To be submitted to
Journal of Physics C

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-80/3(P)
17 Gennaio 1980

E. Brézin and G. Parisi: EXPONENTIAL TAIL OF THE ELECTRONIC DENISTY OF LEVELS IN A RANDOM POTENTIAL.

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-80/3(P)
17 Gennaio 1980

E. Brézin[(x)] and G. Parisi: EXPONENTIAL TAIL OF THE ELECTRONIC DENSITY OF LEVELS IN A RANDOM POTENTIAL.

ABSTRACT.

The density of levels of a disordered system falls off exponentially at large negative energies in the region of localized states if the randomness is Gaussian. We compute the absolute normalization of the pre-exponential factor in order to make contact with available two dimensional numerical results.

The problem of the density of states of an electron moving in a random potential has received recently very much attention, especially near the mobility edge. Far from this point,. for large negative energies in the localized region the theory is well understood. The exponential tail of the density of levels was determined by Halperin and Lax 1966, 1967, and by Langer and Zittarz 1966.

A recent numerical calculation was performed by Thouless and Elzain 1978 in two dimensions and a comparison was done with the theoretical estimates. However the normalization of the asymptotic exponential tail was unknow and this made the comparison somewhat

(x) DPhT, CEN Saclay, BP2, 91990 Gif-sur-Yvette, France .

uneasy. In this note we report the results of the calculation of this normalization. In fact it can be deduced without further work from the asymptotic estimates made by Brézin and Parisi 1978 for the N-component Landau-Ginzburg for arbitrary N. The density of levels in a random potential is related to the N = 0 limit. This fact has already been noted by several authors ; in particular Cardy 1978 studied recently this problem from exactly the same viewpoint. However he used a renormalized energy which is different from that of Thouless and Elzain ; in this approach the pre-exponential power is drastically modified in two dimensions. Furthermore he did not compute the overall normalization.

For the N-component Landau-Ginzburg model in d dimensions, the effective "Hamiltonian" being

$$\mathcal{H} = \int d^d x \left\{ \sum_1^N {}_i \left[\frac{1}{2}(\partial_\mu \varphi_i)^2 + \frac{m^2}{2} \varphi_i^2 \right] + \frac{1}{4} g |m|^{4-d} \left(\sum_1^N {}_i \varphi_i^2 \right)^2 \right\} \quad (1)$$

we know the large order behaviour of the perturbative series for any correlation function G(g) ; i. e. if we expand it in powers of g as :

$$G(g) = \sum_0^\infty {}_k (-g)^k G_k \quad , \qquad (2)$$

one finds for large k :

$$G(k) \sim k!\, a^k k^b c \left[1 + 0(1/k) \right] \quad , \qquad (3)$$

where the numbers a, b, c have been computed by Brézin and Parisi 1978 for all correlation functions, d = 2 and d = 3, arbitrary N.

Equivalently this information can be summarized in the following way : G(g) is analytic in a plane cutted along the negative g axis with an essential singularity at g = 0. The discontinuity across the cut behaves for small negative g as :

$$\operatorname{Im} G(g) \underset{|g| \to 0}{\sim} \frac{\pi c}{|ag|^{b+1} \exp(1/|ag|)} \qquad (4)$$

In order to see how these results apply to the random electron problem we briefly recall how the replica method works in this framework. We consider the resolvent kernel

$$G(x,y,E) = \langle x | 1/(E-H) | y \rangle \,, \qquad H = -\Delta + V(x) \tag{5}$$

in which $V(\vec{x})$ are Gaussian independent variables :

$$\overline{V(x)\,V(y)} = w^2 \delta(x-y) \,. \tag{6}$$

We then write the path integral :

$$G(x,y,E) = \\ = -\frac{\int \mathcal{D}\varphi\, \varphi(x)\varphi(y) \exp\left[-\frac{1}{2}\int d^dz\, \varphi(z)\left[-E-\Delta+V(z)\right]\varphi(z)\right]}{\mathcal{D}\varphi \exp\left[-\frac{1}{2}\int d^dz\, \varphi(z)\left[-E-\Delta+V(z)\right]\varphi(z)\right]} \,. \tag{7}$$

The complicated average over $V(x)$ of this ratio is simplified by the replica trick

$$G(x,y,E) = -\lim_{N\to 0} \int \Pi_1^N{}_j\, \mathcal{D}\varphi_j\, \varphi_1(x)\, \varphi_1(y) \cdot \\ \cdot \exp\left\{-\frac{1}{2}\Sigma_1^N{}_i \int d^dz\, \varphi_i(z)\left[-E-\Delta+V(z)\right]\varphi_i(z)\right\} . \tag{8}$$

The average over V is now easy :

$$G_R(x-y,E) \equiv \overline{G(x,y,E)} = \lim_{N\to 0} -\int \Pi_1^N{}_j\, \mathcal{D}\varphi_j\, \varphi_1(x)\, \varphi_1(y) \cdot \\ \cdot \exp\left\{-\int d^dz\left[\frac{1}{2}\Sigma_1^N{}_i\left[(\partial_\mu \varphi_i)^2 - E\varphi_i^2\right] - \frac{w^2}{8}\left(\Sigma_1^N{}_i\, \varphi_i^2\right)^2\right]\right\} . \tag{9}$$

The density of levels is given by

$$n(E) = \frac{1}{\pi}\, \mathrm{Im}\, G_R(x,x,E) \,. \tag{10}$$

We are thus studying the imaginary part of the two point Green function corresponding to the effective "Hamiltonian" eq. (1) if $m^2 = -E$ and the dimensionless coupling constant is given by

$$g = -\frac{1}{2} w^2 / |E|^{2-d/2} \quad . \tag{11}$$

When $E \to -\infty$, g goes to zero and we can apply eq. (4).

The only point which remains to be discussed concernes the energy variable. The asymptotic extimates of Brézin and Parisi 1978 are made in terms of a renormalized mass squared $m_R^2 = -E_R$ and we have to express E in terms of E_R; this involves a trivial one loop diagram: one finds the following results for negative E_R:

$$\tilde{E} \equiv |E - E_o| = |E_R| - \frac{w^2}{4\pi}\left[1 + \ln(4\pi |E_R| / w^2)\right] \quad ; \quad d = 2$$

$$\tilde{E} \equiv |E - E_o| = (|E_R|^{1/2} - \frac{w^2}{8\pi})^2 \quad ; \quad d = 3 \, . \tag{12}$$

Following Thouless and Elzain 1978 E_o has been chosen in such a way that at the one loop level we have:

$$\left.\frac{dE}{dE_R}\right|_{E=E_o} = 0 \, . \tag{13}$$

This definition of E_o is nothing but the edge of the band in the CPA approximation. Eqs. (13) can be readly inverted for large negative E.

The number a, b, c in eq. (3) has been obtained using the saddle point method (Lam 1968, Lipatov 1977): one must find the solution of minimal "action" of the non linear differential equation

$$-\Delta \varphi_c(x) + \varphi_c(x) = \varphi_c^3(x) \quad , \tag{14}$$

and compute the moments $I_k = \int d^D x \, \varphi^k(x)$. One should also compute the renormalized determinants of the operators

$$-\Delta + 1 - \varphi_c^2(x) \; ; \qquad -\Delta + 1 - 3\,\varphi_c^2(x) \, . \tag{15}$$

All these quantities have been extimated by Brézin and Parisi 1978 ; using the notations and the results of this paper, we find for large negative E :

$$\frac{n(E)}{n_o(-E)} \rightarrow \frac{I_6 - I_4}{8\pi^2} \left[I_4 \bar{D}_R(1/3)/\bar{D}_R(1) \right]^{1/2} \left(\frac{4\pi \tilde{E}}{w^2} \right)^{\frac{3}{2} - \frac{I_4}{8\pi}} \exp\left(- \frac{\tilde{E} I_4}{2 w^2} - \frac{I_4}{8\pi} \right)$$

$$\simeq 0.120 \left(\frac{4\pi\tilde{E}}{w^2} \right)^{0.5689} \exp\left[- \frac{4\pi\tilde{E}}{w^2} \, 0.9311 \right] \qquad \text{if } d = 2 \, ,$$

(16)

$$\frac{n(E)}{n_o(-E)} \longrightarrow I_4^{1/2} \left(\frac{I_6 - I_4}{3 I_4} \right)^{\frac{3}{2}} \exp\left(- \frac{I_4}{16\pi} \right) \left[\bar{D}_R(1/3)/\bar{D}_R(1) \right]^{1/2} \frac{\tilde{E}}{w^4} \, \cdot$$

$$\cdot \, \exp\left[- \frac{I_4 \tilde{E}^{1/2}}{2 w^2} \right] \simeq 3.092 \left(\frac{8\pi \tilde{E}^{1/2}}{w^2} \right)^2 \exp\left(- 1.504 \, \frac{8\pi \tilde{E}^{1/2}}{w^2} \right)$$

$$\text{if } d = 3 \, .$$

Where $n_o(E)$ is the density of levels of free electrons ($w^2 = 0$) which of course vanishes for negative E.

If we compare our results with the numerical experiment in two dimensions, the agreement is satisfactory, though it is difficult to extract from the numbers a detailed determination of the pre-exponential factors.

E. Brézin is happy to thank the Laboratories of Frascati and the Institute of Physics of Rome for their kind hospitality.

REFERENCES.

Brézin E., Parisi G. 1978, J. Stat. Phys. 19, 269.

Cardy J.L. 1978, J. Phys. C11, L321.

Halperin B.I., Lax M. 1966, Phys. Rev. 148, 722.

Halperin B.I., Lax M. 1967, Phys. Rev. 153, 802.

Lam C.S. 1968, Nuovo Cimento 55, 258.

Lipatov L.U. 1977,

Thouless D.J., Elzain M.E. 1978, J. Phys. C11, 3425.

Zittartz J., Langer J.S. 1966, Phys. Rev. 148, 741.