Invited talk given at the
"2nd Oxford Quantum
Gravity Conference",
Oxford, April 1980.

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-80/31(P)
16 Giugno 1980

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AN OVERVIEW ON BROKEN SUPERGRAVITY MODELS.

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S. Ferrara[(x)]: AN OVERVIEW ON BROKEN SUPERGRAVITY MODELS.

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1. - WHY SUPERGRAVITY ?

The present theoretical approach to high-energy particle physics is based on some general principles which have received considerable experimental support in recent years. These guidelines can be summarized as follows: the dynamical framework of our theoretical understanding of the basic forces of Nature relies on Lagrangian Quantum Field Theory. This framework is supplemented by symmetry principles, i.e., the dynamics should not change when we perform some symmetry operations on the field variables of a model under consideration. These symmetry principles, when the symmetry operations are continuous, are further constrained by the requirement that symmetries should be made local. More interestingly this last requirement gives often some additional informations on the underlying dynamics. In fact in recent time there has been increasing evidence that gauge quantum field theory may have some unconventional properties which may show up beyond perturbation theory such as confinement, dynamical Higgs mechanism and topological effects. If one accepts such framework for modern particle physics it seems natural to further use some economy principle, that is, different low energy symmetries are eventually unified at energies higher than the energy scales which correspond to the strength of the interactions needed today in order to understand "low energy" phenomena (J. Iliopoulos, 1979).

In the above mentioned framework a possible scenario for the unification of particle interactions emerges:

a) Unification of electromagnetic and weak forces at 100 GeV;

b) Grand Unification of electroweak and nuclear (strong) forces at 10^{14}-10^{15} GeV;

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c) Superunification of electroweak, strong and gravitational forces at the Planck energy $\sim 10^{19}$ GeV.

Although this framework is highly questionable, especially in b) and c), it is important that the requirements of Grand and Superunifications can already give constraints and predictions on physical phenomena at present energies. Examples in Grand Unified Theories are the proton decay rate, neutrino oscillations and cosmological implications (J. Ellis, 1979 ; J. Ellis, 1980a; R. Barbieri, 1980; D. V. Nanopoulos, 1980). In Superunified Theories these constraints are more speculative but in some models one can get a modification of the Newton law at short distances which may give detectable effects in laboratory experiments (J. Scherk, 1979 ; 1980). Another prediction arises in a preconstituent model for a superunified theory based on spontaneously broken supergravity with a local SU(8) invariance (J. Ellis et al. 1980a). This model restricts to three the number of quark-lepton families in a GUT based on SU(5). SU(5) is also the maximal allowed subgroup of this theory which may remains unbroken in a first step of sequential symmetry breakings. We recall that SU(5) is the minimal simple group which contains $SU(3)_{COLOR} \otimes (SU(2) \otimes U(1))_{ELECTROWEAK}$.

In the attempts at superunifications, in the framework of gauge quantum field theory, it is inevitable that the unifying gauge group must contain the space-time symmetry group and the internal symmetry group all together. The former is related to gravitational interactions, the latter to the non gravitational sector which is described by a Yang-Mills theory. Once this gauge principle is accepted the only known ways these groups can be related is at present in the framework of supergravity (D. Z. Freedman et al., 1976; S. Deser et al., 1976).

Supergravity is the gauge theory of a graded (super) algebra. Graded (super) algebras whose even (bosonic) part contains the space-time symmetry algebra (Poincaré, Conformal or De Sitter) are usually called supersymmetries (Fermi-Bose symmetries). Supergravity is therefore the gauge theory of supersymmetry. What is supersymmetry? Supersymmetry (Yu. A. Gol'fand et al., 1971 ; J. Wess et al., 1974 ; L. Corwin et al., 1975 ; P. Fayet et al., 1977 ; A. Salam et al., 1978) is a set of symmetry operations which transform particles of different statistics into each others. More specifically it is a continuous symmetry of local quantum field theory, based on a GLA (Graded Lie Algebra), which enables us to put bosons and fermions in irreducible multiplets of particle states. In supersymmetry the odd part of the GLA contains N-spinorial charges of Majorana type Q^i_α $\alpha = 1 \ldots 4$, $i = 1 \ldots N$. These charges form the so called grading representation of the GLA. They are a representation of the even part of GLA. The Lie algebra part of GLA contains the Poincaré algebra and one has

$$\left[Q^i_\alpha , P_\mu\right] = 0 , \tag{1}$$

$$\left[Q^i_\alpha , M_{\mu\nu}\right] = i(\sigma_{\mu\nu})_\alpha^{\ \beta} Q^i_\beta , \tag{2}$$

$$\left\{Q^i_\alpha , \bar{Q}^j_\beta\right\} = -2\gamma^\mu_{\alpha\beta} P_\mu \delta^{ij} + Z^{ij}\delta_{\alpha\beta} + Z'^{ij}\gamma^5_{\alpha\beta} , \tag{3}$$

the generators Z^{ij} (Z'^{ij}), both antisymmetric in the i, j indices, belong to the centre of GLA and are called central charges. $M_{\mu\nu}$, P_μ are the usual generators of the Poincaré group. In local quantum field theory the central charges correspond to additional quantum numbers. They may correspond to global or local symmetries depending on the particular theory under consideration. These operators have non-vanishing eigenvalues only when they act on massive supermultiplets. Moreover when supersymmetry is gauged, i. e. in supergravity, these massive multiplets are minimally coupled to some spin 1 partners (graviphotons) of the graviton with a dimensionless coupling constant g_z proportional to their mass M and the gravitational constant K (S. Ferrara et al., 1977 ; K. Zachos, 1978 ; J. Scherk, 1979) $g_z \propto MK$. This peculiar relation, as pointed out by Scherk (1979), can give a substantial modification of the Newton law at short distances for identical particles because the gravitational force is attractive, the vectorial force is repulsive and being of comparable strength, they may compensate at short distances.

We now come back to the supersymmetry algebra whose relevant commutation rules are given by (1), (2) and (3). In local quantum field theory the fermionic generators Q^i_α are obtained as space-integrals of local current densities

$$Q^i_\alpha = \int d^3\underline{x}\, J^i_{o\alpha}(\underline{x}, t) \qquad (4)$$

and current conservation

$$\partial^\mu J^i_{\mu\alpha}(x) = 0 = \frac{d}{dt} Q^i_\alpha = 0$$

ensures that the corresponding action

$$I = \int d^4x\, \mathscr{L}(x) \qquad (5)$$

is invariant under supersymmetry transformations

$$\delta_\epsilon I = \int d^4x\, \delta_\epsilon \mathscr{L} = 0\,, \qquad \delta_\epsilon \mathscr{L} = [\epsilon^i Q^i, \mathscr{L}] \qquad (6)$$

We note that in order to define $\delta_\epsilon \mathscr{L} = (\delta\mathscr{L}/\delta\Phi)\,\delta_\epsilon\Phi$ anticommuting spinorial parameters ϵ^i_α , having the same trasnformation properties of Q^i_α, must be introduced. These parameters are odd elements of a Grassman algebra. They anticommute with any fermionic quantity while commute with ordinary c-numbers as well as with bosonic operators.

We now give some motivations for the introduction of supersymmetry in local quantum field theory. There are indeed several reasons and the one alluded at the beginning of this section, namely the unification program, is just one of them.

1) Supersymmetry is the only known symmetry consistent with relativistic QFT which relates particles with different spin. This is a major breakthrough because it gives a way to overcome previous no-go theorems on the possibility of non trivial mixing between space-time and internal symmetries.

2) When supersymmetry is realized as a global (rigid) symmetry, supersymmetric theories are the less divergent QFT's known today. For instance the super Yukawa theory, which is a particular combination of renormalizable couplings among scalars and spin -1/2 fermions, needs only one (logarithmically divergent) renormalization constant. Other remarkable examples are the supersymmetric Yang-Mills theories: the maximally extended N = 4 theory has a vanishing $\beta(g)$ function at the first two loops and it is a completely finite theory up to this order of perturbation theory.

3) At the local level gauge supersymmetry requires gravity. It gives a new type of extension of the Einstein theory in which a subtle interplay between the quantum mechanical concept of spin and of space-time geometry occurs. General coordinate invariance is not the primary (local) symmetry principle: coordinate transformations are obtained by merely repeating twice a local supersymmetry transformation.

4) Supergravity theories have better quantum properties than Einstein theory (P. Van Nieuwenhuizen et al., 1977). They offer examples of model field theories in which interaction of matter receives finite gravitational radiative corrections in the first two order of perturbation theory. This is a major progress with respect to all previous unsuccessful attempts in ordinary Einstein theory.

5) Supergravity and especially extended supergravities offer possible schemes for superunification (M. Gell-Mann, 1977). Yang-Mills and Einstein Lagrangians are linked under the invariance principle of local supersymmetry.

6) Supergravity theories provide the unification of interactions of gauge particle of different spin like the graviton, Rarita Schwinger spin-3/2 fields (gravitinos) and Yang-

Mills vector bosons. Gauge particles of half-integral spin therefore emerge for the first time. As a byproduct supergravity provides the first example of a consistent interaction for Rarita-Schwinger fields. In extended supergravity spin 0 and 1/2 matter fields are unified with gauge fields in a single irreducible multiplet. Matter fields and geometrical fields are therefore unified through the local supersymmetry principle.

II. - MULTIPLET STRUCTURE.

Supersymmetric theories and in particular supergravity theories are described in terms of field supermultiplets. These are collections of ordinary fields with different spin, statistics and internal symmetry properties. In this section we will consider the multiplet structure of one particle states which are supposed to be described by the asymptotic fields of supersymmetric quantum field theories.

We consider the representation of the supersymmetry algebra given by (1), (2), (3) on one-particle states. The construction of particle supermultiplets uses the Wigner method of induced representations (A. Salam et al., 1974a, 1975; W. Nahm, 1978; D. Z. Freedman, 1978). We will temporarely consider the supersymmetry algebra without the central charge operators Z^{ij}, Z'^{ij}. Because these operators belong to the centre of the algebra it is consistent to set them equal to zero.

We consider first the stability subalgebra of a time-like momentum $P^{\mu} = (M, \vec{0})$. M is the common mass of the different spin states of a massive multiplet. If we use two component Weyl spinors the stability subalgebra is

$$\{ Q^i_{\alpha}, \bar{Q}_{\beta j} \} = \delta^i_j \delta_{\alpha\beta} \qquad \alpha, \beta = 1, 2 \qquad i, j = 1 \ldots N \tag{7}$$

$$\{ Q^i_{\alpha}, Q^j_{\beta} \} = 0 \tag{8}$$

It is nothing but the Clifford algebra for 2N creation and destruction fermionic operators.

If $M = 0$ we may choose $P^{\mu} = (1, 0, 0, -1)$ and we have

$$\{ Q^i_1, \bar{Q}_{1j} \} = \delta^i_j , \tag{9}$$

$$\{ Q^i_1, \bar{Q}^j_1 \} = 0 , \tag{10}$$

$$\{ Q^i_{\alpha}, Q^j_{\beta} \} = 0 . \tag{11}$$

From (10), (11) we can put $Q^i_1 = 0$, $Q^i_2 = Q^i$ and then we obtain the Clifford algebra for N creation and destruction operators. If we define the Clifford vacuum Ω as the one-particle state having the property

$$\bar{Q}^i_{\alpha} \Omega = 0 \tag{12}$$

then we obtain irreducible representations of the supersymmetry algebra of dimension $2^{2N} \times d_{\Omega}$ for $M \neq 0$ and $2^N \times d_{\Omega}$ for $M = 0$. d_{Ω} is the dimensionality of the representation of the Clifford vacuum. This is a representation of the even part of the stability supersymmetry algebra. The smallest representation (fundamental representation) is obtained when Ω is a singlet ($d_{\Omega} = 1$). The generic state of the fundamental representation is obtained by repeatingly applying the creation operators Q^i_{α} to the vacuum state Ω.

$$\Omega, \; Q^i_{\alpha} \Omega , \; Q^i_{\alpha} Q^j_{\beta} \Omega , \; \ldots\ldots \tag{13}$$

For $M \neq 0$ we get states

$$Q^{i_1}_{\alpha_{i_1}} \ldots\ldots Q^{i_{in}}_{\alpha_{i_{in}}} \Omega \quad . \tag{14}$$

The state of given spin $s = \frac{N-n}{2}$ $(0 \leq n \leq N)$ belongs to the n-fold (traceless) antisymmetric representation of Sp(2N) (S.Ferrara et al, 1979a): $[2N\text{x}\ldots 2N]_n$, 2N is the defining spinor representation of Sp(2N). The highest spin state is a singlet with spin S = = N/2. If Ω is not a singlet but has spin J and belongs to a given representation R of Sp(2N) then the generic state given by (14) will belong to a reducible representation of Sp(2N) given by $[2N\text{x}\ldots 2N]_n \times R$ and it will contain several spin states from $\frac{N-n}{2} + J$ down to $\left|\frac{N-n}{2} - J\right|$. Irreducible representations of the supersymmetry algebra which are not the fundamental one have therefore the property that a state of given spin S lies in a reducible representation of Sp(2N). For M = 0 the generic state

$$Q^{i_1} \ldots Q^{i_n} \Omega \tag{15}$$

(Q^{i_1} is a lowering helicity operator) belongs to the N-fold antisymmetric representation of SO(N) if Ω is a singlet. SO(N) can be enlarged to SU(N) if we allow the internal symmetry part not to commute with parity. In this case Q^i belongs to the $\bar{N}$ representation of SU(N). If the Clifford vacuum Ω_λ has helicity λ the state (15) has helicity $\lambda - n/2$. If we require to a given massless representation to be TCP self-conjugate we must add to the states given by (15) the states

$$\Omega_{-\lambda}, \; \bar{Q}_{i_1} \Omega_{-\lambda}, \; \bar{Q}_{i_1} \bar{Q}_{i_2} \Omega_{-\lambda}, \; \ldots\ldots \tag{16}$$

or equivalently the states

$$\Omega_{\frac{N}{2}-\lambda}, \; Q^{i_1} \Omega_{\frac{N}{2}-\lambda}, \; Q^{i_1} Q^{i_1} \Omega_{\frac{N}{2}-\lambda} \; \ldots \tag{17}$$

Note that $\bar{Q}^i$ belongs to the N representation of SU(N) and $[N\text{x}\ldots N]_n = [\bar{N}\text{x}\ldots\bar{N}]_{N-n}$ We have then that a TCP conjugate massless supermultiplet has dimension 2^{N+1} whith helicity content $|\lambda| \ldots |\lambda - N/2|$. There are however special cases in which the representation given by (15) is automatically PCT self-conjugate. This happens when $\lambda = \frac{N}{2} - \lambda$ i.e. $\lambda = \frac{N}{4}$ In this case the representation has dimension 2^N. Particular cases of self-conjugate multiplets are the $\lambda = 1$ gauge supermultiplet of N = 4 maximally extended Yang-Mills theory and the $\lambda = 2$ gauge supermultiplet of N = 8 maximally extended supergravity theory. We note, as far as internal symmetry properties are concerned, that SO(N) is the maximal internal symmetry of a massless supermultiplet which gives states which are parity preserving. Any SO(N) representation in a given CPT self-conjugate supermultiplet is automatically parity preserving. However if we enlarge SO(N) to SU(N), the SU(N) representations are generally not invariant under parity. A given SU(N) representation acts on a state of a given chirality. For instance if the Clifford vacuum has helicity (chirality) λ and belongs to the representattion R of SU(N), the generic state given by (15) will have helicity (chirality) $\lambda - \frac{n}{2}$ and will belong to the representation $[\bar{N}\text{x}\bar{N}\text{x} \ldots \bar{N}]_n \times R$ of SU(N). It follows that a given SU(N) representation (irreducible or not) can be parity preserving if and only if it is self-conjugate. This implies that in a given supermultiplet one can get states of both chiralities only if the corresponding SU(N) representation is self-conjugate. An irreducible CTP self-conjugate supermultiplet is never SU(N) self-conjugate so it cannot be invariant under parity because it will contain some states of fixed chirality. However one can add several massless supermultiplets in order to obtain only SU(N) self-conjugate representations and therefore states with both chiralities. This is for instance what happens if we decompose a massive representation of N-extended supersymmetry with respect to states of given helicities. We obtain in this way a set of massless (CPT self-conjugate) multiplets which are clasified according to SU(N) representations coming from the reduction $Sp(2N) \supset SU(N)$.

We can give an explicit representation of the Sp(2N) and of SU(N) generators for massive and massless supermultiplets if we consider the enveloping algebras of the stability subalgebras given by (7) - (11).

Consider first $M \neq 0$, then the following matrices give a realization of the N(2N+1) dimensional compact Lie algebra

$$S^{ij}_{\alpha} = Q^{i}_{\alpha}\, \varepsilon^{\alpha\beta}\, Q^{j}_{\beta} \tag{18}$$

$$A^{ij} = \frac{1}{2}\,(Q^{i}_{\alpha}\delta^{\alpha\dot{\alpha}}\,\bar{Q}^{j}_{\dot{\alpha}} - \bar{Q}^{j}_{\dot{\alpha}}\delta^{\dot{\alpha}\alpha}\,\bar{Q}^{i}_{\alpha}) \tag{19}$$

Any Sp(2N) generator can be written in the form

$$\Lambda = \begin{pmatrix} iA^{ij} & S^{ij} \\ S^{*ij} & -iA^{ij} \end{pmatrix} \qquad \begin{array}{l} S^{ij} \text{ complex symmetric NxN matrix} \\ A^{ij} \text{ hermitian NxN matrix} \end{array} \tag{20}$$

(20) is in fact the most general form of an element which belongs to the SU(2N) algebra and which satisfies the symplectic condition $\Lambda^{T}\Omega + \Omega\Lambda = 0$, $\Lambda \subset SU(2N)$, $\Omega = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$.

For massless representations $S^{ij} = 0$, because $Q^{i}_{1} = 0$ and we have only the U(N) subalgebra of Sp(2N). The SO(N) subalgebra of U(N) is given by

$$\frac{i}{2}\,(Q^{i}_{2}\,\bar{Q}^{j}_{2} - Q^{j}_{2}\,\bar{Q}^{i}_{2})\,. \tag{21}$$

The matrices S^{ij}, A^{ij} satisfy the following set of commutation relations

$$\begin{aligned} \left[S^{ij}, S^{*lm}\right] &= A^{im}\delta^{jl} + A^{jm}\delta^{il} - A^{jl}\delta^{im} - A^{il}\delta^{jm} \\ \left[S^{ij}, S^{lm}\right] &= 0 \\ \left[A^{lm}, S^{ij}\right] &= S^{lj}\delta^{im} - S^{li}\delta^{jm} \\ \left[A^{ij}, A^{lm}\right] &= \delta^{jl}A^{im} - \delta^{im}A^{lj}\,. \end{aligned} \tag{22}$$

From the previous considerations we obtain the following general result : in N-extended supersymmetry massless representations contain a set of particle states whose helicity must reach at least $\frac{N}{4}\left(\frac{N+1}{4}\right)$. It follows that scalar spinor multiplets can exist up to N = 2 and scalar-spinor-vector multiplets up to N = 4.

Perturbatively renormalizable QFT limits the spin content of fields to 0, 1/2 and 1. We conclude that these theories can tolerate at most N = 4 extended supersymmetry. However if we allow helicity 2 states, suitable to describe the graviton, as it happens in supergravity theory, we have much larger supermultiplets. Supergravities can tolerate at most N = 8 extended supersymmetry. We also note that the maximally extended theories, which occur at N = 4 for Yang-Mills interactions and at N = 8 for gravitational interactions correspond to PCT self-conjugate particle supermultiplets.

In Tables I and II the list of all massless multiplets which correspond to maximal helicities $\lambda = \pm 1$ and $\lambda = \pm 2$ are reported.

TABLE I - Particle spectrum of extended Yang-Mills theories.

N \ λ	1	$\frac{1}{2}$	0
1	1	1	
2	1	2	$2 = 1 \oplus 1$
3	1	$3 \oplus 1$	$3 \oplus 3$
4	1	4	$3 \otimes 1 \oplus 1 \otimes 3$

The gauge group G is arbitrary and commutes with the supersymmetry generators.

TABLE II - Particle spectrum of extended supergravity.

N \ λ	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
1	1	1			
2	1	2	1		
3	1	3	3	1	
4	1	4	$3 \otimes 1 \oplus 1 \otimes 3$	4	$2 = 1 \oplus 1$
5	1	5	10	$10 \oplus 1$	$5 \oplus 5$
6	1	6	$15 \oplus 1$	$20 \oplus 6$	$15 \oplus 15$
7	1	$7 \oplus 1$	$21 \oplus 7$	$35 \oplus 21$	$35 \oplus 35$
8	1	8	28	56	$35 \oplus 35$

From Table II we see the remarkable fact that the (± 1) helicity states embeded in the gravitational multiplet occur in the adjoint representation of SO(N) as a consequence of the fact that the maximal helicity state (± 2) is a singlet. This circumstance suggests that, using the supermultiplet of the graviton, one can unify gravity with an SO(N) Yang-Mills theory whose gauge fields just correspond to the ± 1 helicity partners of the graviton. This possibility has been shown to occur (D. Z. Freedman et al., 1977, 1978 ; E. Fradkin et al., 1977) in extended supergravities up to N = 4 by explicit construction of the Lagrangian. For $N > 4$ the Lagrangians with an SO(N) gauge coupling have not been constructed so far and it remains to be seen whether this is possible. As far as the representation content is concerned exceptional cases are $N = 6$ and $N = 7$ extended supergravities. In these cases the ± 1 helicity states belong to reducible representations of SO(N). For $N = 6$ the adjoint + a singlet $(15+1)$, for $N = 7$ to the adjoint + the fundamental $(21+7)$ representations respectively. In these special cases one expect the symmetries to be enlarged to $SO(6) \times SO(2)$ and SO(8) respectively.

The unification picture which comes out by gauging SO(N) is not satisfactory with the present understanding of "low energy" particle phenomenology (M. Gell-Mann, 1977). If we want to identify the (± 1), $(\pm 1/2)$ and 0 helicity partners of the gravitons with the fundamental particles of a unified theory of electromagnetic, weak and strong interactions we must recover at least $SU(3)_{COLOR} \times (SU(2) \times U(1))_{ELECTROWEAK}$ but this is not a subgroup of SO(8), the orthogonal symmetry group of the maximally extended supergravity theory. Also if we confine ourselves to the unbroken gauge group $SU(3)_{COLOR} \otimes U(1)_{em} \subset SO(8)$ many light particles (like the muon) are missing in the fundamental gauge multiplet (M. Gell-Mann, 1977).

We now consider again massive representations. We have seen that the fundamental representation, in absence of central charges, has dimension 2^{2N}. The spin content runs from 0 up to N/2, therefore in N-extended supersymmetry with massive multiplets without central charges, one must have at least particle states with spin N/2.

Massive representations can be smaller in presence of central charges (R. Haag et al., 1975 ; P. Fayet, 1979). We give few examples : In $N = 2n$ extended supersymmetry, if one central charge does not vanish, one can get massive multiplets of dimension 2^{2n+1} (instead of 2^{4n}), whose maximum spin is n/2 (instead of n). These 2^{2n+1} states are a doublet of massive representations of n-extended supersymmetry without central charges. The internal symmetry breaks from Sp(4n) down to $Z \times Sp(2n)$. Z is the central charge operator which rotates the two real multiplets of maximum spin $S = n/2$. In Tables II and III we report some massive representations of extended supersymmetry without and with central charge respectively.

TABLE III - Some massive representations of N-extended supersymmetry (without central charges).

N \ J	5/2	2	3/2	1	1/2	0
1					1	2
				1	2	1
			1	2	1	
		1	2	1		
2				1	4	5
			1	4	5 ⊕ 1	4
		1	4	5 ⊕ 1	4	1
3			1	6	14	14
		1	6	14 ⊕ 1	14 ⊕ 6	14
4		1	8	27	48	42
5	1	10	44	110	165	132

TABLE IV - Some massive representations of N-extended supersymmetry (with central charges). Complex representations.

N \ J	3/2	1	1/2	0
2			1	2
		1	2	1
	1	2	1	
4		1	4	5
	1	4	5 ⊕ 1	4
6	1	6	14	14

The central charge acts as a phase transformation on these complex representations.

Massive representations of extended supersymmetry with non vanishing central charges occur in the N = 8 broken supergravity model through dimensional reduction (J. Scherk et al., 1979 ; E. Cremmer et al., 1979a). From this model one gets massive multiplets with central charge for N = 6, 4, 2 extended supersymmetry (S. Ferrára et al., 1979a). Other examples for N = 4, 2 are given by Yang-Mills theories with a spontaneous breakdown of the Yang-Mills group (P. Fayet, 1979).

III. - SPONTANEOUS SUPERSYMMETRY BREAKING : THE SUPERHIGGS EFFECT.

The basic feature of supersymmetric field theories lies in the possibility of having interacting multiplets unifying particles of different spin and internal quantum numbers. This remarkable property has its counterpart in the fact that all particles in a given multiplet are degenerate in mass. This is a bad property because such a degeneracy has not been observed in Nature. If supersymmetry is relevant for the physical world it must have two properties :

a) It must be a broken symmetry ;
b) It must be realized as a gauge symmetry.

The symmetry breaking can be either spontaneous or explicit but renormalizability and simplicity arguments favor a spontaneous symmetry breaking. In fact non supersymmetric interactions would probably spoil good renormalizability properties and lose all predictive power of the symmetry. The requirement b) is due to the particular role which gravity plays as gauge theory of the Poincaré group. The supersymmetry algebra given by (1), (2), (3) implies that if we gauge the Poincaré group then also supersymmetry must be gauged. In fact gravity + global supersymmetry imply local supersymmetry and viceversa local supersymmetry implies gravity. From the previous arguments we can conclude that only spontaneously broken realization of supergravity may have the chance of being candidates for the superunification of the fundamental interactions.

When supersymmetry is spontaneously broken, particles arise in the spectrum of the theory which are massless and carry the same quantum numbers of the broken generators Q^i_α. The signal for spontaneous breakdown of supersymmetry is a non linear term in the transformation law of some spin 1/2 fields (R. V. Volkov et al., 1973a ; J. Iliopoulos et al., 1974 ; A. Salam et al., 1974b ; P. Fayet et al., 1974 ; L. O'Raifeartaigh, 1975)

$$\delta\lambda^i_\alpha(x) = a\,\epsilon^i_\alpha + \text{q-number terms} . \qquad (23)$$

The parameter a which has the dimension of (mass)^2 is related to the size of spontaneous symmetry breaking. In model field theories the parameter a is proportional to the vacuum expectation value of some auxiliary field H of the basic field representation of the supersymmetry algebra

$$\langle 0|H|0\rangle \sim a . \qquad (24)$$

The spin -1/2 particle fields which transform as in (23) are the Goldston fermions (Goldstinos) of the theory. These massless particles can be identified with new types of neutrinos or are eventually eaten up by the spin 3/2 gauge particles (gravitinos) of local supersymmetry. It is evident from (23) that if $\epsilon^i_\alpha(x)$ is space-time dependent we can perform a supersymmetry transformation such that $\lambda^i_\alpha(x) = 0$. The lower $\pm 1/2$ helicity states of the massive gravitinos are nothing but the would be Goldstone fermions of spontaneously broken supersymmetry. This mass generation for spin 3/2 particles is called the Super-Higgs mechanism (D. V. Volkov et al., 1973b ; S. Deser et al., 1977) and is entirely analogous to the more conventional Higgs-Kibble mechanism for vector particles in spontaneously broken Yang-Mills theories. Because Rarita-Schwinger fields can have consistent interactions only through gravitons we encounter the remarkable situation that the Higgs mechanism for spin 3/2 particles can only occur in curved space-time. Due to this fact it is important to have the possibility that the spontaneous breakdown does not induce a cosmological constant which would affect the particle interpretation of the physical states It will turn out that the contraint of zero cosmological term in broken supergravity is a quite strong one and it will have additional consequences on the properties of the broken theory.

If the gravitino is a real particle one may wonder which is the magnitude of its mass. This is controversial. If spontaneous supersymmetry breaking takes place in the energy range of hadron interactions then the mass of the gravitino is very small (S. Deser et al. 1977)

$$m_\psi \sim K m^2_{PROTON} \sim 10^{-19}\ \mathrm{GeV} \tag{25}$$

This is also the case in the supersymmetric version of electroweak interactions where Fayet (1977, 1980) found

$$m_\psi \sim \frac{KM_W^2}{e} \sim \frac{Ke}{G_F} \sim 10^{-5} - 10^{-6}\ \mathrm{eV}\ . \tag{26}$$

A supersymmetry breakdown at scales of electroweak interactions would imply the existence of new families of hadrons (R-hadrons) associated to a new (R) quantum number in the GeV-mass range. More importantly gravitational amplitudes involving massive gravitinos would be of strength comparable to electroweak amplitudes involving the would be Goldstone fermions and they could give dectable effects at moderate energies.

The alternative picture is that supersymmetry breaking occurs at the Planck scale. This would in particular explain why there is no trace of a supersymmetry pattern at present energies. In this case exact supersymmetry would manifest at the superunification scale and in the gauge hierarchy it would be prior to the grand unification scale which is believed to be in the 10^{14} - 10^{15} GeV range. In such a situation the gravitino, if not confined, would have conceivably a mass $m_\psi \gtrsim 10^{19}$ GeV. An intermediate possibility, which may occur in extended supergravity, is that the different supersymmetry generators Q^i_α are broken at different scales. In particular some of them at superhigh energies ($10^{15} \sim 10^{19}$ GeV range) and some other at "low energies" ($\sim 10^2$ GeV).

Recently it has been suggested (E. Cremmer et al., 1978a, 1979b; J. Ellis et al., 1980a) that the elementary fields of N = 8 extended supergravity, with the exception of the graviton, may be preconstituent fields of a superunified theory. In particular the gravitino would be superconfined and would not correspond to any physical state. We will comment on this possibility in the last section of this review.

We now return to the question of spontaneous breaking and superHiggs effect in supergravity. Using a non-linear realization of supersymmetry (D.V. Volkov et al., 1973a), which is suitable to describe Goldstone fermions (D.V. Volkov et al., 1973a; J. Iliopoulos et al., 1974; A. Salam et al., 1974b; P. Fayet et al., 1974; L. O'Raifeartaigh, 1975), it has been shown, to lowest order in the gravitational constant K, that the spin 3/2 gravitino can become massive with vanishing cosmological term. Moreover the gravitino mass m_ψ is related to the symmetry breaking parameter a (see eq. (23)) by the following universal relation (S. Deser et al., 1977)

$$m_\psi = \frac{1}{\sqrt{6}} Ka \tag{27}$$

In the class of models considered in (P. Fayet, 1977, 1980) one has

$$a = \frac{\Delta m^2}{e_G} \tag{28}$$

where Δm^2 is the fermion-boson (mass)^2 splitting of a given multiplet and e_G is the Yukawa or vector coupling of the Goldstone spinor λ.

The superHiggs effect and the general validity of eq. (27) has been explicitely exhibited (E. Cremmer et al. ,1978b, 1979c) to all order in K in the minimal N=1 supergravity model which describes the interaction of the gauge multiplet of helicity content (± 2, $\pm 3/2$) with the (super) Higgs multiplet of spin content ($1/2$, 0^+, 0^-). The most general interaction is described by a real function of two variables ($A(0^+)$, $B(0^-)$). If a canonical energy term for the scalar fields is demanded, then the degree of arbitrariness is reduced to a function of the complex variable $A+iB$. After spontaneous breakdown of supersymmetry and superHiggs effect (without induced cosmological constant) the spectrum consists of a massless graviton (± 2 helicity states), a massive gravitino and two massive superHiggs particles. Moreover the following general mass formula holds

$$4m_{\psi}^2 = m_A^2 + m_B^2 \tag{29}$$

It is interesting to note that eq. (29) ensures that the one-loop induced cosmological term by radiative corrections is at most logarithmically divergent, in constrast with non supersymmetric field theories where it is quantically divergent. We recall that the one-loop induced cosmological term for a particle of spin J and mass M_J is given, according to B. S. De Witt (1965)

$$G = - \frac{(-)^{2J}(2J+1)}{8\pi^2} \left[\frac{1}{\sigma^2} - \frac{M_J^2}{2\sigma} - \frac{M_J^4}{4} \left(\frac{1}{2} \lg \left| 2M_J^2 \sigma \right| + \gamma - \lg 2 - \frac{5}{4} \right) \right] \tag{30}$$

where $\Lambda = 1/\sigma$ is an ultraviolet cutoff. The previous mass formula (eq. (29)) is a particular case of a more general mass formula which holds in a large class of supersymmetric theories (S. Ferrara et al., 1979b)

$$\sum_J (-)^{2J} (2J+1) \, M_J^2 \, d_J \quad 0 \tag{31}$$

d_J is the degeneracy of the state of mass M_J and spin J. Eq. (31) is the statement that the graded trace of the square mass matrix vanishes

$$\mathrm{Gr\,Tr}\, M^2 = 0 \, . \tag{32}$$

It is interesting to note that in spontaneously broken (renormalizable) globally supersymmetric models (eq. (31)) has been shown to be preserved by radiative corrections up to finite terms of second order in the symmetry breaking parameter (L. Girardello et al., 1979).

The question of an induced cosmological constant is a standard problem in ordinary gauge theories and especially in Grand Unified Theories. These theories would predict a huge cosmological constant in contrast with experimental evidence. One could cure this by adding to the Lagrangian a compensating cosmological term which however must be tuned with a fantastically high accuracy. To our knowledge supersymmetry is the only principle which implies (B. Zumino, 1975)

$$\langle 0 | T_{\mu\nu}(x) | 0 \rangle = 0 \tag{33}$$

provided supersymmetry is unbroken. The very simple explanation of (33) is that $T_{\mu\nu}(x)$ belongs to a supermultiplet which contains the supersymmetry current $J^i_{\mu\alpha}(x)$ and under a supersymmetry transformations

$$\delta J^i_{\mu\alpha}(x) \sim \delta^{ij} (\bar{\varepsilon} \gamma^{\nu})_{\alpha} T_{\mu\nu}(x) + \text{q-number terms} \tag{34}$$

by taking the V.E.V. of (34) and using the fact that $Q^i_\alpha|0\rangle = 0$ one gets (33). It is more important that eq. (33) is preserved by the spontaneous breakdown of supersymmetry. This is precisely what happens in the "minimal" model previously considered with one superHiggs multiplet. It must be pointed out that the vanishing of the cosmological term in a spontaneously broken theory is not automatic but one has to select some potential which has this property. The mass relations (29) and (31) are automatically fulfilled once this choise has ben done.

The superHiggs mechanism considered in this section is the simplest case of spontaneously broken supergravity model. This mechanism can be probably enlarged to more complicated (and interesting) situations like $N = 1$ supergravity with Yang-Mills multiplets or $N = 2$, $N = 4$ supergravity coupled to matter multiplets. Work along these lines has not been done yet but is strongly recommended.

IV. - ALTERNATIVE WAYS OF SPONTANEOUS BREAKDOWN IN SUPERGRAVITY.

The spontaneous supersymmetry breaking and superHiggs effect discussed in the previous section is the natural generalization of the standard Higgs mechanism of spontaneously broken Yang-Mills theories. It takes place because the scalar potential has minima which correspond to non-supersymmetric solutions of the equations of motion. The field which takes a non-vanishing expectation value is one of the auxiliary fields of the theory and it provides the symmetry breaking parameter which is in turn related to the gravitino mass.

Other two types of spontaneous supersymmetry breaking can occur in extended supergravity models. The first one uses generalized dimensional reduction to get masses in lower dimensions (J. Scherk et al., 1979). The second one (J. Ellis, 1980a) advocates a dynamical mechanism responsable for the supersymmetry breakdown and it is based on the assumption that (extended) supergravity theories make sense as quantum field theories. The latter is a bold hypothesis because supergravity theories fall in the class of (power-counting) non-renormalizable field theories.

Let us first consider the case of supersymmetry breaking through dimensional reduction (J. Scherk et al., 1979). This method uses the fact that if a Lagrangian has a global compact symmetry G in D+E dimensions, this symmetry can be used to obtain masses in D dimensions by requiring that the D+E dimensional fields depend on the extra E coordinates through an element of the group G. If the global symmetry is non compact then G is a maximal compact subgroup. If the theory has general covariance in D+E dimensions, the dimensionally reduced theory (in D dimensions) has a mass matrix which depends at least on a number of parameters which equal the rank of $G \times SO(E)$, $SO(E)$ being the maximal compact subgroup of $SL(E,R)$. Dimensional reduction from D+E to D dimensions in a theory invariant under general coordinate transformations induces in addition a minimal (broken) gauge symmetry whose global Lie algebra has dimension E. The E vector fields gauging this internal symmetry are the vector fields coming from the gravitational field in D+E dimensions. In ordinary dimensional reduction (without dependence of fields on the extra E coordinates) this gauge group is unbroken and reduces to $U(1)^E$. In the case of generalized dimensional reduction this group is non trivial and if one demands that no cosmological term is induced in the 4-dimensional theory, one obtains a "flat group" according to Scherk and Schwarz (1979). We observe that this group always contains an unbroken U(1) subgroup whose gauge field has been called by Scherk (1979) graviphoton. In extended supergravity the flat group has dimension higher than E because the theory has additional (abelian) gauge symmetries in D+E dimensions. For instance in $D = 4$, $N = 8$ spontaneously broken supergravity the flat group has dimensions 28 and in lower N-extended models it has dimension $N(N-1)/2$. Spontaneously broken $N = 8$ supergravity in four-dimensions can be obtained through generalized dimensional reduction of $N = 8$ extended supergravity in $D = 5$ dimensions (E. Cremmer et al., 1979a).

In five dimensions the supersymmetry algebra has a global Sp(N) symmetry (E. Cremmer et al., 1979a)

$$\left\{ \bar{Q}^{a}_{\alpha}, Q^{b}_{\beta} \right\} = \Omega^{ab} \gamma^{\mu}_{\beta\alpha} P_{\mu} \qquad \begin{array}{r} a,b = 1 \ldots N \\ \alpha,\beta = 1 \ldots 4 \\ \mu = 0 \ldots 4 \end{array} \tag{35}$$

N has to be even because the charges satisfy a generalized Majorana condition

$$\begin{aligned} & Q^{a}_{\alpha} = C\bar{Q}^{Ta}_{\alpha} , \quad C = \gamma_{o}\gamma_{5} , \quad \bar{Q}^{a}_{\alpha} = (Q^{+}_{a}\gamma_{o})_{\alpha} , \\ & Q_{a} = \Omega_{ab} Q^{b} , \quad \Omega_{ab} = - \Omega_{ba} . \end{aligned} \tag{36}$$

Massless five-dimensional multiplets of $N = 8$ supersymmetry have the same spin content of massive four-dimensional multiplets of $N = 4$ supersymmetry (without central charges). This is because the little group (SO(3)) of a massless particle of given momentum in $D = 5$ dimensions is the same as the little group of a massive particle in $D = 4$ dimensions.

The particle fields of the supergravity multiplet occur in antisymmetric traceless representations of Sp(N). For $N = 8$ we have a singlet graviton, an octet of spin 3/2, a 27-plet of spin 1, a 48-plet of spin 1/2 and a 42-plet of spin 0.

$$e_{\mu a}, \quad \psi^{a}_{\mu\alpha}, \quad A^{ab}_{\mu}, \quad \chi^{abc}_{\alpha}, \quad \Phi^{abcd} . \tag{37}$$

In the full non linear theory this Sp(8) is the diagonal subgroup of $Sp(8)_{global} \otimes Sp(8)_{local} \subset E_{6\,global} \otimes Sp(8)_{local}$, the last being the full group of invariance of the five-dimensional Lagrangian.

The mass matrix of the spontaneously broken four-dimensional theory is obtained by introducing x_5-coordinate dependence on the five-dimensional fields in the form (J. Scherk et al., 1979; E. Cremmer et al., 1979a)

$$\Phi_{J}(x, x_5) = \exp(i\, \mathcal{M}_{J} x_5)\, \Phi_{J}(x) \tag{38}$$

$\mathcal{M}_J$ being a representative of the Cartan subalgebra of Sp(8) in the representation appropriate to the particle field of spin J. The generic element can be written as follows

$$\mathcal{M} = \sum_{k=1}^{4} m_k \lambda_k \tag{39}$$

where λ_k is a basis in the (4-dimensional) Sp(8) Cartan subalgebra. The mass spectrum of the broken theory depends therefore on four arbitrary (real) parameters. They equal the rank of the global symmetry group $G = Sp(8)$. The mass spectrum is given in Table V. We list some properties of the $N = 8$ broken theory:

a) The mass spectrum satisfies three mass relations (for all values of m_k)

$$\sum_{J} (-)^{2J} (2J+1) M^{2r}_{J} = Gr\, Tr\, \mathcal{M}^{2r} = 0 \qquad r = 0,1,2,3 \tag{40}$$

In general the number of these mass relations depend on the rank of G. These mass relations are an extension, for $r > 1$, of the mass formula (31).

b) The theory has an unbroken U(1) gauge group whose gauge field is the (4μ) component of the five dimensional graviton. This field (graviphoton) is minimally coupled to all massive particles with charge coupling

$$Q_i = \pm 2 M_i K \qquad (41)$$

c) If n parameters out of the m_k, s vanish the theory has an unbroken N = 2n local supersymmetry and massive multiplets have non vanishing central charge (S. Ferrara et al., 1979a). These multiplets contain charged particles which lie in representations of Sp(2n).

d) The introduction of the parameter m_k in the broken theory breaks the SO(8) global symmetry of the N = 8 unbroken theory down to SO(2) $\sim$ U(1). However if h of the m_k, s ($h \leq 4$) are equal, the theory has an additional SU(h) global symmetry and the 28-dimensional flat group contains at least an unbroken $U(I)^{h^2}$ factor.

TABLE V - Mass spectrum of the broken N = 8 theory.

Spin	Mass	Degeneracy	Number of states
2	0	1	2
$\frac{3}{2}$	$\lvert m_i\rvert$	2	32
1	0	4	8
	$\lvert m_i \pm m_j\rvert \quad i<j$	2	72
$\frac{1}{2}$	$\lvert m_i\rvert$	4	32
	$\lvert m_i \pm m_j \pm m_k\rvert \quad i<j<k$	2	64
0	0	6	6
	$\lvert m_i \pm m_j\rvert \quad i<j$	2	24
	$\lvert m_1 \pm m_2 \pm m_3 \pm m_4\rvert$	2	16

We now make some comments on the properties listed above. Property a) ensures that, according to (30) the one-loop induced cosmological term is finite for N = 8 and N = =6 broken supergravity. The vector field minimally coupled to massive particles is the gauge field of the central charge operator which appears in the supersymmetry algebra for N = 2, 4 and 6. This explains eq. (41). The property alluded in c) and the symplectic structure of $\mathcal{M}$ are responsable for the mass relations given in a). The quantity $\mathrm{Gr\,Tr}\,\mathcal{M}^{2r}$ is in fact an homogeneous symmetric polynomial of order 2r in the m_k variables. $\mathrm{Gr\,Tr}\,\mathcal{M}^{2r} = 0$ for all r if at least one of the m_k, s vanishes. Then it follows that $\mathrm{Gr\,Tr}\,\mathcal{M}^{2r} = 0$ identically for $r = 0 \ldots n-1$ for an Sp(2n) group. As a byproduct any truncation of N = 8 supergravity with 2n < 8 broken supersymmetries will fulfill n-1 mass relations.

In the final part of this report we would like to mention the possibility of a dynamical spontaneous symmetry breaking in extended supergravity. This situation has been recently envisaged by Ellis, Gaillard, Maiani and Zumino (1980a, b) following a previous result obtained by Cremmer and Julia (1978a; 1979b; 1980) for the unbroken N-extended supergravity models.

Cremmer and Julia have pointed out that in N-extended supergravities there are hidden internal symmetries which do not commute with the supersymmetry charges. These symmetries are always of the form $G_{global} \otimes G_{local}$ where G_{global} is non compact and G_{local} is compact. Moreover $G_{global} > G = G_{local}$ as its maximal compact subgroup. For example in $N = 8$ supergravity in $D = 5$ dimensions $G_{global} = E_6$, $G_{local} = Sp(8)$. In $N = 8$ supergravity in $D = 4$ dimensions $G_{global} = E_7$, $G_{local} = SU(8)$. In particular the scalar fields always belong to the coset space G_{global}/G. Note that for $N < 4$ $G_{global} = G$. In $N = 8$, $D = 4$ supergravity the scalar fields can be represented by a 56×56 matrix, a group element of E_7, of the form

$$V = \begin{pmatrix} U_{[AB]}{}^{[MN]} & V_{[AB]\,[MN]} \\ \bar{V}^{[AB]\,[MN]} & \bar{U}^{[AB]}{}_{[MN]} \end{pmatrix} \tag{42}$$

where square brackets mean antisymmetrization of indices and $(A,B) = 1\ldots8$, $(M,N) = 1\ldots8$ transform under $SU(8)$ and E_7 respectively. From (42) one can construct an E_7 invariant composite vector field

$$\partial_\mu V V^{-1} = \begin{pmatrix} 2Q_{\mu}{}_{[A}^{[G}\,\delta_{B]}^{D]} & P_{\mu\,[ABCD]} \\ \bar{P}_{\mu}^{[ABCD]} & 2\bar{Q}_{\mu}{}_{[A}^{[C}\,\delta_{B]}^{D]} \end{pmatrix} \tag{43}$$

where $Q_{\mu A}{}^{B}$ and $P_{\mu [ABCD]}$ transform according to the self-conjugate (adjoint) 63 and 70 dimensional representations of $SU(8)$. $Q_{\mu A}{}^{B}$ is a connection for $SU(8)$ so that the Lagrangian for the scalar fields can be written

$$(D_\mu V V^{-1})^2 \sim P_{\mu\,[ABCD]}\,\bar{P}^{\mu [ABCD]} \tag{44}$$

and contains only 70 degrees of freedom. From (44) it is evident that the scalar sector of the theory has a σ-model structure and the theory has an $SU(8)$ local invariance. This is analogous to the CP^{N-1} models in $D = 2$ dimensions when $G_{global} = SU(N)$ and $G_{local} = SU(N-1) \otimes U(1)$.

Cremmer and Julia further conjectured in analogy to CP^{N-1} models that the composite operator $Q_{\mu A}{}^{B}$ may become dynamical because of quantum effects. In CP^{N-1} models this indeed happens (A. D'Adda et al., 1978; 1979; E. Witten, 1979) as a non-perturbative phenomenon and can be studied using the $1/N$ expansion thanks to the arbitrariness of the $SU(N)$ symmetry of these models. Moreover the renormalizability of CP^{N-1} models is crucial. In extended supergravity none of these two points are fulfilled but one may take the optimistic point of view that $N = 8$ supergravity makes sense as a quantum theory and then one might explore the consequences of such an hypothesis.

In a supersymmetric field theory the $SU(8)$ gauge connection $Q_{\mu A}{}^{B}$ must belong to some $N = 8$ supermultiplet. In particular if $Q_{\mu A}{}^{B}$ becomes a propagating field and its propagator develops a pole at zero mass one should get quantum excitations which fall in a $N = 8$ massless supermultiplet which contains helicity ± 1 states in the adjoint representation of $SU(8)$. This is under the assumption that the quantum effect responsable for these new particle states is prior to supersymmetry breaking. Ellis et al. (1980a; 1980b) conjecture that this composite supermultiplet contains the standard gauge fields of low energy physics, the photon, colored gluons, the $W^{\pm}$, Z vector bosons of weak interactions as well as the gauge fields of the GUT and additional gauge fields associated with the generation group. They also conjecture that the same supermultiplet must contain all leptons and quarks of electroweak and strong interactions.

The fundamental multiplet of the N = 8 supergravity Lagrangian is assumed to be a multiplet of preconstituent fields (preons) with the exception of the ± 2 helicity states which corresponds to the graviton. Therefore the interpretation of the basic multiplet of extended supergravity is entirely different from the models considered in the previous section and the spin 3/2 gravitinos as well as the other elementary fields are supposed to be superconfined at least for energies up to the Planck mass 10^{19} GeV.

The composite CPT self-conjugate multiplet which contains the adjoint representation of SU(8) is assumed to be the following (J. Ellis, 1980a, 1980b).

Helicity	$\mp\frac{5}{2}$	∓ 2	$\mp\frac{3}{2}$	∓ 1	$\mp\frac{1}{2}$	0	
SU(8)	8	28	$\bar{8}$	63	8	28	
content		36	56	1	216	420	(45)
			168	70	$\overline{56}$	$\overline{28}$	
				378	$\overline{504}$	$\overline{420}$	

Ellis et al. (1980b) show that extended supergravities with $N \leq 6$ have not sufficient states in an analogous supermultiplet to accomodate the observed spin-1/2 spectrum of fermions, namely three $(10+\bar{5})$ SU(5) representations of left-handed fermions. They focus the attention on the spin-1/2 and SU(5) content of (45) and neglect the SU(5) representations which are not contained in the SU(8) representations of the type $\bar{8} \times [8 \times \ldots 8]_N - [8 \times \ldots 8]_{N-1}$. Under these circumstances these authors conclude that the maximal unbroken subgroup of SU(8) below 10^{19} GeV is SU(5) and that the maximal anomaly free renormalizable gauge theory which can be constructed out of the multiplet given by (45) contains exactly three families of $(10+\bar{5})$ SU(5) representations for left-handed spin-1/2 particles plus a set of self-conjugate representations

$$(45+\overline{45})_L + 4(24)_L + 9(10+\overline{10})_L + 3(5+\bar{5})_L + 9(1)_L \tag{46}$$

which may acquire a big SU(5) invariant mass. The other unwanted states in the supermultiplet given by (45) are supposed to have became massive with masses $\gtrsim 10^{19}$ GeV through an as yet unspecified dynamical symmetry breaking and to be decoupled from the low energy (10^{15} GeV) renormalizable theory (GUT). This last argument (Veltman theorem) is required because the full SU(8) supermultiplet would have anomalies both at the composite and at the preon level. Moreover many unwanted high spin states ($J \geq 1$) could not become massive through conventional symmetry breaking due to the absence in (45) of the required helicity partners with the same properties under the exact gauge group $SU(3)_{color} \otimes U(1)_{e.m.}$.

It is interesting to further remark that the requirement of chirality (left-handed spin-1/2 states without their right-handed partners) and a vector-like subgroup $SU(3)_{color} \otimes U(1)_{e.m.}$ of the bigger group SU(8) almost uniquely favor the anomaly free subset of spin-1/2 particles given by $3(\bar{5}+10)$ plus the self-conjugate states given by (46). If one considers for instance a subset of (45) without SU(6) (instead of SU(5)) anomalies and a vector like subgroup $SU(3)_{color} \otimes U(1)_{e.m.}$ then the only solutions are completely vector like. Another possibility would be to go to the low energy theory through a sequential breaking which does not contain SU(5), for example:

$$SU(8) \supset Sp(8) \supset SU(3) \otimes SU(2) \otimes U(1) .$$

The Sp(8) group is of same interest in supergravity because it plays a role in attempts to formulate this theory in a consistent way off mass-shell (E. Cremmer et al., 1980). Unfortunately this decomposition is completely vector-like and it would also imply a wrong charge assignement for quarks and leptons.

Some possible alternatives can be envisaged in the framework considered by Ellis et al. One possibility is to look for anomaly free subset of (45) without disregarding the SU(8) trace-representations contained in $\bar{8} \times [8 \times \ldots 8]_N$. Another possibility is to try to restore conventional symmetry breaking by considering more supermultiplets other than (45). It is evident that if one add sufficiently many massless supermultiplets to (45) one can give in fact a (supersymmetric invariant) mass to every state by enlarging SU(8) to Sp(16) (see section II). This is too much but one could contemplate intermediate situations and it is to be seen if one can give masses to the unwanted (would be) massless states in a way which is still consistent with the constraints imposed by "low energy" phenomenology.

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