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M. Greco: SOFT GLUON EFFECTS IN QCD PROCESSES.

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ABSTRACT

The effect of emission of soft radiation in hard processes is examined to all orders in QCD. This paper summarizes some recent results obtained in the framework of the coherent states formalism.

I will discuss the effects of the emission of soft radiation to all orders in QCD, for various processes. To this aim a convenient framework is provided by the formalism of coherent states, which has been recently proposed[1,2] to discuss jet phenomena in QCD, and is particularly useful to resum most of the leadind logs one encounters in perturbation theory.

I will briefly review some of the main results found[2] in e^+e^- annihilation, in particular a simple formula for a general jet cross section in terms of an infinite number of gluons and massless quarks, and a probability distribution for the jet transverse momentum which includes correlations induced by momentum conservation.

Then I will discuss the case of deep inelastic scattering and Drell-Yan process, where large infrared corrections have been found[3] to the leading order results.

After the work of Sterman and Weinberg[4] jet phenomena have been considered in great detail. The basic idea is to compute suitably defined jet cross sections which are free of infrared and mass singularities and can be safely compared with experiments.

For the reaction $e\bar{e} \to q_{jet}\,\bar{q}_{jet}$, defining $P(\epsilon, \delta)$ as the probability of finding a fraction $\epsilon \equiv \Delta\omega/Q$ of the total energy Q outside a pair of oppositely directed cones of half angle δ, or equivalently of maximum transverse momentum $k_{\perp max} \simeq Q\delta/2$, one has to compute theoretically the cross section for the emission of all kind of radiation (quarks and gluons, soft and hard, real and virtual) which satisfy the same kinematical bounds.

In the coherent state picture one calculates the inclusive cross section for $e\bar{e} \to \tilde{q}\tilde{\bar{q}}$, where

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$$|\tilde{q},\tilde{\bar{q}}> \equiv |q,\bar{q}> + |q,\bar{q},g> + |q,\bar{q},gg> + |q,\bar{q},(q\bar{q})> + \ldots \qquad (1)$$

and the emitted radiation is either soft ($k \leq \Delta\omega$) or hard and collinear ($k > \Delta\omega$, $k_\perp \leq k_{\perp max}$). Then, all infrared and mass singularities can be shown[(1)] to cancel and one finds[(2)], in the leading logarithmic approximation (L.L.A.)

$$P(\varepsilon, k_{\perp max}) = \exp\{-\frac{1}{\pi^2}\int_{2\varepsilon}^{1-2\varepsilon} dx\, P_{gq}(x) \int_{k_{\perp max}}^{Q/2} \frac{d^2k_\perp}{k_\perp}\, \alpha(k_\perp)\}\,, \qquad (2)$$

where the gluon distribution due to quarks is

$$P_{gq}(x) = C_F \frac{1+(1-x)^2}{x}\,, \qquad (3)$$

$C_F = (N_C^2-1)/2N_C$ for $SU(N_C)$ colour and $\alpha(k_\perp)$ is the running coupling constant $\alpha(k_\perp) = 12\pi/(33-2N_f)\ln(k^2/\Lambda^2)$.

The physical interpretation of eq. (2) is rather transparent. If corresponds to multiple independent gluon emission from the leading $q\bar{q}$ pair, with an effective coupling constant given by the "final state interaction" of gluons (i.e. $g \longrightarrow q\bar{q}$, gg,...). First order expansion of (2) directly gives the result of Sterman and Weinberg, and also includes finite ε terms, proportional to $\ln\delta$, which agree with the calculation of Stevenson[(6)].

Eq. (2) can be simply generalized to gluon jets by replacing $P_{gg}(x)$ with

$$P_{gg}(x) + N_f P_{gg}(x) = N_C\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right] + N_f \frac{x^2+(1-x)^2}{x}\,, \qquad (4)$$

corresponding to the sum of probabilities that a leading gluon radiates gluons or $N_f(q\bar{q})$ pairs. Again eq. (2) so modified agrees with explicit perturbative calculations[(7)].

The experimental investigation of the transverse momentum properties of jets provides a rather clean test of resummation formulae for soft effects in QCD. In $e\bar{e}$, the distribution of the total transverse momentum with respect to the jet axis, which is a particularly simple quantity to measure, is predicted[(2)]

$$\frac{d^2P(\varepsilon)}{d^2K_\perp} = \frac{1}{2\pi}\int_0^\infty x\, dx\, J_0(xK_\perp)\exp\{-\frac{2}{\pi}\int_0^{Q/2}\frac{dk_\perp}{k_\perp} I(\varepsilon)\,\alpha(k_\perp)\,[1 J_0(xk_\perp)]\}\,, \qquad (5)$$

where $I(\varepsilon) = \int_{2\varepsilon}^{1-2\varepsilon} dx\, P_{gq}(x)$. Eq. (5) improves eq. (2) by taking explicitly into account transverse momentum conservation in the multiple gluon emission. Furthermore

$$<K_\perp^2(\varepsilon)> = \frac{1}{\pi}\int_0^{Q/2} I(\varepsilon)\,\alpha(k_\perp)\, dk_\perp^2\,. \qquad (6)$$

Whenever ε is not experimentally determined, namely one sums over all hadrons contained in the jet, then $I(\varepsilon) \simeq C_F \ln(k_\perp^2/Q^2)$, in the L.L.A.[(8)].

The transverse momentum distribution of all charged particles produced in a $q\bar{q}$ jet, measured by the PLUTO collaboration at PETRA at various c.m. energies, is plotted in Fig. 1, against the theoretical expectations given by eq. (5). The normalization is fixed by eq. (6) at each energy, and in both eqs. (5-6) Q/2 is replaced by some upper limit $\bar{k}_\perp$, due to the fact that only charged particles are observed. The agreement is quite impressive. The increase of the average transverse momentum with the c.m. energy is also evident from Fig. 1.

Let us turn now to deep inelastic scattering, in the soft region $x \simeq 1$[(3)]. In that limit it is known that parton densities are also exponentiated, in addition to the usual exponentiation of the moments. In fact one finds[(9)], for the valence densities,

$$q(x,\xi) \quad \frac{\exp\,[(\frac{3}{4} - \gamma_E)C_F\xi]}{\Gamma(C_F\xi)}\,(1-x)^{C_F\xi - 1}\,, \qquad (7)$$

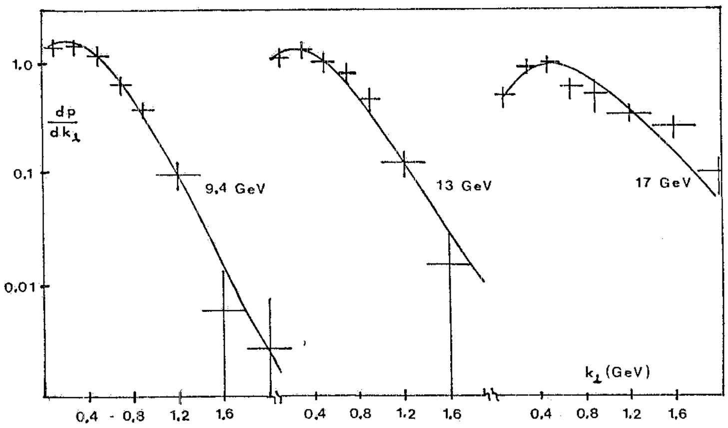

FIG. 1

where

$$\xi(k^2_{\perp max}) = \int_{\lambda^2}^{k^2_{\perp max}} \frac{dk^2_\perp}{k^2_\perp} \frac{\alpha(k_\perp)}{\pi} , \tag{8}$$

and $k^2_{\perp max} \simeq Q^2$. This result is valid for $(\frac{\alpha(Q^2)}{\pi}) \ln(\frac{1}{1-x}) \ll 1$ and $(\alpha(Q^2)/\pi) \ln Q^2 \lesssim 1$. Remarkably it coincides with the energy distribution of the radiation emitted from the valence quark, in the coherent state picture, taking into account the constraint of energy conservation(10)

$$q(x,Q^2) \equiv \frac{dP}{d\omega} = \frac{1}{2\pi} \int_{-\infty}^{\infty} db\, e^{ib\omega} \exp\{ \frac{C_F}{2} \int_0^1 \frac{dz}{z} [1+(1-z)^2]\ \xi(Q^2)\ [e^{-ibz}-1]\}, \tag{9}$$

where $\omega = 1-x$ is the fraction of energy carried out by the soft gluon radiation. More generally the equivalence of the n-th moment of the distribution (9) with the conventional parton density moments for n large, becomes straightforward after rewriting eq. (9) in the form

$$q(x,Q^2) \simeq \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dn\, x^{-n} \exp\{ \frac{C_F}{2} \int_0^1 dy\, (\frac{1+y^2}{1-y}) \xi(Q^2)\ [y^n-1]\} , \tag{10}$$

having approximated $(1-x) \sim -\ln x$ for $x \simeq 1$.

In the very soft region, when x gets so close to 1 that $\ln(1/(1-x))$ is no longer negligible with respect to $\ln Q^2$, the corrections to the leading order result (7) become quite sizeable, as explicitly found in first order perturbation theory(11). This fact casts obvious doubts on the significance of the resummation of the leading logs and the relative importance of higher order corrections.

The solution to this problem has been shown(3) to be directly connected to the appropriate use of exact kinematical constraints in the various processes. More in detail the most important corrections to the leading order results, namely terms proportional to $1/(1-x) \ln(1/(1-x))$ are also found to exponentiate in a simple manner. In the coherent state picture this result can be put in the form

$$q(x,Q^2) \simeq \frac{1}{2\pi} \int_{-\infty}^{\infty} db\, e^{ib(1-x)} \exp\{ C_F \int_0^1 \frac{dz}{1-z} \int_{\lambda^2}^{k^2_{\perp max}=Q^2(1-z)} \frac{dk^2_\perp}{k^2_\perp} \alpha(k_\perp)\ [e^{-ib(1-z)}-1]\} , \tag{11}$$

to be compared with (9). Asit is clear the use of the appropriate kinematical bound in the $k_\perp$ space of the soft gluons takes simply into account the most important next to leading corrections.

The Altarelli-Parisi evolution equation(5) for the non singlet quark density gets also modified in this limit

$$\frac{dq(x,Q^2/Q_o^2)}{d\ln(Q^2/Q_o^2)} = \frac{1}{2\pi}\int_x^1 \frac{dy}{y}\{P(\frac{x}{y})\,\alpha\,[\frac{Q^2}{Q_o^2}(1-\frac{x}{y})]\}_+ q(y,Q^2/Q_o^2), \qquad (12)$$

where Q_o^2 is a normalization scale and $P(z) = C_F(1+z^2)/(1-z)$.

A similar result is found in the Drell-Yan process:

$$\frac{d\sigma^{DY}}{dQ^2} = \frac{4\pi\alpha^2}{9SQ^2}\int\frac{dx_1}{x_2}\frac{dx_2}{x_2}\left[\sum_i e_i^2 q_i(x_1,Q^2)\,\bar{q}_i(x_2,Q^2)+(1\leftrightarrow 2)\right] f(z,Q^2) \qquad (13)$$

where

$$f(z,Q^2) = \frac{1}{2\pi}\int db\, e^{ib(1-z)} \exp\{2C_F\int_o^1\frac{dy}{1-y}\int_{Q^2(1-y)}^{Q^2(1-y)^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha(k_\perp)}{\pi}\,.\,[e^{-ib(1-y)}-1]\}\exp\{\frac{\alpha(Q^2)}{2\pi}C_F\pi^2\}, \qquad (14)$$

and $z = \tau/x_1x_2$. In this case the kinematical upper limit is $k_{\perp max}^2 \simeq Q^2(1-y)^2$. The Q^2 dependence in the parton densities appearing in eq. (13) is in agreement with eq. (11).

Notice that the factor $\exp\{(\alpha(Q^2)/2\pi)\,C_F\pi^2\}$ in eq. (14), which is due to the continuation of the Q^2 dependence in the quark form factor from the spacelike to time like regions(12), plays an important rôle in renormalizing the naive Drell-Yan cross section by about a factor of two at present energies, for all τ values. More detailed phenomenological implications of our results are in progress.

To conclude, the formalism of coherent states is quite appropriate to discuss soft gluon effects at all orders in various process. Important next to leading corrections can also be resummed in a rather simple way. Preliminary results on e^+e^- jets at PETRA give strong support to these resumming formulae.

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