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A. Turrin: THEORY OF QUASILINEARLY TAPERED COUPLERS HAVING WEIGHTED COUPLING STRENGTH.

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A. Turrin : THEORY OF QUASILINEARLY TAPERED COUPLERS HAVING WEIGHTED COUPLING STRENGTH.

ABSTRACT

A calculation is presented for the efficiency of quasilinearly tapered couplers that have a finite coupling phase (nonadiabatic couplers). The resulting very simple formula bridges the gap between the two well-known (and opposite) cases were (i) an infinite coupler length is assumed (nearadiabatic coupler) and (ii) phase-matched guides are considered. It turns out that these short nonadiabatic couplers are quite tolerant of parameters variations.

INTRODUCTION

In this communication we present a theory on coupling between modes in a tapered coupler having a coupling coefficient K(z) which decreases near both its ends and a mismatch $\Delta(z) = \beta_1(z) - \beta_2(z)$ between the propagation coefficients which is a quasilinear function of the distance z along the coupler's axis. It will be shown that

i) complete exchange of power between the two modes can be obtained, provided the coupler's parameters are chosen properly;

ii) power transfer remains still insensitive even to wide variations of the coupler's parameters;

iii) efficient coupler operation can be achieved even with short coupler lengths.

Before we start with our discussion, we first look at what is to be found in the literature on this subject.

The simplest formulation of a non perturbative theory on power-transfer produced in a two-mode coupler by slowly varying $\Delta(z)$ through the cross-over point $z = 0$ (where $\Delta = 0$) is embodied in the nearadiabatic model[1, 2, 3]. In this model a constant coupling coefficient K_o and a linearly varying mismatch

$$\Delta_o(z) = \Delta'_o z \qquad (1)$$

are assumed. If one takes all the power to be in one mode only at $z = -\infty$, than the resulting asymptotic coupler efficiency becomes[1, 2, 3]

$$\eta_{o \atop z=+\infty} = 1 - \exp(-2\pi f_o) , \qquad (2)$$

where

$$f_o = K_o^2 / \Delta'_o . \qquad (2a)$$

On this model we shall make two remarks :

i) the degree of adiabaticity of the coupler is described by one independent parameter only (f_o) ;

ii) the assumption of a constant coupling coefficient $K(z) = K_o$ puts us in a certain difficulty, because it does not allow to define conveniently the "length" of the coupling region[1, 3] required to get an efficient power transfer.

One would like therefore to have a more complete theory which synthesizes the coupler by two parameters, namely the (finite) coupling phase

$$s = \int_{-\infty}^{+\infty} K(z)\, dz \qquad (3a)$$

and

$$f = K^2(0) / \Delta'(0) \qquad (3b)$$

(as stated above, the mismatch $\Delta(z)$ is assumed to be zero at $z = 0$).

This new model should have its extreme domains of validity coincident with those of the nearadiabatic model[1, 2, 3] in the limit $s = \infty$.

With the model presented here, one can easily calculate the coupler's efficiency in terms of s and f, bypassing thereby difficulties like those encountered in Refs. (1, 3) in defining properly the coupler's length.

The present communication addresses itself on this problem.

THE NONADIABATIC MODEL.

K(z) is assumed here to be an even, positive, arbitrary function of z, going to zero at $z = \pm\infty$.

For $\Delta(z)$ we write, in place of eq. (1),

$$\Delta = \Delta(z) = \left[(s/\pi)/f\right] K \tan\left[(\pi/s)\sigma\right] \tag{4a}$$

where

$$\sigma = \int_0^z K\,dz \tag{4b}$$

$(\sigma(\pm\infty) = \pm s/2)$.

To convey a feeling for the suitability of our approach, we give an example which is representative of the conditions actually occurring in weighted coupling applications[4, 5]. Thus the form of $\Delta(z)$ as given by eqs. (4a, b) is shown explicitly in Fig. 1 for the special case where a gaussian profile of width z_c (FWHM),

$$K(z) = K(0)\exp(-\tau^2), \tag{5a}$$

$$\tau = \sqrt{2\ln 2}\; z/z_c, \tag{5b}$$

is adopted for K.

Incidentally, note that in this case z_c can be chosen as a parameter which characterizes the coupler "length".

The model described by eqs. (4a, b), with K(z) arbitrary, will be referred to as the "nonadiabatic model".

ANALYSIS.

Once the nonadiabatic model above is adopted, the object is to solve the conventional first-order coupled-mode equations

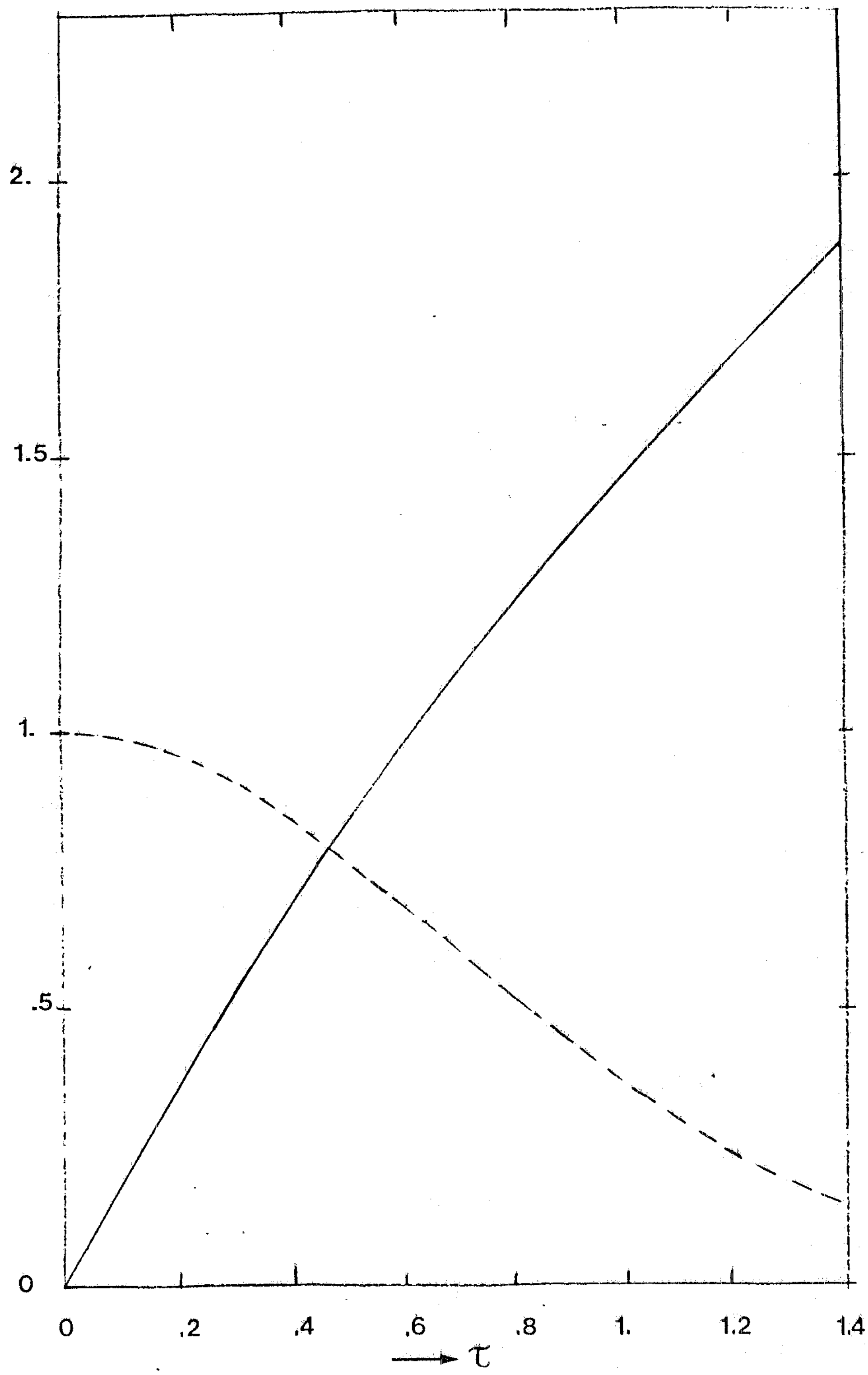

FIG. 1 - Dashed curve: shape of a gaussian profile for the coupling coefficient (eqs. (5a, b)).
Solid curve: $\exp(-\tau^2)\tan((\pi/2)\,\mathrm{erf}(\tau))$, which, by eqs. (4a, b) of the text, exhibits the features of the (corresponding) sweeping program of the mismatch $\Delta(z)$ through the cross-over point.

$$ie' = Kg \exp(-i \int_0^z \Delta dz) \tag{6a}$$

$$ig' = Ke \exp(i \int_0^z \Delta dz) \tag{6b}$$

for the nearly stationary mode amplitudes, e and g, of the electromagnetic field in the coupler. We set for the initial-values conditions

$$g = 0, \quad e = 1 \quad \text{at} \quad z = -\infty \tag{6c}$$

($gg^* + ee^* = 1$ is our normalization condition).

The coupled differential equations (6a, b) can readily be separated by differentiation. Thus, we have the equation for g:

$$g'' - (i\Delta + K'/K)g' + K^2 g = 0. \tag{6d}$$

Now, we insert the nonadiabatic model $\Delta(z)$ of eqs. (4a, b) in eq. (6d) and introduce the new independent variable

$$x = (1/2)\left[1 + \sin((\pi/s)\sigma)\right]. \tag{7}$$

We get

$$x(1-x)d^2g/dx^2 + (\gamma/2 - \gamma x)dg/dx + \varepsilon g = 0, \tag{8}$$

where

$$\varepsilon = (s/\pi)^2 \quad \text{and} \quad \gamma = 1 + i\varepsilon/f. \tag{9a, b}$$

As z goes from $-\infty$ to $+\infty$, x spans the interval $0 \leftarrow x \rightarrow 1$.

Eq. (8) is the hypergeometric differential equation[6]

$$x(1-x)d^2g/dx^2 + \left[c - (1+a+b)x\right]dg/dx - abg = 0 \tag{10}$$

with the three constants a, b, c

$$a = i\varepsilon/(2f) \pm \sqrt{\varepsilon - (\varepsilon/(2f))^2} \tag{11a}$$

$$b = i\varepsilon/(2f) \mp \sqrt{\varepsilon - (\varepsilon/(2f))^2} \tag{11b}$$

$$c = i\varepsilon/(2f) + 1/2. \tag{11c}$$

Thus, the general solution of eq. (8) can be written in the form

$$g = C\,F(a,b;c;x) + D x^{1-c} F(a-c+1, b-c+1; 2-c; x) \, , \tag{12}$$

where C and D are integration constants and the F's are hypergeometric functions (eqs. (15.5.3) and (15.5.4) of ref. (6)).

In order that the boundary conditions expressed by eqs. (6c) be satisfied we must set $C = 0$. To determine D, we reconsider eq. (6b) which, in terms of the new independent variable x, transforms into the equation

$$i(\pi/s)\sqrt{x(1-x)}\, dg/dx = e \exp\left(i\int_0^z \Delta\, dz\right) . \tag{13}$$

This gives, in the limit $z \rightarrow -\infty$, i.e. $x \rightarrow 0$,

$$(dg/dx)(dg/dx)^{*}_{x \rightarrow 0} = \varepsilon/x \, . \tag{14}$$

Differentiating eq. (12) (with $C = 0$) we have

$$dg/dx = D(1-c)x^{-c} F(a-c+1, b-c+1; 1-c; x) \tag{15}$$

(use of the differentiation formula given by eq. (15.2.4) of ref. (6) has been made); combining eq. (14) and (15), we obtain, in the limit $x \rightarrow 0$,

$$DD^{*} = 4\varepsilon / \left[1 + (\varepsilon/f)^2\right] . \tag{16}$$

Now, in the limit $z \rightarrow +\infty$, we find

$$\eta_{z \rightarrow +\infty} = gg^{*}_{z \rightarrow +\infty} = DD^{*} \left| \Gamma(2-c)\Gamma(c-a-b) / \{\Gamma(1-a)\Gamma(1-b)\} \right|^2 \tag{17}$$

where the Γ's are gamma functions.

In deriving this expression for the coupler efficiency $\eta_{z \rightarrow +\infty}$ we have made use of eq. (15.1.20) of ref. (6).

Finally, we develop the squared modulus entering in eq. (17) and use eq. (16). We get for the desired coupler efficiency

$$\eta_{z=+\infty} = 1 - \cos^2\left[s\sqrt{1-((s/\pi)/(2f))^2}\right] \mathrm{sech}^2\left[\pi(s/\pi)^2/(2f)\right] \tag{18a}$$

for $s/\pi \leq 2f$

$$\eta_{z=+\infty} = 1 - \cosh^2\left[s\sqrt{((s/\pi)/(2f))^2 - 1}\right]\operatorname{sech}^2\left[\pi(s/\pi)^2/(2f)\right] \tag{18b}$$

$$\text{for} \quad s/\pi \geq 2f.$$

RESULTS.

By looking first at eq. (18a), we see that it reduces precisely to the well-known solution[4] $\sin^2(s)$ at $f = \infty$.

On eq. (18b), we note that it reduces precisely to eq. (2) for s/π large; this can be very quickly derived by retaining only the zero-th and first order terms in the binomial expansion of the $\sqrt{1-(2f/(s/\pi))^2}$ expression entering in eq. (18b).

To gain full insight into the performance of the nonadiabatic coupler in the case where the coupling phase s is relatively small, we have plotted, in Fig. 2, a diagram showing the $2s/\pi$ and $1/f$ values required to obtain almost complete power transfer.

The conclusion can be drawn that there is nothing in coupled-mode theory that restricts efficient tapered coupling operation to high coupling phases, and that no critical values of the coupler's parameters s and f are involved anyway.

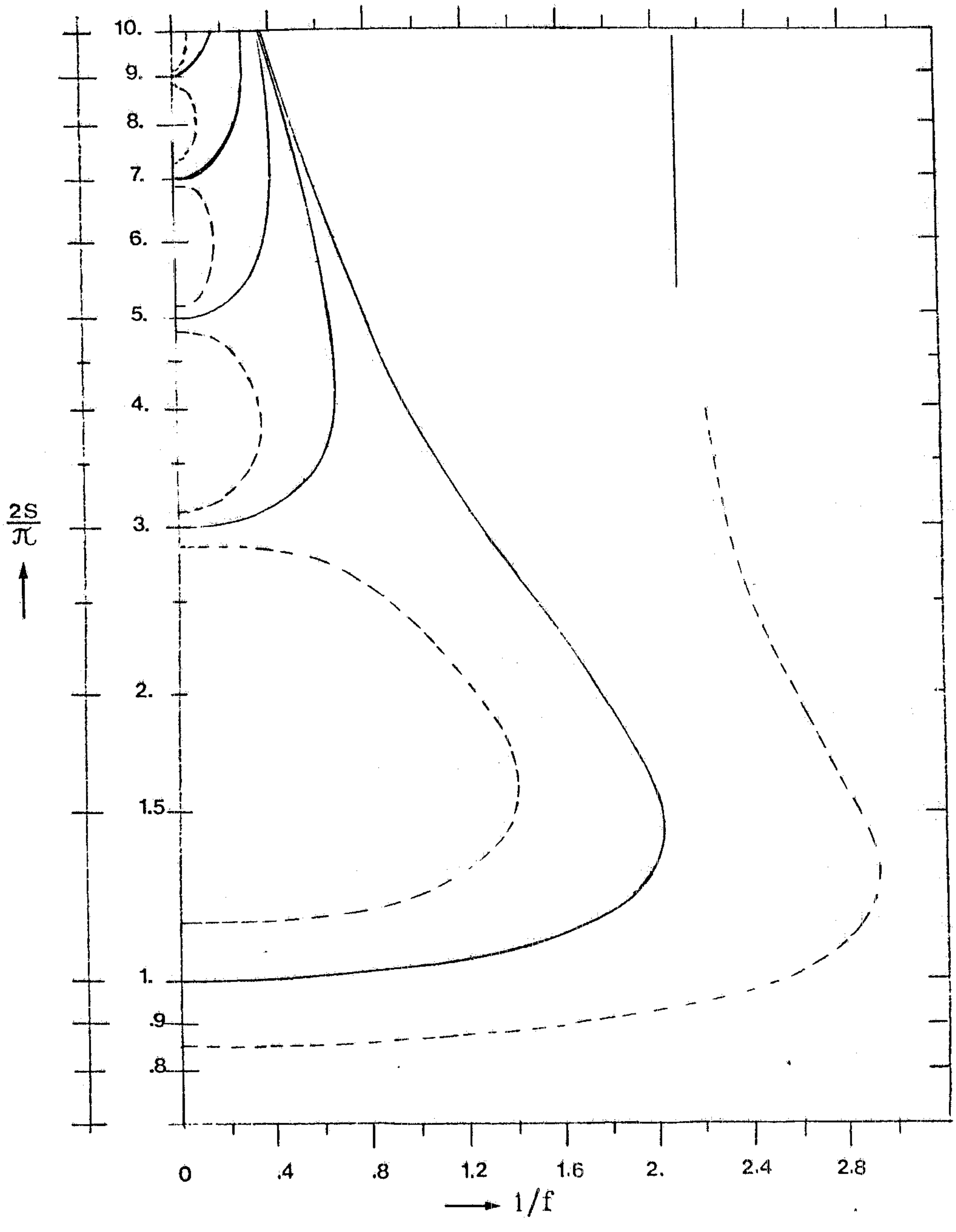

FIG. 2 - Diagram showing the performance of the nonadiaba- tic coupler. The solid curves are the loci of complete power- -transfer conditions ($\eta_{z=+\infty} = 1$). The loci where $\eta_{z=+\infty} =$ $= 0.95$ are represented by the dashed curves. The vertical straight line at the top of the diagram represents the asymptotic behaviour ($(2s/\pi) \rightarrow \infty$) of the $\eta_{z=+\infty} = 0.95$ curve.

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