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ASYMPTOTIC PREDICTIONS FOR EXCLUSIVE PROCESSES

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It is argued that in QCD it is possible to compute the asymptotic behaviour of exclusive processes such as form factors, threshold behaviour of structure functions, production of low-mass jets, elastic scattering, ... Only a few phenomenological parameters are needed as an input.

It is often stated that no firm predictions can be made in QCD for exclusive processes. This point of view is too pessimistic: in this note we will argue that it is possible to compute the asymptotic behaviour of form factors, structure functions at threshold, the production of two fixed-mass jets in e^+e^- collisions, elastic scattering at a fixed angle and similar exclusive processes.

Let us concentrate our attention on the simplest case, i.e. the behaviour of the form factor as a function of q^2 when $q^2 \to \infty$. The renormalization group techniques have been used to compute the form factor of the parton [1,2]; at first order in perturbation theory dimensional counting rules may be derived [3,4] for the bound-state form factor: the difficult problem is to find the corrections to these rules. Particularly large corrections may be expected in gauge theories: the electromagnetic form factors of the electron and the quarks are respectively given by [2,5]:

$$F_e(q^2) \to \exp[-(\alpha/4\pi)\ln^2 q^2]\,,$$

$$F_q(q^2) \to \exp[-\tfrac{8}{25}\ln q^2 \ln(\ln q^2)]$$

$$= [q^2]^{-(8/25)\ln(\ln q^2)}\,. \qquad (1)$$

Both go to zero faster than any power of q^2. If we assume that the form factor of the bound state cannot be larger than the form factor of its constituents, the bound-state form factor too must decrease faster than any power. The same conclusion has been reached using the prediction that the structure function at fixed mass goes to zero faster than any power of q^2 [6].

Both arguments are wrong: the correct result for the form factor of the pion is [7,8]:

$$F_\pi(q^2) \to (8\pi/q^2)\,\alpha(q^2) f_\pi^2\,, \qquad (2)$$

where $f_\pi = 132$ MeV.

It is easy to understand why the first argument is wrong [‡1]: let us consider the case of QED. The elastic electron form factor corresponds to the process in which the electron is accelerated without emitting photons. The probability for such an event is given by $\exp(-\langle n\rangle)$, where $\langle n\rangle$ is the mean number of emitted photons which can be estimated semiclassically: $\langle n\rangle \propto \alpha \ln^2 q^2$ [9]. During the elastic scattering of positronium, it is possible that the electron is accelerated when near to the positron and immediately after the collision the electron shares its momentum with the positron. The amount of classical radiation is small because of local charge compensation. This event may be

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[‡1] The second argument is wrong because twist-4 operators have been neglected in the light-cone expansion. They do not have large anomalous dimensions and their contribution is proportional to $1/q^2$; in the really asymptotic region the structure function at fixed mass will be dominated by the twist-4 and not by the twist-2 operators, as is often assumed.

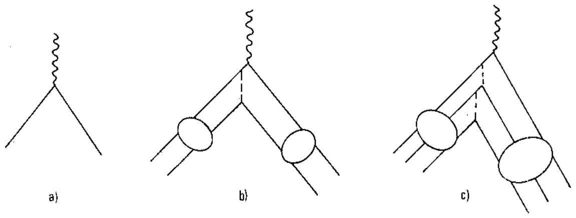

Fig. 1. In (a) we show a diagram for the parton elastic form factor, these diagrams are strongly suppressed by soft gluon emission; in (b) and (c) we show the leading diagrams contributing to the pion and the proton form factor, respectively; these diagrams are not suppressed by soft gluon emission.

represented by diagram b of fig. 1 and its contribution is proportional to the wave function at the origin. As far as the behaviour of the form factors of singlet states is concerned, there is no serious difference between gauge theory and the other renormalizable theories [10].

The corrections to the dimensional rules have been extensively studied in renormalizable non-gauge theories [11]. Their results may be summarized in the following form factor expansion, which we write directly in the case of an asymptotically free theory:

$$J(p-p') \xrightarrow[(p-p')^2 = q^2 \to \infty]{} \sum_{a,b} C_{a,b}[\alpha(q^2)] \times q^{2[(\tau_a+\tau_b)/2-1]} (\ln q^2)^{\gamma_a+\gamma_b} \times O_a(p)|0\rangle\langle 0|O_b(p'), \tag{3}$$

where all the references to Lorentz indices have been suppressed for simplicity; O_a is a generical local operator, τ_a is the twist of O_a, γ_a is the anomalous dimension and $\alpha(q^2)$ is the running coupling constant. The coefficient function $C_{a,b}$ can be computed in perturbation theory. If we apply eq. (3) to the asymptotic behaviour of the transition form factor from A to B we find:

$$\langle \mathrm{A}|J|\mathrm{B}\rangle = F_{\mathrm{AB}}(q^2) \to \sum_{a,b} C_{a,b}(\alpha(q^2)) \times \frac{\langle \mathrm{A}|O_a|0\rangle\langle 0|O_b|\mathrm{B}\rangle}{q^{2[(\tau_a+\tau_b)/2-1]}} (\ln q^2)^{-(\gamma_a+\gamma_b)/2}. \tag{4}$$

The operator O_a is an interpolating field for the state A and eq. (4) tells us that the form factor is dominated by the interpolating fields having lower twist, and among those having the same twist, by the one having minimal anomalous dimension [12].

If we apply eq. (4) to the partons themselves we find:

$$F_{\mathrm{P}}(q^2) \to 1/(\ln q^2)^{\gamma_{\mathrm{P}}}, \tag{5}$$

where γ_{p} is the anomalous dimension of the parton field.

Eqs. (4), (5) cannot hold in gauge theories: they clash with eq. (1). However, in gauge theories γ_{p} is not gauge invariant. It has been noticed that a gauge-invariant definition of $\gamma_{\mathrm{p}}^{\mathrm{GI}}$ [13] can be given by [14]:

$$\gamma_{\mathrm{p}}^{\mathrm{GI}} = \lim_{n\to\infty} \gamma_n, \tag{6}$$

where γ_n is the anomalous dimension of the operator O_n of twist 2 and spin n; indeed, eq. (6) holds in all renormalizable theories, gauge theories excluded. An explicit computation shows that $\gamma_n \sim \ln n$. The gauge-invariant anomalous dimension is infinite; if we interpret this result by saying that the effective gauge-invariant anomalous dimension increases like $\log q^2$, we recover eq. (1). We conjecture that using this definition of anomalous dimensions of non-singlet operators, eqs. (3), (4) are valid also for gauge theories.

The asymptotic behaviour of the form factor is dominated by the twist-2 operators, in particular by the axial current; that explains the presence of f_π in eq. (3).

We can apply eq. (4) to the study of the threshold behaviour of the electromagnetic structure function $F_2^\pi(x, q^2)$ of the pion. The structure function can be written as the sum of all transition form factors from the pion to any state having mass $W^2 = m_\pi^2 + ((1-x)/x)|q^2|$. In the limit $q^2 \to \infty$ at fixed mass W^2, one obtains:

$$F_2^\pi(W^2, q^2) \to C' f_\pi^2 (\alpha^2(q^2)/q^2)\, \mathrm{Im}\,\Pi_{\mathrm{A}}(W^2), \tag{7}$$

where $\mathrm{Im}\,\Pi_{\mathrm{A}}(W^2)$ is the imaginary part of the axial current propagator. In the region where W^2 is not too small, it may be convenient to add also the contributions of the other operators and use the asymptotic estimate for their imaginary part. One finds:

$$F_2^\pi(W^2, q^2) \to f_\pi^2 C_n' (\alpha^2(q^2)/q^2)\, [\ln W^2/\ln Q^2]^{\gamma_n}, \tag{8}$$

where C_n' are computable constants and γ_n are the anomalous dimensions of the twist-2 operators.

Similar results may be obtained for the proton:

$$F_p(q^2) \to C'' f_p^2 \alpha^2(q^2)/[(q^2)^2 (\ln q^2)^{\gamma_{O_3}}] ,$$

$$F_2^p(W^2, q^2) \to C'' \alpha^2(q^2) F_p(q^2) \times \mathrm{Im}[\Pi_3(W^2)]/(\ln q^2)^{\gamma_{O_3}} , \quad (9)$$

where $f_p = \langle 0|O_3|p\rangle$, O_3 being the three-quark operator of twist-3 having minimal anomalous dimensions γ_3 and $\Pi_3(W^2)$ is the O_3 propagator. Unfortunately, in this case f_p is not known, although its order of magnitude may be estimated in the bag model. Notice that f_p cancels in the predictions for the threshold behaviour of the structure function.

We can also use the form factor expansion (eq. (3)) in order to compute the cross section to produce, in e^+e^- collisions, two clusters of mass squared W_1^2, $W_2^2 \ll Q^2$. The same familiar chain of arguments leads to:

$$\frac{1}{\sigma_T} \frac{d^2\sigma}{dW_1^2\, dW_2^2} \to \sum_{n,m} \frac{C'_{n,m} (\ln W_1^2)^{\gamma_n} (\ln W_2^2)^{\gamma_m} \alpha^2(Q^2)}{Q^4 (\ln Q^2)^{\gamma_n + \gamma_m}} , \quad (10)$$

where the constants $C'_{n,m}$ are computable.

As noticed in ref. [8] a heuristic interpretation of eq. (2) can be given in the effective parton language. Let us denote by $\phi^\pi(x, 1-x; q^2)$ the effective q^2-dependent amplitude to find a quark of momentum xP and an antiquark of momentum $(1-x)P$ inside a pion in the infinite-momentum frame. This amplitude may be written as the sum of contributions of operators of twist 2 (twist 3 for the proton) and when q^2 goes to infinity only the lowest dimension operator (i.e. the axial current) survives. In this limit we know the explicit form of the wave function and eq. (4) can be simply obtained by plugging the asymptotic expression for the wave function in the first-order diagrams of fig. 1. This observation suggests that the same procedure can be used to obtain parameter-free asymptotic predictions for large-angle scattering.

We conjecture that the correct results are obtained by computing perturbatively the large-angle scattering using as an input the same form of the wave function of the hadron as for the electro-magnetic form factor. In this way we reproduce the result of the dimensional counting rules [15]; for proton–proton scattering we obtain:

$$d\sigma/dt \to t^{-10} f_p^8 h[s/t, \alpha(t)]/(\ln t)^{4\gamma_{O_3}}$$

$$= t^{-2} F_4(p)\, \tilde{h}[s/t, \alpha(t)] . \quad (11)$$

The function $\tilde{h}[s/t, \alpha(t)]$ is now computable because we know the asymptotic form of the proton wave function, the only unknown parameter being f_p. In the conventional approach the wave function is unknown and the function $\tilde{h}[s/t, \alpha(t)]$ is incomputable; this great simplification is an effect of anomalous dimensions.

Some complication may arise from the so-called Landshoff diagrams [16]. In perturbation theory they violate the dimensional scaling laws and give a contribution proportional to t^{-8} to large-angle proton scattering. However, these diagrams represent the independent scattering of partons in different points of space–time. No local charge compensation is present and strong gluon bremsstrahlung is expected. The Landshoff process is depressed by $[F_q(t)]^{12}$ and it is negligible at large energies [17].

We are led to the conclusion that most of the large transverse momentum exclusive processes are computable in QCD, absolute normalization included, using as an input only a very small number of phenomenological parameters. The verification of all these predictions would be a very serious test of QCD.

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