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ABSTRACT.

A semimicroscopic model of the nucleus is presented in which spin-isospin order is realized as zero-point oscillation of spin-up protons and spin-down neutrons against spin-down protons and spin-up neutrons. This phase is shown to be energetically favored by the O.P.E. potential in lighth deformed oblate nuclei. The model is characterized by isospin admixtures in the ground state and enhanced M2 transition probabilities.

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1. - INTRODUCTION.

Several years ago it has been suggested⁽¹⁾ that the O.P.E. potential should give rise to a spin-isospin ordered phase in nuclear matter. Any spin-isospin order, however, requires some localization of the nucleons. In a number of specific models⁽²⁾ the localization takes place only along one direction, the direction of spin quantization, giving rise to a laminated structure where spin and/or isospin alternate their orientation going from one layer to the next one.

The actual occurrence of this phase depends on the balance between the kinetic energy increase necessary in order to localize the nucleons, the attraction coming from the O.P.E. potential and the change in the short range interaction energy. This last effect is most difficult to evaluate and makes uncertain the determination of the critical density. It is therefore hard to establish theoretically whether this ordered phase is actually present in nuclei.

We want to explore the possibility of a realization of spin-isospin order in actual nuclei which does not require the localization of the nucleons in layers. Being nuclei finite it is in fact sufficient to correlate nucleons with different spin-isospin in appropriate way in order to get a nonvanishing contribution from the O.P.E. potential. Such a correlation can be realized by allowing protons with spin-up and neutrons with spin-down to oscillate with respect to protons with spin-down and neutrons with spin-up. Spin-isospin order is thus obtained in the zero-point motion.

In this model there is no kinetic energy increase with respect to the disordered phase. The contribution of the O.P.E. potential, however, will results attractive only for lighth deformed (oblate) nuclei. This reflects the fact that this spin-isospin order, unlike the laminated structure, is specific of finite nuclei and cannot be extrapolated to infinite nuclear matter.

The present model might be relevant to the interpretation of precursor phenomena⁽³⁾ predicted in the framework of neutral pion condensation⁽⁴⁾. The laminated structure in nuclear matter is the source for a macroscopic pion field⁽⁵⁾, and therefore the onset of the

spin-isospin ordered phase coincides with the onset of pion condensation. Many authors, however, agree that the critical density for neutral pion condensation is higher than the density of actual nuclei. They have accordingly followed two lines: Either to create higher densities in heavy ions collisions⁽⁶⁾ or to look for precursor phenomena in nuclei. Some success has been obtained along the second line because it seems that there is some evidence of precursor phenomena in light nuclei. Precursor phenomena, however, are possible only in the vicinity of a second order phase transition, while pion condensation requiring spatial order has the typical feature of a first order phase transition. It is therefore tempting to relate precursor phenomena to the present model. In both cases one deals only with spin-isospin order, without spatial order.

We will not investigate here this aspect of our model. We will confine ourselves to its presentation in Section 2, to the evaluation of the energy contribution by the O. P. E. potential to the ground state in Section 3 and to the analysis of its e. m. properties in Section 4.

2. - THE MODEL.

Consistently with the procedure adopted in refs. (1, 2) we introduce displaced creation operators

$$(C_{a\lambda}^{(\alpha)})^+ = \exp\left(-\frac{i}{\hbar} \varepsilon_{\lambda}^{\alpha} \frac{\vec{d}}{2} \cdot \vec{p}\right) C_{a\lambda}^+ \exp\left(\frac{i}{\hbar} \varepsilon_{\lambda}^{\alpha} \frac{\vec{d}}{2} \cdot \vec{p}\right) \quad (1)$$

where we labeled the spatial and spin-isospin quantum numbers of the single particle (s. p.) states by a and λ respectively. We will use the labels h or p instead of a when we need to specify whether the s. p. states are occupied or not. As regard to the other symbols \vec{p} is the s. p. momentum, \vec{d} a displacement parameter and $\varepsilon_{\lambda}^{\alpha}$ the "sign" of the displacement which specifies the spin-isospin order of the phase α . The phase $\alpha = 4$ of ref. (2), for instance, is realized by putting $\varepsilon_{\lambda}^{(4)} = \tau_3$ while the "sign" parameter determining the spin-isospin order of the phase $\alpha = 6$ is $\varepsilon_{\lambda}^{(6)} = \sigma_3 \tau_3$.

Out of the operators (1) we construct a Slater determinant

$$\Lambda^{(\alpha)} = \prod_{h\lambda} (C_{h\lambda}^{(\alpha)})^+ |0\rangle \quad (2)$$

$|0\rangle$ being the physical vacuum.

This state vector depends on the "intrinsic" s. p. variables as well as on the "collective" displacement parameter \vec{d} . Its normalized expansion up to second order in \vec{d} is

$$\begin{aligned} \Lambda^{(\alpha)} \simeq & \left(1 - \frac{1}{2} \sum_{i=1}^3 d_i^2 \langle \Lambda_i^{(\alpha)} | \Lambda_i^{(\alpha)} \rangle\right) \Lambda + \\ & + \sum_{i=1}^3 d_i \Lambda_i^{(\alpha)} + \sum_{ij} d_i d_j \Lambda_{ij}^{(\alpha)} \end{aligned} \quad (3)$$

where

$$\begin{aligned} \Lambda &= \Lambda^{(\alpha)} \Big|_{\vec{d}=0} = \prod_{h\lambda} C_{h\lambda}^+ |0\rangle, \\ \Lambda_i^{(\alpha)} &= -\frac{1}{2} \sum_{ph\lambda} \varepsilon_{\lambda}^{(\alpha)} \langle p\lambda | \partial_i | h\lambda \rangle C_{p\lambda}^+ C_{h\lambda} \Lambda, \\ \Lambda_{ij}^{(\alpha)} &= \frac{1}{8} \sum_{\substack{p_1 p_2 \lambda \\ h_1 h_2 \lambda}} \varepsilon_{\lambda_1}^{\alpha} \varepsilon_{\lambda_2}^{\alpha} \langle p_1 \lambda_1 | \partial_i | h_1 \lambda_1 \rangle \langle p_2 \lambda_2 | \partial_j | h_2 \lambda_2 \rangle \cdot \\ & \cdot C_{p_1 \lambda_1}^+ C_{p_2 \lambda_2}^+ C_{h_2 \lambda_2} C_{h_1 \lambda_1} \Lambda + \frac{1}{8} \sum_{ph\lambda} \langle p\lambda | \partial_i \partial_j | h\lambda \rangle C_{p\lambda}^+ C_{h\lambda} \Lambda \end{aligned} \quad (4)$$

Consistently with the traditional approach to the macroscopic description of collective vibrations⁽⁷⁾, we redefine \vec{d} as collective variable and assume a total wave function of the form

$$\Psi_{(n)}^{(\alpha)} = \Phi_{(n)}(\vec{d}) \Lambda^{(\alpha)} \quad (5)$$

where $\Phi_{(n)}(\vec{d})$ can be considered independent of the phase α . The expectation value of the nuclear Hamiltonian H with respect to the in

trinsic variables gives in the harmonic approximation, aside from a constant

$$H_\alpha = \langle \Lambda_n^{(\alpha)} | H | \Lambda_n^{(\alpha)} \rangle = - \frac{\hbar^2}{2M_\alpha} \Delta_\alpha + \frac{1}{2} \sum_{i=1}^3 (k_i + k_i^{(\alpha)}) d_i^2 \quad (6)$$

where it is emphasized that the restoring force constant is composed of a term independent of the phase and a (much smaller) part which depends on it. This collective Hamiltonian describes the relative oscillation of two fluids, whose nature is specified by the kind of spin-isospin order actually established in the ground state. We will confine ourselves to nuclei with $N = Z$. For these nuclei the reduced mass is independent of the phase so that the only dependence of the Hamiltonian (6) on the phase is through the term $k_i^{(\alpha)}$. The ground state of the nucleus will therefore exhibit the spin-isospin correlation of the phase α_0 for which $k_i^{(\alpha_0)}$ is largest in magnitude and negative. If this happened for the phase $\alpha_0 = 4$, then the zero point motion should result from the relative oscillation of protons against neutrons, as in the Goldhaber-Teller model⁽⁸⁾. If instead $k_i^{(\alpha=6)}$ came out to be the most attractive term, then the ground state correlation should manifest as oscillation of spin-up protons and spin-down neutrons against spin-down protons and spin-up neutrons. This is the case of interest to us since V_{OPE} favors this phase. Let us therefore suppose that the lowest energy is obtained for $\alpha = 6$. The ground and first excited states of the system belonging to this phase are of the form

$$\Psi_{LM}^{(6)} = \Phi_{LM}(\vec{d}) \Lambda^{(6)} \quad L = 0, 1 \quad (7)$$

where $\Lambda^{(6)}$ is given by eq. (3) when we put $\varepsilon_j^6 = \sigma_3 \tau_3$. The states belonging to phases other than $\alpha = 6$ however cannot be represented by vectors of the same form.

The $\Lambda^{(\alpha)}$'s in fact, although linearly independent, are far from being mutually orthogonal. Let us assume for sake of simplicity that only one alternative phase is possible, say $\alpha = 4$. The Schmidt orthogonalization procedure will give for the lowest states of such a phase

up to first order in \vec{d}

$$\Psi_{0\mu}^{(4)} \approx \Phi_0(\vec{d}) \Xi_{\mu}^{(4)} \quad \mu = 0, -1, 1 \quad (8)$$

where $\Xi_{\mu}^{(4)}$ are spherical components of the form:

$$\begin{aligned} \Xi_{\mu}^{(4)} = n_{\mu}^{-1/2} \left\{ (\Lambda_{\mu}^{(4)} - \Lambda_{\mu}^{(6)}) + \sum_{\nu=0, -1, 1} d_{\nu} \left[(\Lambda_{\mu\nu}^{(4)} - \Lambda_{\mu\nu}^{(6)}) - \right. \right. \\ \left. \left. - \Lambda(\langle \Lambda_{\mu}^{(6)} | \Lambda_{\nu}^{(4)} \rangle - \langle \Lambda_{\mu}^{(6)} | \Lambda_{\nu}^{(4)} \rangle) \right] \right\} \end{aligned} \quad (9)$$

and the normalization constant has the expression:

$$\begin{aligned} n_{\mu} &= \frac{1}{4} \sum_{ph\lambda} (\varepsilon_{\lambda}^4 - \varepsilon_{\lambda}^6)^2 |\langle p\lambda | \partial_{\mu} | h\lambda \rangle|^2 = \\ &= \frac{1}{2} \sum_{ph\lambda} (1 - \sigma_3) |\langle p\lambda | \partial_{\mu} | h\lambda \rangle|^2. \end{aligned} \quad (10)$$

If an harmonic oscillator s. p. basis is used, the zero order expression of $\Xi_{\mu}^{(4)}$ is easily derived:

$$\Xi_{\mu}^{(4)} \approx n_{\mu}^{-1/2} (\Lambda_{\mu}^{(4)} - \Lambda_{\mu}^{(6)}) = \quad (11)$$

$$= N_{\mu} \sum_{ph\lambda} \tau_3 (1 - \sigma_3) \langle p\lambda | r_{\mu} | h\lambda \rangle C_{p\lambda}^{+} C_{h\lambda} \Lambda$$

where

$$N_{\mu} = \left[\sum_{ph\lambda} (1 - \sigma_3)^2 |\langle p\lambda | r_{\mu} | h\lambda \rangle|^2 \right]^{-1/2} \quad (12)$$

This is the familiar form of the particle-hole states derived within the schematic model⁽⁹⁾. The lowest state (8) must therefore be close in energy to the first excited states (L = 1) of the phase dominant in the ground state.

3. - V_{OPE} CONTRIBUTION TO THE GROUND STATE.

In order to ascertain whether the spin-isospin order of phase $\alpha = 6$ is present in the ground state or not we should compute the contribution of the nucleon-nucleon interaction to the restoring force constant for each phase. To start with, we will confine ourselves to evaluate the contribution of V_{OPE} to $k_i^{(6)}$ in order to see if there is any charge for the phase $\alpha = 6$ to be realized.

We assume spin quantization along the symmetry axis which we take as the Z axis. We further assign to the index λ the values 1, 2, 3, 4 for spin-up protons, spin-down protons, spin-up neutrons, spin-down neutrons respectively.

The direct part of the expectation value of V_{OPE} in the state $\Phi_0 \Lambda^{(6)}$ is:

$$\langle V_{\text{OPE}} \rangle_6 = (\Phi_0, W_{\text{OPE}}^{(6)}(\vec{d}) \Phi_0) \quad (13)$$

where:

$$W_{\text{OPE}}^{(6)}(\vec{d}) = (\Lambda^{(6)}, V_{\text{OPE}} \Lambda^{(6)}) = \frac{1}{2} \sum_{\lambda_1 \lambda_2} \gamma_{\lambda_1 \lambda_2} \int d\vec{r}_1 d\vec{r}_2 \cdot \quad (14)$$

$$\cdot \left[V_C(r_{12}) + \left(3 \frac{z_{12}^2}{r_{12}^2} - 1 \right) V_T(r_{12}) \right] e_{\lambda_1}(\vec{r}_1, \vec{d}) e_{\lambda_2}(\vec{r}_2, \vec{d}),$$

$$(\gamma_{\lambda_1 \lambda_2}) = \begin{vmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{vmatrix},$$

$$V_C(r_{12}) = v_0 \frac{\exp(-\mu r_{12})}{\mu r_{12}},$$

$$V_T(r_{12}) = v_0 \left(1 + \frac{3}{\mu r_{12}} + \frac{3}{(\mu r_{12})^2} \right) \frac{\exp(-\mu r_{12})}{\mu r_{12}},$$

$$v_0 = \frac{1}{3} \frac{p^2}{\hbar c} m_n c^2 = 3.65 \text{ MeV}, \quad \mu = 0.70 \text{ fm}^{-1} \quad (15)$$

$$\varrho_\lambda(\vec{r}, \vec{d}) = \sum_h \varphi_{h\lambda}^*(\vec{r}, \vec{d}) \varphi_{h\lambda}(\vec{r}, \vec{d}),$$

$$\varphi_{h\lambda}(\vec{r}, \vec{d}) = \left(e^{-\frac{i}{\hbar} \sigma_3 \tau_3 \vec{p} \cdot \vec{d}} \varphi_{h\lambda} \right) (\vec{r}).$$

We disregard the exchange term according to the estimates of refs. (1, 2) in nuclear matter.

In order to get a workable expression for the mean value (14) we approximate the one body density by a gaussian of the form:

$$\varrho_\lambda(\vec{r}, \vec{d}=0) = \varrho(\vec{r}) = \varrho_0 \exp\left(-\frac{x^2+y^2}{R_1^2} - \frac{z^2}{R_2^2}\right) \quad (16)$$

where R_1 and R_2 can be shown to be related to the deformation parameter δ and the nuclear radius R through:

$$R_1 = \left[\frac{2}{5(1+\delta)} \right]^{1/2} R, \quad R_2 = \left[\frac{2}{5(1-\delta)} \right]^{1/2} R \quad (17)$$

After a lengthy but straightforward calculation we get from eq. (14) to first order in δ :

$$\langle V_{\text{OPE}} \rangle = \frac{1}{2} K^{(6)} \langle d^2 \rangle = \frac{1}{2} \left[K_0^{(6)} + \delta K_1^{(6)} \right] \langle d^2 \rangle \quad (18)$$

where:

$$K_0^{(6)} = K \int_0^\infty dx x p_2(x) \exp\left(-\frac{2}{\sqrt{5}} \mu R x - x^2\right),$$

$$K_1^{(6)} = K \int_0^\infty dx x p_4(x) \exp\left(-\frac{2}{\sqrt{5}} \mu R x - x^2\right), \quad (19)$$

$$x = \frac{\sqrt{5}}{2} \frac{r_{12}}{R}, \quad K = \frac{5}{4} \sqrt{\frac{5}{\pi}} v_0 \frac{A^2}{\mu R^3} \quad (20)$$

$$p_2(x) = 1 - \frac{2}{3}x^2, \quad p_4(x) = \left(\frac{3}{4} + \frac{13}{3(\mu R)^2}\right) - \quad (21)$$

$$- \left(\frac{19}{18} - \frac{26}{3\sqrt{5}\mu R}\right)x + \left(\frac{52}{45} - \frac{4}{3(\mu R)^2}\right)x^2 + \left(\frac{2}{9} - \frac{8}{3\sqrt{5}\mu R}\right)x^3 - \frac{16}{45}x^4$$

It is easy to check that V_{OPE} gives a repulsive contribution in spherical as well as prolate deformed nuclei. Only light deformed nuclei with an oblate shape have a chance of getting an attractive contribution. This was confirmed by the quantitative estimates we made by evaluating the integrals (19) numerically.

For $A = 28$ and $\delta = -0.4$ we get $K^{(6)} = -0.673 \text{ MeV/fm}^2$, and for $A = 12$ and $\delta = -0.44$ we get $K^{(6)} = -3.909 \text{ MeV/fm}^2$.

In order to get the corresponding energy contributions we evaluated $\langle d^2 \rangle$ by assuming the fluids to be rigid bodies as in the Goldhaber-Teller model⁽⁸⁾. Disregarding the small dependence on α the expression for $\langle d^2 \rangle$ is then:

$$\langle d^2 \rangle = 3 \left(\frac{2\pi}{5}\right)^{1/4} \left(\frac{\hbar^2}{m} \frac{r_0}{v_0} R\right)^{1/2} A^{-1} \simeq 4.92 A^{-5/6} \text{ fm}^2 \quad (22)$$

m being the nucleon mass and having assigned the value of 40 MeV to the two nucleon separation potential v_0 and 2 fm to its range r_0 . We then get for $A = 28$ $\langle V_{\text{OPE}} \rangle = -100 \text{ keV}$ and for $A = 12$ $\langle V_{\text{OPE}} \rangle = -1.21 \text{ MeV}$.

These values are obtained for the experimental density, $R = 1.2 A^{1/3}$. If we double the density we get an attraction one order of magnitude larger.

It is to be noted that unlike the laminated structure in nuclear matter, the central part of the O. P. E. potential is always repulsive.

The tensor part, being linear in the deformation parameters δ , is attractive only for oblate nuclei. This attractive contribution prevails over the central repulsive one for enough light nuclei.

For prolate nuclei it is convenient to orientate the spins in the plane perpendicular to the symmetry axis. In this way the tensor contribution remains attractive but it is halved while the central contribution becomes more repulsive. The net contribution is always repulsive at experimental density, though becomes attractive at higher densities.

4. - GROUND STATE PROPERTIES AND E. M. TRANSITIONS.

Given the small energy gain and the approximations inherent to the model, we cannot draw the conclusion that light deformed oblate nuclei are in a spin-isospin ordered phase. The previous results leave only open such a possibility. It is therefore useful to implement the energy calculation with the analysis of other properties of the states so correlated. Let us first of all mention that, in spite of spin-isospin correlations, the average value of the source of the pion field vanishes. Nevertheless the ground state contains small admixtures of isospin components other than $T = 0$, as required⁽¹⁰⁾ for laminated structure and pion condensation. We get in fact, disregarding deformation effects

$$\langle T^2 \rangle_6 = (\Phi_0 A_N^6, T^2 \Phi_0 A_N^6) \approx \frac{(N+1)(N+2)(N+3)}{2A} \quad (23)$$

N labeling the highest occupied mayor shell. This equation gives

$$\langle T^2 \rangle_6 \sim 1 \text{ for } A = 28 \text{ and } A = 12.$$

The features of such a state are further evidentiated by its e. m. properties. The e. m. transition probabilities from the ground to the first excited state of the same phase ($\alpha = 6$) are given by:

$$B^{(6 \rightarrow 6)}(\lambda; K = I = 0 \rightarrow KI) = \frac{2}{1 + \delta_{k0}} \left| (\Phi_{1k}, \mathcal{M}^{(6)}(\lambda = I, \nu = k) \Phi_0) \right|^2 \quad (24)$$

where :

$$\mathcal{M}^{(6)}(I, K) = (\Lambda^{(6)}, \mathcal{M}(I, K) \Lambda^{(6)}) \quad (25)$$

Its expression to first order in \vec{d} is

$$\mathcal{M}^{(6)}(I, K) = \frac{i}{2\hbar} \sum_{i=1}^3 d_i \sum_{h\lambda} \sigma_3 \tau_3(\varphi_{h\lambda}, [p_i, \mathcal{M}(I, K)] \varphi_{h\lambda}) \quad (26)$$

It is easy to show from this equation that the lowest nonvanishing multipole operator is the magnetic quadrupole one. Its expression is

$$\mathcal{M}_d^{(6)}(M2, K) \approx \frac{1}{8} \sqrt{\frac{15}{2\pi}} \langle 1K 1 0 | 2K \rangle (g_p - g_n) (-)^K d_k \frac{e\hbar}{2mc} \quad (27)$$

The corresponding transition probabilities are in Weisskopf units

$$\begin{aligned} B(M2, K=0 \rightarrow K=0) &\approx 2.19 (1 + \frac{1}{4} \delta) A^{1/3} \\ B(M2, K=0 \rightarrow |K|=1) &\approx 2.33 (1 - \frac{1}{4} \delta) A^{1/3} \end{aligned} \quad (28)$$

More specifically for $A = 28$, $B(M2; K=0 \rightarrow K=0) \sim 5.98$ w.u.,

$B(M2; K=0 \rightarrow |K|=1) = 7.78$ w.u., and for $A = 12$, $B(M2; K=0 \rightarrow K=0) \sim$

~ 4.51 w.u., $B(M2; K=0 \rightarrow |K|=1) \approx 5.87$ w.u.

The model therefore predicts large M2 transition probabilities to states of the same phase. Enhanced transition probabilities to states belonging to phase other than $\alpha = 6$ are not excluded however. The E1 transition probability to states (8) belonging to phase $\alpha = 4$, for instance, is determined by the matrix element

$$\begin{aligned} (\bar{E}_K^{(4)}, \mathcal{M}(E1, K) \Lambda) &\approx \\ &\approx \frac{e}{4} \sqrt{\frac{4\pi}{3}} \frac{\sum_{ph\lambda} \tau_3 (1 - \tau_3) (1 - \sigma_3) |\langle p\lambda | r_K | h\lambda \rangle|^2}{\left[\sum_{ph\lambda} (1 - \sigma_3)^2 |\langle p\lambda | r_K | h\lambda \rangle|^2 \right]^{1/2}} \end{aligned} \quad (29)$$

which is essentially the expression determining the E1 giant resonant transition probability within the schematic model.

5. - CONCLUSIONS.

Summarizing the model predicts that a spin-isospin ordered phase induced by the O.P.E. potential may be energetically favored only in light deformed oblate nuclei.

In order to confirm such a result one needs to evaluate the energy contributions coming from all components of the nucleon-nucleon interaction and, perhaps, a refinement of the semimicroscopic model developed. Even in the present form, however, the predictions concerning isospin admixtures in the ground state and enhancement of M2 transition probabilities can be taken in our view as qualitatively significant.

The model can be entirely reformulated within the more elegant microscopic framework of R.P.A.. This improvement is also desirable in connection with the possible relation between the specific spin-isospin ordered considered and the precursor phenomena mentioned in the Introduction.

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