽⁸⁾ in terms of the renormalization group β -function. What we do is to consider Majorana spinors and investigate whether the dynamically generated boson fields become supersymmetric partners to the original Majorana fields. Hence the title.

We consider the following classical 4-fermion (non-derivative) Lagrange density \mathcal{L}_1 , which is globally SU(2) symmetric⁽⁹⁾:

$$\mathcal{L}_1 = \frac{i}{2} (\vec{\bar{\psi}} \sigma \cdot \vec{\psi}) + \frac{G}{4} (\vec{\bar{\psi}} \gamma_\mu \times \vec{\psi}) \cdot (\vec{\bar{\psi}} \gamma^\mu \times \vec{\psi}), \quad (1)$$

where $\vec{\psi}$ is a Majorana spinor and an isovector. We now introduce the auxiliary fields $A_\mu(x)$ such that

$$\mathcal{L}_2 = \frac{i}{2} (\vec{\psi} \vec{\sigma} \cdot \vec{\psi}) - \frac{g}{2} (\vec{\psi} \gamma^\mu \times \vec{\psi}) \cdot \vec{A}_\mu - \frac{1}{2} (\delta\mu^2) (\vec{A}_\mu \cdot \vec{A}^\mu), \quad (2)$$

has the same generating functional as does \mathcal{L}_1 provided

$$G = \frac{g^2}{2\delta\mu^2}. \quad \text{Performing the gaussian integral in eq. (2) we find}^{(10)}$$

$$\mathcal{Z} = \int (d\vec{A}_\mu) \exp i \left\{ -\frac{1}{2} \delta\mu^2 (\vec{A}_\mu \cdot \vec{A}^\mu) - i \text{Tr} \left[1 - (i\vec{\sigma})^{-1} ig\vec{T} \cdot \vec{A} \right] + \bar{\eta} \frac{1}{(i\vec{\sigma} - ig\vec{T} \cdot \vec{A})} \eta \right\}. \quad (3)$$

In eq. (3), η is the (Majorana) source for the spinor ψ , the 4-dim. space-time integral in the exponent has been suppressed and an overall normalization constant in the generating functional Z has been omitted. \vec{T} denotes the iso-spin 1 generator.

In terms of Feynman diagrams, $-i \text{Tr} [1 - (i\vec{\sigma})^{-1} ig\vec{T} \cdot \vec{A}]$ is the sum of all one-fermion loop contributions with any number of external \vec{A}_μ legs. Only diagrams up to four external legs give divergent results. Thus, we obtain the effective Lagrange density

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{1}{2} (-\delta\mu^2 + g^2 I_2) (\vec{A}_\mu \cdot \vec{A}^\mu) + \left(\frac{2}{3} g^2 I_1 \right) \left(-\frac{1}{4} \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} \right) + \\ & + \bar{\eta} \frac{1}{(i\vec{\sigma} - ig\vec{T} \cdot \vec{A})} \eta + \mathcal{L}_{\text{conv}}(\vec{A}_\mu, g), \end{aligned} \quad (4)$$

where $\mathcal{L}_{\text{conv}}$ is a non-polynomial Lagrange density with finite coefficients. $I_{1,2}$ stand for the divergent expressions

$$I_1 = 4i \int_{\epsilon}^{\Lambda} \frac{(d^4q)}{(2\pi)^4} \frac{1}{(q^2)^2}, \quad I_2 = 4i \int_{\epsilon}^{\Lambda} \frac{(d^4q)}{(2\pi)^4} \frac{1}{q^2} \quad (5)$$

$$\text{Also, } \vec{F}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g(\vec{A}_\mu \times \vec{A}_\nu).$$

Imposing the condition

$$\delta\mu^2 - g^2 I_2 = 0, \quad (6a)$$

and rescaling

$$\vec{\hat{A}}_\mu = \left(\frac{2g^2 I_1}{3}\right)^{1/2} \vec{A}_\mu \quad (6b)$$

and

$$\hat{g} = \left(\frac{2g^2 I_1}{3}\right)^{-1/2} g, \quad (6c)$$

we find

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} \vec{\hat{F}}_{\mu\nu} \cdot \vec{\hat{F}}^{\mu\nu} + \bar{\eta} \frac{1}{(i\delta - ig\vec{T} \cdot \vec{\hat{A}})} \eta + \mathcal{L}_{\text{conv}}(\vec{\hat{A}}_\mu, \hat{g}) \quad (7)$$

Functionally $\mathcal{L}_{\text{conv}}(\vec{A}_\mu, g) = \mathcal{L}_{\text{conv}}(\vec{\hat{A}}_\mu, \hat{g})$ since $g\vec{A}_\mu = \hat{g}\vec{\hat{A}}_\mu$.

We now turn to a local SU(2) invariant Lagrange density^(11, 12)

$$\mathcal{L}_3 = -\frac{1}{4} \vec{\hat{F}}_{\mu\nu} \cdot \vec{\hat{F}}^{\mu\nu} + \frac{i}{2} \vec{\hat{\psi}} \gamma^\mu \cdot (\overrightarrow{D}_\mu \hat{\psi}), \quad (8)$$

where $\overrightarrow{D}_\mu \hat{\psi} = \partial_\mu \hat{\psi} - g' \vec{\psi} \times \vec{A}_\mu$.

It is known that this theory is also invariant under the supersymmetry transformation

$$\delta \vec{\hat{A}}_\mu = i \bar{\epsilon} \gamma_\mu \vec{\hat{\psi}}, \quad (9a)$$

$$\delta \vec{\hat{\psi}} = \vec{\hat{F}}_{\mu\nu} (\sigma^{\mu\nu} \epsilon), \quad (9b)$$

ϵ being a constant anticommuting Majorana spinor.

Considering the generating functional of \mathcal{L}_3 and performing the ψ integration, one is led precisely to the effective Lagrange density given by eq. (7), once the coupling constant and the fields have been rescaled. It is understood that here some supersymmetric regularization procedure⁽¹³⁾ has to be employed in order not to produce spurious mass terms, nor destroy supersymmetry.

This establishes the equivalence and hence proves the assertion made in the introduction that the model defined through eq. (1), at the quantum level, is equivalent to the theory defined by eq. (8).

In interpreting these results it is logical to consider eqs. (1), (2) and (8) in succession as three stages of passage from the purely classical to the fully quantum level. At stage 1, one has neither a local symmetry nor supersymmetry and certainly no chance of perturbative renormalizability. Non-renormalizability in this sense may now be understood as saying that for any G however small but not zero there is always a bound state and hence a perturbative expansion necessarily fails. Thus, at stage 2 one may suspect renormalizability. Moreover, at this stage the number of degrees of freedom for Bose and Fermi fields is equal and one may even hope for supersymmetry. In fact, stage 3 achieves all three: local $SU(2)$, renormalizability as well as supersymmetry.

We point out that the mere equality of Bose and Fermi degrees of freedom at stage 2 of a given theory does not necessarily signal a hidden supersymmetry to be unveiled at stage 3. An illustrative example follows.

If one were aiming for the massless Wess-Zumino model⁽¹⁴⁾, an evident candidate for the corresponding stage 2 Lagrangian would appear to be

$$\mathcal{L}'_2 = \frac{i}{2} (\bar{\psi} \not{\sigma} \psi) - g \bar{\psi} (A - i\gamma_5 B) \psi - \frac{1}{2} \delta\mu^2 (A^2 + B^2), \quad (10)$$

which is invariant under an Abelian chiral group. \mathcal{L}'_2 gives the same generating functional as does the stage 1 Lagrangian

$$\mathcal{L}'_1 = \frac{i}{2} (\bar{\psi} \not{\sigma} \psi) + G (\bar{\psi} \psi)^2, \quad (11)$$

with $G = g^2 / (\delta\mu^2)$. We have used the identity

$$(\bar{\psi} \psi)^2 = (\bar{\psi} i\gamma_5 \psi)^2$$

valid for a Majorana field ψ .

Following the procedure outlined above we would find that the equivalent Lagrangian is

$$\begin{aligned} \mathcal{L}'_3 = & \frac{1}{2} \left[(\partial_\mu A)(\partial^\mu A) + (\partial_\mu B)(\partial^\mu B) \right] + \\ & + \frac{i}{2} (\bar{\psi} \not{\partial} \psi) - g \bar{\psi} (A - i \gamma_5 B) \psi - \frac{\lambda}{2} (A^2 + B^2)^2, \end{aligned} \quad (12)$$

with $\lambda = 4g^2$ instead of $\lambda = g^2$ as required by supersymmetry in the Wess-Zumino model. Thus, there is no supersymmetry. This model does, however, possess a stable, semi-classical vacuum⁽¹⁵⁾.

In conclusion, the present analysis reveals that as the purely "matter" Lagrangian develops into a gauge theory it may also generate supersymmetry. The sufficient conditions for the latter, however, are not known yet. Once we find these conditions we may also be able to see the path towards obtaining local supersymmetry. Another possible application of the present approach lies in the realm of grand unified models. In such theories, after symmetry breakdown at a given mass scale, to obtain a lower energy effective Lagrangian one integrates over the superheavy flavors ending with 4-fermion interactions which are then discarded. It would be interesting to investigate this problem thoroughly in light of the discussion given here.

FOOTNOTES AND REFERENCES.

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