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A.F. Grillo: PRIMORDIAL BLACK HOLES AND BARYON PRODUCTION IN GRAND UNIFIED THEORIES.

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A. F. Grillo: PRIMORDIAL BLACK HOLES AND BARYON PRODUCTION IN GRAND UNIFIED THEORIES.

ABSTRACT: It is suggested that baryon production through primordial black holes in Grand Unified Theories is not likely to generate an important number of baryons.

Grand Unified Interactions (GUT's) of particle interactions[1,2] allow in principle to discuss the evolution of the very early universe up to the threshold of "classical" cosmology, i.e. the Planck time ($t_p \approx 10^{-43}$ sec) when quantum gravitational effects are likely to be dominant. This is possible because, apart from technical difficulties, quantities relevant to particle dynamics can be in principle computed even at the very high temperature (i.e. $T \sim T_p \sim \hbar/Kt_p \sim 10^{19}$ GeV) characteristic of this epoch[3, 4, 5].

In this framework it is possible to study, for instance, the evolution of density fluctuations that can be related to the actual structure of the Universe[5] and to estimate the ratio of baryons to photons (refs. (6, 7).

As for the last point a possible competing mechanism has been advocated, through the formation of primordial black holes (p. b. h.'s)[8, 9] and their subsequent evaporation[10] into (CP and B violating) radiation, for instance Grand Unified superheavy gauge and Higgs bosons[11,12].

We want to argue here that, in the framework of GUT's, this mechanism is not likely to give an appreciable contribution to the observed baryon to photon ratio of the Universe.

In order to specify our argument, we consider the following scenario for the evolution of the very early Universe[(x)]:

a) $1 \lesssim t \lesssim t_o \approx 10^5$.

The "age" of the Universe is related to its temperature in the standard F.R.W. big-bang cosmology[13] through

$$t \approx \left(\frac{3}{32\pi gB}\right)^{1/2} T^{-2} \approx (44\, T^2)^{-1} \tag{1}$$

where g is the number of elementary particle helicity states ($g \sim 160$ in the minimal SU(5) model) and B appears in the energy density

$$\varrho = BgT^4 \tag{2}$$

$B \simeq 0.36$ at equilibrium.

The mean free time for G.U. interactions is[5]

$$\tau_{GU} \approx (2gA\alpha^2 T)^{-1} \approx \frac{46}{T} \quad . \tag{3}$$

A being defined in the number density

$$n = AgT^3 \; ; \qquad A \simeq \frac{1}{3} B \; . \tag{4}$$

and $\alpha \approx 1/50$ is the G.U. coupling constant.

We see that in this epoch

$$\tau_{GU} > t \tag{5}$$

i.e. G.U. interactions are much slower than the expansion rate of the Universe.

(x) From now on, we will use Planck units $\hbar = k = c = G = 1$ (i.e. $t_p = T_p = 1$).

Down to a temperature $T_{GUT} \sim M_X$, X being the heaviest gauge boson ($M_X \sim 10^{-4}$ in SU(5)) all particles can be regarded as massless[(x)] so that the fluid filling the expanding Universe can be regarded as an ultrarelativistic gas ($p = \frac{1}{3}\varrho$).

It can also be regarded as non interacting, at least for what concerns large scale evolution, due to the slowness of G. U. interactions, which only reflects itself in the presence of viscosity terms in the evolution of fluctuations[(5)].

b) $t > t_o$.

From t_o on, G. U interactions become "strong" (i. e. faster than the expansion rate). Some of the particles acquire a mass (at $T \sim M_X$) so that the above picture no longer holds.

What does the above scenario tell us about p. b. h. production ?

Primordial black holes form in the very early Universe if fluctuations are present such that an overdense region has density contrast of order one within a radius of the order of the Jeans length, so that gravitational potential energy overcomes the kynetic energy and the region begins to collapse[(8)].

If the equation of state is "hard", i. e. $p = \frac{1}{3}\varrho$, then the Jeans and the Schwarzschild radius essentially coincide, so that a collapsing region becomes almost immediately a black hole.

The number distribution of p. b. h. 's can be computed as a function of their mass[(9)].

In the two epochs defined above we have the following situations:

a) $1 < t < t_o$.

Let forget viscosity effects for the moment. The spectrum of p. b. h. 's is then, according to Carr[(9)]

$$n(m) = \frac{3\zeta}{32\pi} m^{-5/2} \tag{6}$$

(x) Note that (at least for SU(5)) $t_o < t_{GUT} = (44\, M_X^2)^{-1}$

since the Universe can be described as a free, ultrarelativistic gas ; ζ is the fraction of matter into p. b. h. 's at t_p (Note that a p. b. h. produced at time t has $m \simeq t$).

b) $t > t_o$.

Here G. U. interactions become strong, and presumably play a rôle in the evolution of fluctuations. The distribution of p. b. h. 's can be taken as[9]

$$n(m) = \frac{3\Phi}{32\pi} m^{-\alpha} \tag{7}$$

with α undetermined ; continuity at t_o requires

$$\Phi \lesssim \zeta m_*^{-3} \tag{8}$$

where m_* is the maximum p. b. h. mass produced in epoch a (m_* 10^5).

What are the observational limits on ζ, Φ, α? Consider first epoch b. Primordial black holes of mass $m \sim 10^{20}$ are exploding right now, essentially into 100 MeV photons. The observed γ-ray background then implies[14]

$$\Phi \lesssim 10^{20\alpha - 75} . \tag{9}$$

In contrast, p. b. h. 's formed in epoch a decay very early ($\tau \sim \sim m^3 < 10^{15} \simeq 10^{-28}$ sec) so that photons coming from them are thermalized. In order not to have too much 3^oK background produced this way, we must have[14]

$$\zeta < 10^{-2} m^{-1} \sim 10^{-7} . \tag{10}$$

If the above scheme is correct, then baryon number production through p. b. h. 's is strongly depressed.

Following the notation of ref. (12) the contribution to the baryon/photon (n_B/n_γ) ratio due to p. b. h. 's of mass between m and m + dm is :

$$d(n_B/n_\gamma) \approx \frac{3}{g^{1/4}} \; \varepsilon N_x \; \zeta \, m^{-5/2} \, dm \qquad m < m_*$$

$$\approx \frac{3}{g^{1/4}} \; \varepsilon N_x \; \Phi \, m^{-\alpha} \, dm \qquad m > m_* \tag{11}$$

assuming that B- and CP-violating decay of X-bosons radiated from p. b. h. 's is responsible for baryon production.

N_x is the number of X bosons emitted in the p. b. h. evaporation ($T_{hole} = (8\pi m)^{-1}$)

$$N_x = \frac{3}{g} m^2 \qquad m < (8\pi M_x)^{-1} = M$$

$$N_x = \frac{3}{g} (8\pi m_x)^{-2} \qquad m > M \tag{12}$$

and ε is the baryon excess per X decay.

Integration then gives

$$\frac{n_B}{n_\gamma} = \frac{9\,\varepsilon}{g^{5/4}} \left\{ \zeta \left[2(M^{1/2} - \mu^{1/2}) - \frac{2}{3} (m_*^{-3/2} - M^{-3/2}) M^2 \right] + \right.$$
$$\left. + \frac{\Phi}{\alpha - 1} M^2 m_*^{1-\alpha} \right\} \tag{13}$$

μ is the either 1 or $g^{-1/6} M_x^{-2/3}$ (12) (We have assumed that $m_* > M$ which need not be the case if X is an Higgs boson - see later).

If X is a gauge boson ($M_x \sim 10^{-4}$, $M \sim 4 \cdot 10^2$), then eq. (13) gives :

$$\frac{n_B}{n_\gamma} \approx 10^{-2} \varepsilon \left\{ 50\, \zeta + \frac{\Phi}{\alpha - 1} \; 5 \cdot 10^{-8} \right\} \approx 0.5\, \varepsilon \zeta \tag{14}$$

requiring $\varepsilon \gtrsim 10^{-2}$ in order to have $n_B/n_\gamma \sim 10^{-9}$. Estimates of ε are in the neighbouring of 10^{-6} (7) so that this possibility seems ruled out.

The situation is slightly different if X is an Higgs boson ($M_x \sim$

$\sim 10^{-6} - 10^{-7}$) so that

$$M \sim 4\,(10^4 - 10^5) \tag{15}$$

The form of eq. (13) is trivially changed if $M > m_*$, without affecting the numerical result which is (for $M_X \sim 10^{-6}$) :

$$\frac{n_B}{n_\gamma} \gtrsim 10^{-2}\,\varepsilon \left\{ 5\cdot 10^2\,\zeta + 5\cdot 10^{-4}\,\frac{\Phi}{\alpha - 1} \right\} \tag{16}$$

implying at best $\varepsilon \gtrsim 10^{-3}$, a value which is only marginally allowed[7].

How the above discussion is changed by viscosity effects ? In lack of a better approximation, we take results from ref. (5) as a basis of discussion.

Compressional modes (which are of interest for p. b. h. production) of frequency ω are damped by G. U. viscosity by a factor (in the amplitude)

$$D(\omega) \approx \exp\left[- \frac{\omega^2}{6T^2}\left(\frac{1}{T} - \frac{1}{T_i}\right)\right] \tag{17}$$

where T_i is some initial temperature. This result holds if $\omega \gg \omega_j$, the Jeans frequency[5].

However, p. b. h. 's are produced when $\omega \sim \omega_j$ so that we are using eq. (17) outside its validity range : the conclusions we'll draw should be considered as highly qualitative[(x)].

This damping can be translated into a function of p. b. h. mass (remember that $m \simeq t$)

$$D(m) \approx \exp\left\{ - 100\,\frac{\sqrt{m} - 1}{m} \right\} \tag{18}$$

where T_i of eq. (17) has been pushed to the Planck time.

Note also that the epoch in which G. U. viscosity is important

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(x) Better approximations for the viscous effects are presently under investigation.

coincides with our epoch a[5].

Since the observational limit on ζ comes from p.b.h.'s produced at the end of this epoch, we expect it to be only slightly affected by damping. Moreover, the effect of viscous damping would be to lower baryon number production.

A few remarks are in order.

In the spirit of order of magnitude estimates, we have considered a sharp transition from epoch a to epoch b, which might not be the case.

For what regards the damping in p.b.h. production, eq.(17) was derived at thermodynamical equilibrium. Near Planck time this cannot be true due to horizon effects[4]; this could reflect itself in a stronger damping for all low masses except the Planck mass. In fact eq.(18) calculated with the cut-off equlibrium parameters given in ref.(4) becomes:

$$D'(m) \approx \exp\left\{ - 800 \frac{\sqrt{m} - 1}{m} \right\} . \tag{19}$$

Finally, we have used a power law behaviour for n(m). It would seem that our description favours a picture in which the p.b.h. spectrum is exponentially decreasing[9].

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