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LARGE INFRA-RED CORRECTIONS IN QCD PROCESSES

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The effect of soft gluon emission in hard processes is examined to all orders in QCD. Large corrections have been found, which sensibly modify the parton model results, and show the relevance of the appropriate kinematical limits in leptoproduction and in the Drell–Yan process.

In this letter we study the problem of soft corrections in hard processes, by using the approach developed in ref. [1], based on the combined use of renormalization group techniques and the formalism of coherent states, which has been applied to discuss jet physics in QCD [2].

In particular, we address ourselves to the question raised in ref. [3], where it has been observed that the soft behaviour of the theory plays an important role in the evaluation of the corrections to the leading order results.

Our results confirm to all orders in perturbation theory, in the double logarithm approximation including the effects of the running coupling constant, the importance of exact kinematical constraints in the various processes as recently emphasized by Parisi [4]. We have explicitly considered deep inelastic electroproduction and the Drell–Yan process [5].

Let us first discuss leptoproduction in the $x \simeq 1$ region. It is known that in that limit the parton densities are also exponentiated, in addition to the usual exponentiation of the moments. In fact one finds [6–8], for the valence densities, upon which we concentrate from now on,

$$q(x, \xi) \simeq \frac{\exp\left(\frac{3}{4} - \gamma_E\right) C_F \xi}{\Gamma(C_F \xi)} (1-x)^{C_F \xi - 1} , \tag{1}$$

where

$$\xi(k_{\perp\max}^2) = \int_{\lambda^2}^{k_{\perp}^2 \max} \frac{dk_\perp^2}{k_\perp^2} \frac{\alpha(k_\perp)}{\pi} , \tag{2}$$

and $k_{\perp\max}^2 \simeq Q^2$. This result is valid for $[\alpha(Q^2)/\pi] \ln [1/(1-x)] \ll 1$ and $[\alpha(Q^2)/\pi] \ln Q^2 \lesssim 1$. Eq. (1) coincides, as can be easily seen, with the energy distribution of the radiation obtained in the coherent state formalism, taking into account the constraint of energy conservation [9]

$$\frac{dP}{d\omega} = \frac{1}{2\pi} \int_{-\infty}^{\infty} db \, e^{ib\omega} \exp \left\{ C_F \int_0^1 dz \frac{1+(1-z)^2}{2z} \xi(Q^2) [e^{-ibz} - 1] \right\} , \tag{3}$$

where $\omega = 1 - x$ is the fraction of energy carried out by the soft gluon radiation.

More generally, in order to show explicitly the equivalence of the nth moment of the distribution (3) with the

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conventional parton density moments for large n, let us rewrite eq. (3) in the form

$$\frac{dP}{dx}=q(x,Q^2)=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dn\, e^{n(1-x)}\exp\left\{\frac{C_F}{2}\int_0^1 dy\,\frac{1+y^2}{1-y}\xi(Q^2)[e^{-n(1-y)}-1]\right\}$$

$$\simeq\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dn\, x^{-n}\exp\left[\frac{C_F}{2}\int_0^1 dy\,\frac{1+y^2}{1-y}\xi(Q^2)(y^n-1)\right]. \tag{4}$$

This result explicitly shows that the moments of the distribution (3) coincide with the conventional moments of parton densities in the large-n limit.

Let us discuss now the very soft region, when $[\alpha(Q^2)/\pi]\ln[1/(1-x)]\lesssim 1$. In order to study the soft singularities, we use as the infra-red regulator the gluon mass $=\lambda$ with zero quark mass. This regularization scheme makes clearer the relevance of the kinematical constraints in the transverse momentum phase space in all processes. Similar results can be obtained using the dimensional regularization scheme[‡1]. This choice of regularization will also be convenient below in discussing the large-Q^2 behaviour of the quark form factor.

To first order, the electromagnetic quark form factor $F(Q^2)$ is given by $(q^2=-Q^2<0)$

$$F(Q^2)\simeq 1-(\alpha/4\pi)C_F\ln^2(Q^2/\lambda^2), \tag{5}$$

in the leading double logarithm approximation. Correspondingly, the real contribution to the valence parton density is

$$q^{\rm real}(x,Q^2)\simeq\delta(1-x)+\frac{\alpha}{\pi}C_F\frac{1}{1-x}\ln\left[\frac{Q^2}{\lambda^2}(1-x)\right]$$

$$=\delta(1-x)+\frac{\alpha}{\pi}C_F\left\{\frac{1}{1-x}\ln\left[\frac{Q^2}{\lambda^2}(1-x)\right]\right\}_+ +\frac{\alpha}{\pi}C_F\delta(1-x)\int_0^{1-\lambda^2/Q^2}\frac{dy}{1-y}\ln\left[\frac{Q^2}{\lambda^2}(1-y)\right]. \tag{6}$$

From eqs. (5) and (6) it follows that the $\ln^2\lambda$ singularity is cancelled by adding the real and virtual contribution to $q(x,Q^2)$, and in addition the first moment is equal to one, as requested by the Adler sum rule [10]. In order to show the exponentiation of the $1/(1-x)\ln(1-x)$ singularity, we will require below the same cancellation of the $\ln^2\lambda$ singularity between real and virtual contributions, after use of the appropriate expression of the quark form factor, demanding in addition the fulfilment of the Adler sum rule.

By ignoring the effect of the running coupling constant, which will be discussed later, and using the result for the exponentiation of the leading $\ln^2(Q^2/\lambda^2)$ singularity of the form factor [11], we obtain for the nth moment

$$q^{(m)}(Q^2)\simeq\exp\left\{\frac{\alpha}{\pi}C_F\int_0^{1-\lambda^2/Q^2}\frac{dy}{1-y}\ln\left[\frac{Q^2}{\lambda^2}(1-y)\right]y^{n-1}\right\}\exp\left(-\frac{\alpha}{2\pi}C_F\ln^2\frac{Q^2}{\lambda^2}\right)$$

$$=\exp\left\{\frac{\alpha}{\pi}C_F\int_0^1\frac{dy}{1-y}\ln\left[\frac{Q^2}{\lambda^2}(1-y)\right](y^{n-1}-1)\right\}. \tag{7}$$

The same result follows directly in the coherent state approach, by simple kinematical bounds in the $k_\perp$ phase space in eq. (2), namely:

$$\frac{dP}{dx}=q(x,Q^2)\simeq\frac{1}{2\pi}\int_{-\infty}^{\infty} db\, e^{ib(1-x)}\exp\left\{C_F\int_0^1\frac{dz}{1-z}\int_{\lambda^2}^{k^2_{\perp\max}=Q^2(1-z)}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha(k_\perp)}{\pi}[e^{-ib(1-z)}-1]\right\}, \tag{8}$$

‡1 See, for example, ref. [3].

which shows, by the argument given above, that the nth moment of eq. (8) coincides with eq. (7) for large n and $\alpha(k_\perp) = \alpha$ = constant.

We now take into account the effect of the running coupling constant. In our regularization scheme and with on-shell normalization conditions, an explicit two-loop calculation [12] shows that the quark form factor satisfies the Callan–Symanzik equation:

$$\left[\lambda\frac{\partial}{\partial\lambda} + \beta\frac{\alpha}{\pi}\alpha\frac{\partial}{\partial\alpha} - \frac{\alpha}{\pi}C_F\left(\ln\frac{Q^2}{\lambda^2} - 3\right)\right] F(\alpha, Q^2/\lambda^2) = 0\,, \tag{9}$$

where $\beta = -\frac{1}{2}(\frac{11}{3}N_c - \frac{2}{3}N_f)$. Assuming the validity of eq. (9) to all orders, we obtain in the leading logarithm approximation the solution

$$F(\alpha, Q^2/\lambda^2) = \exp\left[-\frac{C_F}{2\beta}\ln\left(1 - \frac{\alpha}{2\pi}\beta\ln\frac{Q^2}{\lambda^2}\right)\ln\frac{Q^2}{\lambda^2}\right]. \tag{10}$$

A similar result has been found using a different regularization scheme in ref. [13].

In analogy with eq. (7), we then find

$$q^{(n)} \simeq \exp\left[C_F\int_0^{1-\lambda^2/Q^2}\frac{dz}{1-z}\int_{\lambda^2}^{Q^2(1-z)}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha(k_\perp^2)}{\pi}z^{n-1}\right]|F|^2$$

$$= \exp\left[C_F\int_0^1\frac{dz}{1-z}(z^{n-1}-1)\int_{\lambda^2}^{Q^2(1-z)}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha(k_\perp^2)}{\pi}\right]. \tag{11}$$

An intuitive derivation of this result simply follows in the coherent state picture, by taking into account the $k_\perp$ dependence of α in eq. (8). So our expression (8) replaces eq. (1) in the very near kinematical limit of $x \simeq 1$. This result has been also obtained by Gribov and Lipatov [6], and Dokshitser [7] by a careful diagrammatic analysis. In particular, in ref. [7] an explicit expression for $q(x, Q^2)$ is given by evaluating eq. (8) by saddle point techniques.

We note also that the Altarelli–Parisi [14] evolution equation for the nonsinglet quark density becomes in this limit

$$\frac{dq(x, Q^2/Q_0^2)}{d\ln(Q^2/Q_0^2)} = \frac{1}{2\pi}\int_x^1\frac{dy}{y}\,[P(x/y)\bar{\alpha}((Q^2/Q_0^2)(1-x/y))]_+\,q(y, Q^2/Q_0^2)\,, \tag{12}$$

where Q_0^2 is a normalization scale, $P(z) = C_F[(1+z^2)/(1-z)]$ and we have implicitly absorbed the mass singularities in the distribution of partons inside the hadron.

We now discuss the implications of our results in the Drell–Yan process. As is well known, in the naive parton model one has

$$\frac{d\sigma^{DY}}{dQ^2} = \frac{4\pi\alpha^2}{9SQ^2}\int\frac{dx_1}{x_1}\frac{dx_2}{x_2}\left[\sum_i e_i^2 q_i(x_1)\bar{q}_i(x_2) + (1\leftrightarrow 2)\right]\delta(1-z)\,, \tag{13}$$

where $\sqrt{S}$ is the invariant mass of the hadronic system, $z = \tau/x_1x_2$ and $\tau = Q^2/S$.

By taking into account the soft gluon emission at all orders and omitting for simplicity the contribution of gluons in the initial state, the delta function in eq. (13) is replaced in the coherent state approach, in the limit $z \simeq 1$, by

$$f(z, Q^2) \simeq \left\{\frac{1}{2\pi}\int db\, e^{ib(1-z)}\exp\left[2C_F\int_0^1 dy\frac{1}{1-y}\int_{\lambda^2}^{Q^2(1-y)^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha(k_\perp)}{\pi}[e^{-ib(1-y)}-1]\exp\{[\alpha(Q^2)/2\pi]C_F\pi^2\}\right.\right.\,, \tag{14}$$

in analogy to eq. (8).

Let us briefly comment on our result. The factor $2C_F$ is simply related to the emission of gluons from two legs in the initial state and the upper limit $Q^2(1-y)^2$ in the integral is due to the kinematical bound $k^2_{\perp\max} = Q^2(1-y)^2/y \simeq Q^2(1-y)^2$. In addition, the factor $\exp\{[\alpha(Q^2)/2\pi]C_F\pi^2\}$ is due to the continuation of the Q^2 dependence in the form factor from space-like to time-like regions [4] [see eq. (5) in exponentiated form].

Eqs. (13) and (14) can now be rewritten as

$$\frac{d\sigma^{DY}}{dQ^2} = \frac{4\pi\alpha^2}{9SQ^2}\int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\left[\sum_i e_i^2 q_i(x_1,Q^2)\bar{q}_i(x_2,Q^2) + (1\leftrightarrow 2)\right]\tilde{f}(z,Q^2), \tag{15}$$

with

$$\tilde{f}(z,Q^2) = \left\{\frac{1}{2\pi}\int db\, e^{ib(1-z)} \exp\left[2C_F\int_0^1 \frac{dy}{1-y}\int_{Q^2(1-y)}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2}\frac{\alpha(k_\perp)}{\pi}\left(e^{-ib(1-y)}-1\right)\right]\right\}\exp\{[\alpha(Q^2)/2\pi]C_F\pi^2\}, \tag{16}$$

having explicitly introduced the Q^2 dependence in the parton densities in deep inelastic scattering. The equivalence of eqs. (15) and (16) with eqs. (13) and (14) follows directly by taking the τ^n moments of $d\sigma/dQ^2$ in both equations and using eq. (8) for the Q^2 dependence of the parton densities. It is easy to check that our final formulae (15), (16) agree with the perturbative calculation of ref. [3] in the limit $z \to 1$.

We note that the term $\exp\{[\alpha(Q^2)/2\pi]C_F\pi^2\}$ in eq. (16) plays an important role in renormalizing the naive Drell–Yan cross section by about a factor of two at present energies and should therefore be taken into account in comparison with experimental data. More detailed phenomenological implications of our results will be discussed elsewhere.

We now comment briefly on the transverse momentum distributions of the μ pairs in the Drell–Yan process. In our framework the $k_\perp$ distributions can be simply derived in analogy with eq. (8) taking into account the constraint of transverse momentum conservation on the multiple gluon emission, as explicitly found in ref. [15]. They coincide with those obtained in ref. [2], once applied to this particular process.

Our results can be summarized as follows. The formalism of coherent states has been shown to be quite appropriate for studying the effects of soft gluon bremsstrahlung at all orders. Large corrections have been found which explicitly show the importance of the soft regions in various processes. An intuitive explanation of these corrections follows from simple kinematical considerations. Similar arguments apply also to the e^+e^- annihilation.

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