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Quantum-chromodynamic radiation and mean scaling for hadronic and current processes

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A recently proposed model for the longitudinal- and transverse-momentum distribution of quantum-chromodynamics (QCD) radiation is shown to lead naturally to mean scaling. Excellent agreement is found for the normalized momentum distribution in $pp \to n\pi + X$ between our predictions and the data from 13 to 1500 GeV/c with n, the number of pions, varying from 4 to 26. We also show that μ-pair production by hadrons exhibits the same transverse-momentum spectrum as do the hadrons produced in e^+e^- annihilation. This transverse scaling in the mean in terms of the dimensionless variable $z_\perp = k_\perp/\langle k_\perp \rangle$ is interpreted as reflecting the primary (soft-gluon) QCD radiation distribution. Again, very good agreement is found for these normalized distributions and our theoretical scaling curves.

I. INTRODUCTION

In a previous paper,[1] we proposed expressions for the four-momentum distribution of the quantum-chromodynamics (QCD) radiation accompanying quark scattering. Assuming that the inclusive hadronic momentum distribution follows that of the QCD radiation, we were able to obtain very satisfactory agreement with the SPEAR data[2] in e^+e^- annihilation both for transverse and longitudinal distribution of pions.

In this paper we give details of the model (Sec. II) and present further experimental evidence supporting our soft-gluon bremsstrahlung formula for QCD. We show that "mean" scaling (or scaling in the mean) found phenomenologically in pp semi-inclusive distributions[3-5]—and held for some time to be without theoretical basis[6,7]—arises naturally in our model. Any scaling underscores the existence of a universal curve—in this case for longitudinal and transverse distributions. In Sec. III we show that our proposed expressions in fact give these curves in excellent accord with the data for both distributions. We then extend this analysis to current processes. In Sec. IV, we present evidence for mean scaling in the variable $z_\perp = k_\perp/\langle k_\perp \rangle$ for the process

$$pp \to \mu^+\mu^- + X$$

as well as for

$$e^+e^- \to \pi^\pm + X.$$

Our discovery of mean scaling for these processes is quite nontrivial and in fact rather surprising. It is nontrivial because the mean transverse momentum for the μ pair is not the usual 300–400 MeV—found also in $e^+e^- \to$ hadrons—but for certain values of the μ-pair mass becomes as large as 1.4 GeV. The apparent surprise may be if one regards μ-pair production to arise exclusively through a single hard quark scattering as implied in the Drell-Yan mechanism. On the contrary, in our approach the above scaling is natural since we expect the μ pair to reflect the transverse-momentum distribution of the primary QCD radiation, just as do the produced hadrons in e^+e^- annihilation[1] or in pp scattering.

II. PHYSICAL PICTURE AND FORMALISM

In analogy with QED reactions, we postulate that in a given hadronic process there is an emission of soft gluons to be dealt with nonperturbatively because of the possibility of an infinite number of zero-energy gluons. In QED, the creation and annihilation of charged matter is always accompanied by the emission of electromagnetic radiation which manifests itself as an energy-momentum imbalance. In QCD soft-gluon emission manifests itself through a reconversion into hadrons, since (the assumed) confinement prevents its detection in any other form, e.g., in a semi-inclusive reaction of the type

$$pp \to pp + n\pi ,$$

we visualize the produced pions as the final visible form of the primary QCD radiation emitted in the scattering of the (hard) quarks constituting the protons. Figure 1 shows this analogy in a pictorial form. In Fig. 1(a) the electromagnetic radiation Γ is a necessary accompaniment of the charged-particle reaction, and in Fig. 1(b) we have the corresponding QCD picture with the radiation accompanying pp scattering and which subsequently converts into pions.

In QED, the Bloch-Nordsieck (BN) theorem provides the proper nonperturbative vehicle to obtain the spectrum of soft photons since a satisfactory analysis of soft bremsstrahlung requires summation to all orders. In Ref. 8 we have given a de-

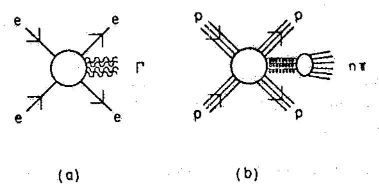

FIG. 1. (a) Radiation (Γ) accompanying the QED process $ee \to ee + \Gamma(K)$. (b) Radiation (Γ) accompanying the QCD process $pp \to pp + \Gamma(K)$.

tailed analysis of the transverse-momentum distribution resulting from this soft radiation. For orientation and to facilitate further developments, we present below the gist of our argument from Ref. 8.

Using the BN theorem and energy-momentum conservation, Etim, Pancheri, and Touschek[9] derived the following expression for the four-momentum distribution $d^4P(K)$ of the emitted radiation in an arbitrary QED process $i \to f + \Gamma(K)$:

$$d^4P(K) = (d^4K) \int \frac{(d^4x)}{(2\pi)^4} e^{iK\cdot x - h(x,\epsilon)}, \qquad (1)$$

where

$$h(x,\epsilon) = \int_0^\epsilon d^3\bar{n}(k)(1 - e^{-ik\cdot x}) \qquad (2)$$

and $d^3\bar{n}(k)$ represents the average number of real, soft quanta emitted in the momentum integral (d^3k). The upper limit ϵ in Eq. (2) required for the convergence of the integral is the maximum energy allowed for a single quantum emitted in a given process. If we write

$$d^3\bar{n}(k) = \beta\left(\frac{dk}{k}\right) f(\hat{n}) d\Omega_n, \qquad (3)$$

then β is seen to represent the spectrum and $f(\hat{n})$ the angular distribution of the single photon.[9] In QED, β is (i) proportional to the coupling constant, (ii) a function of the momenta of the emitting particles (but not the photon momentum), and (iii) differs from process to process. For annihilation or creation of an e^+e^- pair, $\beta_{\rm QED} \simeq (4\alpha/\pi)\ln(W/m_e)$, where W is the total energy of the pair, m_e is the mass of the electron, and $\alpha \simeq (137)^{-1}$ is the fine structure constant.

In Ref. 8, we developed an approximate analytic form for the transverse-momentum distribution $d^2P(K_\perp)$ which preserves the normalization and $\langle K_\perp^2\rangle$ of the exact distribution of Eq. (1).

Now let us consider the corresponding situation in QCD where two immediate differences from QED arise. First, the non-Abelian nature of QCD introduces the three-gluon vertex which has no counterpart in QED. Second, the coupling constant $\alpha_{\rm strong}$ is presumably large in contrast to the QED α. As to the first point, fortunately it has been shown by many authors[10-12] that in the leading-logarithm approximation, the infrared (IR) divergences cancel for color singlets and that an exponentiation very similar to that in QED occurs as well for QCD. In this approximation, the non-Abelian nature of QCD manifests itself essentially in replacing the coupling constant by the running coupling constant; otherwise, the results are practically unchanged, apart from obvious group-theory factors. Regarding the second point (large $\alpha_{\rm strong}$), it should be noted that Eq. (1) is derived whether or not α is small—it assumes only independent emission and global energy-momentum conservation. Thus, we assume, as seems reasonable, that Eqs. (1) and (2) remain valid also for QCD and represent a summation of the soft-gluon spectrum. In order to use them, however, we need to know what is the analogous β for QCD which appears in Eq. (3). Owing to a lack of our understanding of confinement in QCD we are unable to calculate β from first principles. An effective β was introduced in Ref. 1 as the average spectrum of a single soft gluon and is our only parameter. We shall allow it to vary and determine its value phenomenologically for each process. No theoretical analysis of β shall be attempted here, but since β *de facto* reflects the intrinsic details of the theory, its experimental determination in different kinematic domains and for different processes may ultimately throw light on the underlying dynamics.

Unlike QED, first-order expressions in β resulting from Eq. (1) are not likely to suffice for QCD. In Ref. 8 our objective was precisely to obtain approximate analytic forms for $d^2P(K_\perp)$ *valid for large* β. Equation (1) leads to

$$d^2P(K_\perp) = \frac{d^2K_\perp}{\epsilon^2} \int \frac{(d^2x)}{(2\pi)^2} e^{-i\vec{K}_\perp\cdot\vec{x}/\epsilon - \beta g(x)}, \qquad (4)$$

where

$$g(x) = \int d\Omega_n f(\hat{n}) \int_0^{-i(\vec{n}_\perp\cdot\vec{x})} \frac{dy}{y}(1 - e^{-y}). \qquad (5)$$

In Ref. 8 we approximated $g(x)$ by

$$\tilde{g}(x) \sim \tfrac{1}{2}\ln\left(1 + \tfrac{1}{4}x^2 \int d\Omega_n f(\hat{n}) n_\perp^2\right). \qquad (6)$$

Now Eq. (4) can be integrated to give the approximate form

$$\frac{dP(K_\perp)}{dK_\perp} = \frac{1}{\Gamma(\beta/2)}\left(\frac{K_\perp}{\epsilon^2 A}\right)\left(\frac{K_\perp}{2\epsilon\sqrt{A}}\right)^{(\beta/2)-1} K_{(\beta/2)-1}\left(\frac{K_\perp}{\epsilon\sqrt{A}}\right), \qquad (7)$$

where $A = \frac{1}{4}\langle n_\perp^2\rangle$ gives the transverse spread in the primordial gluon radiation and $K_\nu(z)$ is the modified Bessel function (see Ref. 8 for details).

A completely analogous analysis can be performed to obtain the longitudinal $K_\parallel$ distribution. We obtain

$$\frac{dP(K_\parallel)}{dK_\parallel}=\frac{1}{\sqrt{\pi}\,\Gamma(\beta/2)}\,\frac{1}{\epsilon\sqrt{B}}\left(\frac{K_\parallel}{2\epsilon\sqrt{B}}\right)^{(\beta-1)/2} K_{(\beta-1)/2}\left(\frac{K_\parallel}{\epsilon\sqrt{B}}\right), \tag{8}$$

where $B=\frac{1}{2}(1-4A)$.

From Eqs. (7) and (8) we find the average values

$$\langle K_\perp\rangle=(\epsilon\sqrt{A})\,\frac{\sqrt{\pi}\,\Gamma\left(\frac{1+\beta}{2}\right)}{\Gamma(\beta/2)} \tag{9}$$

and

$$\langle K_\parallel\rangle=(\epsilon\sqrt{B})\,\frac{2\Gamma\left(\frac{1+\beta}{2}\right)}{\Gamma(\beta/2)}\,. \tag{10}$$

If we define the two dimensionless variables

$$z_\perp\equiv\frac{K_\perp}{\langle K_\perp\rangle}\,, \tag{11}$$

$$z_\parallel\equiv\frac{K_\parallel}{\langle K_\parallel\rangle}\,, \tag{12}$$

then Eqs. (7) and (8) can be rewritten using (11) and (12) as

$$\frac{dP_\beta}{dz_\perp}=\frac{\sqrt{\pi}\,\Gamma\left(\frac{1+\beta}{2}\right)}{2^{(\beta/2)-1}\Gamma^2(\beta/2)}\,z_T^{\beta/2}K_{(\beta/2)-1}(z_T)\,, \tag{13}$$

$$\frac{dP_\beta}{dz_\parallel}=\frac{\Gamma\left(\frac{1+\beta}{2}\right)}{2^{(\beta-3)/2}\pi\Gamma^2(\beta/2)}\,z_L^{(\beta-1)/2}K_{(\beta-1)/2}(z_L)\,, \tag{14}$$

where

$$z_T=\frac{\sqrt{\pi}\,\Gamma\left(\frac{\beta+1}{2}\right)}{\Gamma(\beta/2)}\,z_\perp \tag{15}$$

and

$$z_L=\frac{2}{\sqrt{\pi}}\,\frac{\Gamma\left(\frac{\beta+1}{2}\right)}{\Gamma(\beta/2)}\,z_\parallel\,. \tag{16}$$

With the above substitutions, Eqs. (13) and (14) describe the normalized probability distributions in terms of dimensionless longitudinal and transverse variables. These dimensionless variables exhibit mean scaling and are our proposed universal expressions from which all references to ϵ, A, and B, which depend on the intrinsic details of the radiation, have disappeared.

For the determination of the remaining parameter β, it is convenient to consider the *dispersion* $\Delta_\perp$

$$\Delta_\perp\equiv\frac{\langle k_\perp\rangle}{(\langle k_\perp^2\rangle-\langle k_\perp\rangle^2)^{1/2}}\,, \tag{17}$$

whose determination may determine β at least in principle. In our model

$$\langle K_\perp\rangle=\left(\frac{\sqrt{\pi}}{2}\right)\frac{\Gamma\left(\frac{1}{2}+\frac{\beta}{2}\right)}{\Gamma(\beta/2)}\,(\langle K_\perp^2\rangle_{\rm Prim})^{1/2} \tag{18}$$

and

$$\langle K_\perp^2\rangle=\tfrac{1}{2}\beta\langle K_\perp^2\rangle_{\rm Prim}\,, \tag{19}$$

where $\langle K_\perp^2\rangle_{\rm Prim}$ denotes the "primordial" transverse-momentum spread of the gluons.[13,14] Thus, we have

$$\Delta_\perp(\beta)=\frac{1}{\left\{\frac{8}{\pi\beta}\left[\frac{\Gamma(\frac{1}{2}\beta+1)}{\Gamma((\beta+1)/2)}\right]^2-1\right\}^{1/2}}\,. \tag{20}$$

In Fig. 2 we have plotted the function $\Delta(\beta)$ vs β to emphasize which domains of β are accurately determined by this procedure. This method is used in Sec. IV to obtain β for current processes where very accurate values of $\langle K_\perp\rangle$ and $\langle K_\perp^2\rangle$ were available.

III. PHENOMENOLOGY OF THE SEMI-INCLUSIVE PROCESS $pp\to n\pi+X$

We now apply Eqs. (13) and (14) to describe a generic semi-inclusive reaction with n pions

$$pp\to n\pi+X\,, \tag{21}$$

in the following way. Define

$$\frac{dP_n}{dz_\perp^{(n)}}\equiv\frac{\langle k_\perp\rangle_n}{n\sigma_n}\,\frac{d\sigma_n}{dk_\perp} \tag{22}$$

and

$$\frac{dP_n}{dz_\parallel^{(n)}}\equiv\frac{\langle k_\parallel\rangle_n}{n\sigma_n}\,\frac{d\sigma_n}{dk_\parallel}\,, \tag{23}$$

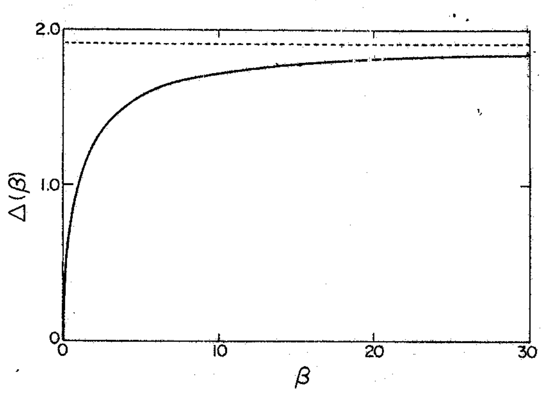

FIG. 2. $\Delta_\perp(\beta)$ vs β from Eq. (20).

where σ_n is the cross section and $\langle k_\perp \rangle_n$, $\langle k_\parallel \rangle_n$ are the mean values for the transverse and longitudinal momentum for process (21).

In view of the discussion in Sec. II, we expect that distributions (22) and (23) are independent of n as well as of the energy, i.e., show mean scaling.[3-5] In our approach, the final pions are produced by the QCD radiation whose four-momentum is then shared by the pions. Thus, the pion four-momentum distribution should reflect that of the original radiation, even though the mean values $\langle p_\parallel \rangle_n$ and $\langle p_\perp \rangle_n$ may be quite different[3] for different n and may even possess energy dependence.

To describe mean scaling we propose Eqs. (13) and (14) as universal forms for it, by setting

$$\frac{dP_n}{dz_\perp^{(n)}} \equiv \frac{dP_\beta}{dz_\perp} \text{ and } \frac{dP_n}{dz_\parallel^{(n)}} \equiv \frac{dP_n}{dz_\parallel}. \tag{24}$$

We now proceed to compare our universal curves with the experimental data. This comparison is done in Figs. 3 and 4. The theoretical curves were obtained upon using the *same* value of $\beta = 3.5$ which was used in our earlier analysis[1] of the SPEAR data.[2] As can be seen, the agreement with the data is very impressive, especially since the only parameter β was not allowed to vary. Notice that the data[3-5] presented in Figs. 3 and 4 cover a range of energy from 13 to 1500 GeV, and n, the number of pions, varies from 4 to 26.

A sensitive test of our scaling curve and the underlying radiation model is provided by evaluation of the fractional rms deviation

$$\Delta_\parallel = \frac{\langle k_\parallel \rangle}{(\langle k_\parallel{}^2 \rangle - \langle k_\perp \rangle^2)^{1/2}}.$$

Our computed value for this quantity is 1.11 (with $\beta = 3.5$). The only experimental values we were able to find were given in Ref. 3 for the 300-GeV data. Their average value turns out to be 1.1 ± 0.02, in excellent agreement with our theoretical value. A similar calculation for the transverse-momentum variable gives

$$\Delta_\perp = 1.46.$$

We have not been able to find any published experimental value for comparison and it would be very interesting to have direct experimental information about this quantity. If we use the phenomenological fit to the $k_\perp$ data made in Ref. 15, we find ≈ 1.54, in good agreement with our value.

As is clear from our formulas, we should observe logarithmic violations of mean scaling due to the energy dependence of β (through its dependence on α_{strong}). The value of $\beta = 3.5$ used in this analysis should be considered as an average value over the entire energy range.

A quantity which is independent of β (and thus also of the main scaling violations) is provided by the ratio of the two coefficients $a_\parallel$ and $a_\perp$ given by Eqs. (11) and (12):

$$a_\perp = \frac{z_T}{z_\perp} = \frac{\sqrt{\pi}\,\Gamma\left(\frac{1+\beta}{2}\right)}{\Gamma(\beta/2)}, \tag{25}$$

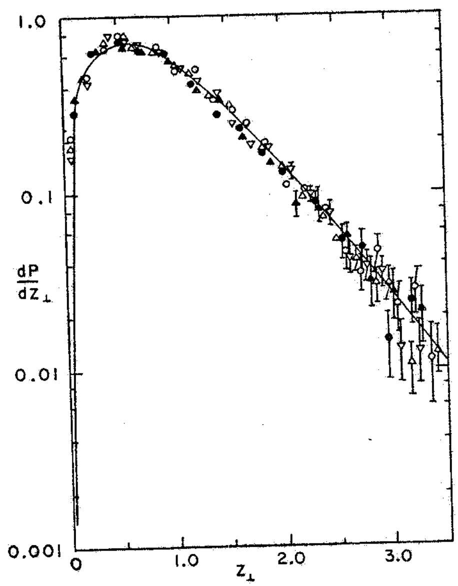

FIG. 3. $dP/dz_\perp$ vs $z_\perp$. All data shown are taken from Ref. 3. For clarity, data from Refs. 4 and 5 which are consistent with it are not shown. The solid curve is our theoretical formula Eq. (13) with $\beta = 3.5$.

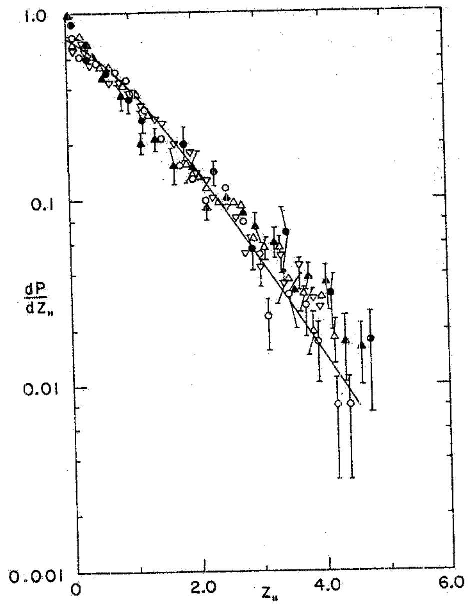

FIG. 4. $dP/dz_\parallel$ vs $z_\parallel$. Data are from Ref. 3. Solid curve is our theoretical formula Eq. (14) with $\beta = 3.5$.

$$a_{\parallel} = \frac{z_\perp}{z_\parallel} = \frac{2}{\sqrt{\pi}} \frac{\Gamma\left(\frac{1+\beta}{2}\right)}{\Gamma(\beta/2)} . \tag{26}$$

As one can see from Eqs. (13) and (14), $a_\perp$ and $a_\parallel$ give the exponential cutoffs $e^{-a_\perp z_\perp}$ and $e^{-a_\parallel z_\parallel}$. The ratio

$$a_\parallel / a_\perp = 2/\pi = 0.64$$

is a constant, independent of any parameter (and in particular of β). The above value compares quite well with the value 0.61 which one can obtain through the phenomenological fit made in Ref. 15. It is not possible to check our value directly against the fits made in Ref. 3 because of a very different functional form for $z_\parallel$ distribution used there.

For large values of z, we expect our model to break down. We can estimate as follows a value $z_{\max}$ beyond which we predict mean scaling to be rather badly violated. From deep inelastic scattering we know that, on an average, each quark (in pp scattering) would carry an energy $E_Q \simeq \sqrt{s}/12$. Thus, $z_{\max} \simeq \sqrt{s}/12\langle k\rangle$. In the transverse case, where $\langle k_\perp\rangle \simeq 0.35$ GeV, $z_{\max} \approx 6$ for 300 GeV and 1.7 for 28 GeV. In the latter case, therefore, we find that mean scaling should become invalid for $k_\perp \gtrsim 0.64$ GeV. This seems to be experimentally verified.[16]

When $z \gtrsim z_{\max}$, it is reasonable to suppose the dominant process to be hard quark scattering with its accompanying power-law damping.[17]

IV. PHENOMENOLOGY OF CURRENT PROCESSES

We begin the phenomenology of current processes by considering the very-high-statistics data (275 000 events) of Ref. 18 for the production of high mass μ pairs ($M_{\mu\mu} > 6$ GeV) by 400-GeV protons on tungsten target. The data covers practically all x_F, a large range of μ-pair mass (6.8–13) GeV and $\langle k_\perp\rangle$ is found to be $\simeq 1.3$ GeV. These authors give a simple expression for the $K_\perp$ dependence, which fits all their data.[18] In terms of the normalized variable $z_\perp$, their formula becomes

$$\left(\frac{dP}{dz_\perp}\right)_{\exp} = \frac{1.845 z_\perp}{(1+0.845 z_\perp{}^2)^6} , \tag{27}$$

where $(dP/dz_\perp)_{\exp}$ is normalized to 1 and represents the transverse distribution upon integration over all the other variables.

In order to compare the above fit with our prediction, Eq. (13), we need to fix the value of the parameter β. Using Eq. (27), the dispersion $\Delta_\perp$ is calculated to be 1.68 which corresponds to $\beta = 8$ (see Fig. 2). In Fig. 5, we show the phenomenological fit, Eq. (27) (dashed curve), together with our expression Eq. (13), for $\beta = 8$ (solid curve). The agreement is excellent. Actually, quite a similar agreement is found for any β lying between (7–11) for this range of $z_\perp$ values.[19]

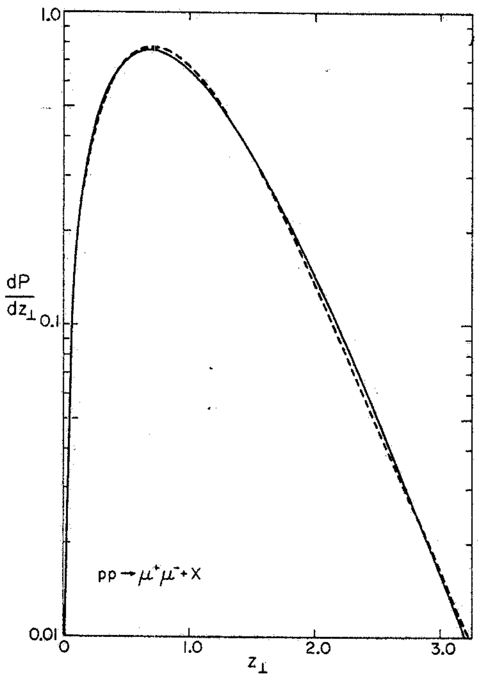

FIG. 5. $dP/dz_\perp$ vs $z_\perp$. Solid line is Eq. (13) with $\beta = 8$. Dashed line is a fit given by Eq. (27) to the μ-pair data of Ref. 18. No uncertainties are shown for the experimental results—since the uncertainty in the functional form for the cross section has not yet been determined by the authors of Ref. 18.

Now we turn to e^+e^- annihilation into hadrons.[2] These data were partially analyzed earlier (for $W = 7$–7.8 GeV), with a prechosen fixed value of $\beta = 3.5$.[1] In Fig. 6 we present all the data from $W = 3$–7.8 GeV in terms of the normalized $dP/dz_\perp$. As one can see there is a clear scaling-in-the-mean effect when the data are plotted as a function of $z_\perp$, even though data at different W are rather different as a function of $k_\perp{}^2$. (After all, the sphericity at 3 GeV is consistent with phase space, while at 7.4 GeV there is a pronounced jet structure.) We also present (solid line) our expression for $\beta = 8$ for comparison. Our formula is seen to provide a good average over all the data.

If we compare Figs. 5 and 6 we see how remarkably similar μ-pair production and e^+e^- annihilation distributions are when studied as a function of the normalized $z_\perp$ variable. Thus, we find strong support for our underlying hypothesis, i.e., that *all* particle production reflects the transverse-momentum distribution of the primary QCD radiation. We conjecture that this structure will show

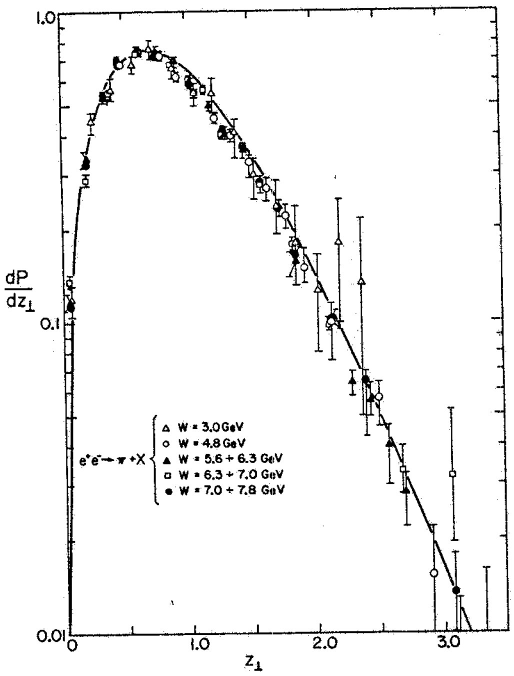

FIG. 6. $dP/dz_\perp$ vs $z_\perp$ for $e^+e^- \to \pi + X$. Data for $W = 3$(△), 4.8(○), 5.6–6.3 (▲), 6.3–7.0(□), and 7.0–7.8(●) GeV are taken from Ref. 2. Solid line is our Eq. (13) with $\beta = 8$.

up in hadronic production accompanying deep inelastic scattering as well.

V. CONCLUSION

We have applied our gluon-radiation model, which was very successful in describing the momentum distribution in e^+e^- annihilation,[1] to study mean scaling in purely hadronic as well as current processes. Clear evidence for mean scaling is found for these processes. In our model it appears quite naturally, since it implies the existence of a basic "internal" process whose four-momentum distribution itself is revealed through mean scaling. We have presented a compact universal formula for this distribution, in terms of β, which describes the effective spectrum of a single gluon—in exact analogy to QED.[8] In Ref. 1 a simple QED-type expression for β was proposed. We now believe that the situation is more complex. For inclusive processes of the type discussed here there occur integrations over the unobserved variables which run over the confinement region thus rendering a theoretical determination of β rather difficult. This problem certainly deserves further study.

We also estimated the range beyond which our model breaks down and which signals the onset of power-law damping.

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[1] G. Pancheri-Srivastava and Y. Srivastava, Phys. Rev. Lett. 43, 11 (1979).

[2] G. Hanson, in *Gauge Theories and Leptons*, proceedings of the XIII Rencontre de Moriond, Les Arcs, Savoie, France, 1978, edited by J. Trân Thanh Vân (Editions Frontières, Gif-sur-Yvette, 1978), p. 15.

[3] F. T. Dao *et al.*, Phys. Rev. Lett. 33, 389 (1974).

[4] H. Böggild *et al.*, in *High Energy Physics*, proceedings of the European Physical Society International Conference, Palermo, 1975, edited by A. Zichichi (Editrice Compositori, Bologna, Italy, 1976).

[5] A. Laasanen *et al.*, Phys. Rev. Lett. 38, 1 (1977).

[6] R. J. Yaes, Phys. Rev. Lett. 36, 821 (1976).

[7] W. Ernst and I. Schmitt, Lett. Nuovo Cimento 16, 39 (1976).

[8] G. Pancheri-Srivastava and Y. Srivastava, Phys. Rev. D 15, 2915 (1977).

[9] E. Etim, G. Pancheri, and B. Touschek, Nuovo Cimento 51, 276 (1967). It was further developed and elaborated in a variety of papers. See, e.g., G. Pancheri, Nuovo Cimento 60, 321 (1969); G. Pancheri-Srivastava, Phys. Lett. 44B, 109 (1973); M. Greco, G. Pancheri-Srivastava, and Y. Srivastava, *ibid.* 56B, 367 (1975); Nucl. Phys. B101, 234 (1975).

[10] See, for example, J. M. Cornwall and G. Tiktopoulos, Phys. Rev. D 13, 3370 (1976); 15, 2937 (1977); T. Kinoshita and A. Ukawa, *ibid.* 15, 1596 (1977); 16, 332 (1977).

[11] Using the coherent-state method, IR-finite, exponential expressions have been obtained by M. Greco, F. Palumbo, G. Pancheri-Srivastava, and Y. Srivastava, Phys. Lett. 77B, 282 (1978). An alternative scheme has been proposed by D. Butler and C. Nelson, Phys. Rev. D 18, 1196 (1978).

[12] Yu. L. Dokshitser, D. I. D'yakonov, and S. I. Troyan, Leningrad Lectures, SLAC Report No. SLAC-Trans-183, 1978 (unpublished); J. Frenkel, M. Shailer, and J. C. Taylor, Nucl. Phys. B148, 228 (1979).

[13] G. Altarelli, G. Parisi, and R. Petronzio, Phys. Lett. 76B, 351 (1978); 76B, 356 (1978); R. Petronzio, in *Phenomenology of Quantum Chromodynamics*, proceedings of XIII Rencontre de Moriond, Les Arcs, Savoie, France, 1978 (Editions Frontières, Gif-sur-

Yvette, 1978), p. 129.

[14]For an excellent review and detailed bibliography, see R. Field, in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979), p. 743.

[15]B. E. Y. Svenson and L. Sollin, Phys. Rev. Lett. 34, 1199 (1975).

[16]L. Gutay, in *Proceedings of the Conference on Hadronic Matter at High Density, Erice, Italy, 1978*, edited by N. Cabibbo and L. Sertorio (Plenum, New York, to be published).

[17]D. Sivers, R. Blankenbecler, and S. Brodsky, Phys. Rep. 23C, 1 (1976).

[18]S. Childress *et al.*, Northeastern University Report No. NUB-2370, 1978 (unpublished), submitted to the 19th International Conference on High Energy Physics, Tokyo, 1978.

[19]We obtained similar results (with $\beta \simeq 11$–13) for the 225-GeV data for $\pi p \to \mu^+\mu^- + X$ of K. Anderson *et al.*, Chicago Report No. EFI 78-38, 1978 (unpublished), submitted to the 19th International Conference on High Energy Physics, Tokyo, 1978.