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QCD JETS FROM COHERENT STATES

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A recently proposed approach to the problem of infrared and mass singularities in QCD, based on the formalism of coherent states, is extended to discuss massless quark and gluon jets. Our results include all leading ($\ln \delta$) terms as well as finite terms in the energy loss ϵ, in addition to the usual $\ln \epsilon$ associated with $\ln \delta$. Our formulae agree with explicit perturbative calculations, whenever available. Explicit expressions for the total K_T distributions are given which take into account transverse-momentum conservation. Predictions are also made for the Q^2 dependence of the mean K_T^2 for quark and gluon jets. Our jet k_T distributions are extrapolated for low k_T and shown to describe with good accuracy the data for $e\bar{e} \to q\bar{q} \to$ hadrons. Numerical predictions are also presented for the forthcoming PETRA, PEP and LEP machines.

1. Introduction

Experimental evidence [1] for jets in e^+e^- annihilation and the observed [2] features of scaling violations in deep inelastic scattering have given great support to the quark parton model and its underlying theory, quantum chromodynamics (QCD). Considerable efforts have been recently devoted to working out quantitative predictions of QCD for hard processes [3]. After the work of Sterman and Weinberg [4], jet phenomena have been considered in great detail. The basic idea is to compute suitable defined jet cross sections which are free of infrared and mass singularities [5]. Much progress has been made in calculating higher-order terms in the leading logarithmic approximation, for various processes. These leading terms have been shown to sum up to an exponential form, for which a very convenient infrared, mass-singularity free coherent-state formalism has been developed [6,7].

Let us briefly summarize the basic idea of this approach. The starting point is

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the well-known fact that all infrared singularities arising from real and virtual soft quanta have to cancel in the physical cross sections [8,9]. This cancellation occurs to all orders of perturbation theory. The formalism of coherent states, developed in ref. [6], indeed provides one with matrix elements free from infrared singularities, at the leading log approximation. This has been obtained by extending from QED [10] to QCD the concept of classical currents associated with the external particles to incorporate two new properties: colour and the appearance of an effective coupling constant $\bar{\alpha}(\tau)$, where τ is the infrared variable. The inclusive cross sections obtained at this level still develop mass singularities in the limit $m \to 0$, where m is the fermion mass. According to the Kinoshita-Lee-Nauenberg theorem [9], these mass singularities have to be cancelled in this limit in the observable cross sections, through the hard emission of quanta collinear with the external particles. The extension of the coherent-state formalism to include this hard component at all orders finally led to a definition of a super-inclusive or jet cross section [7] which is finite and may be confronted with experiment.

In the present paper we extend the work of ref. [7] in many aspects. By a more accurate kinematical analysis of the emission of hard quanta, we are able to include those finite ϵ corrections which are proportional to $\ln \delta$ (see below for definitions) for quark jets.

Using the Altarelli-Parisi [11] quark and gluon probabilities we extend our formulae to include gluon jets, in agreement with lowest-order calculations [12]. Finite ϵ terms are explicitly taken into account. A general formula for an arbitrary number of quark and gluon jets is then obtained.

The transverse-momentum properties of the jets are further investigated. In particular we propose new criteria to test the QCD jet predictions in e^+e^- annihilation based on properties which rest on the exponentiated forms obtained upon summation of all orders in perturbation theory. More explicitly, by taking into account the conservation of transverse momentum, we consider the total K_T distribution of the jets which can be directly measured in the actual experiments. We predict the Q^2 dependence of $\langle K_T^2 \rangle$ for quark and gluon jets. We explicitly confirm the general findings [12,13] that gluon jets are broader than quark jets.

Finally, we present a phenomenological extrapolation of our formalism to a K_T region not directly accessible to perturbation theory and compare our predictions with the transverse momentum distributions of single hadrons measured at SPEAR [1]. Our analysis shows good agreement with data at various energies. Predictions are also made for energies accessible in the near feature with PETRA and PEP machines.

A short presentation of our results has been given in ref. [14].

2. Jet cross sections

Our analysis begins with a formula for a jet cross section, derived in ref. [7], for

$e^+e^- \to q\bar{q}$:

$$d\sigma_{\text{super}} = d\sigma_0 \exp\left\{-\frac{1}{\pi^2}\int_{\Delta\omega/E}^{1} dx\, \mathcal{P}_{gq}(x) \int_{k_{1T}}^{k_{2T}} \frac{d^2k_T}{k_T^2}\bar{\alpha}(k_T)\right\}, \tag{1}$$

where the gluon distribution due to the quarks is [11]

$$\mathcal{P}_{gq}(x) = C_F\left\{\frac{1+(1-x)^2}{x}\right\}, \tag{2}$$

$C_F = (N_c^2 - 1)/2\,N_c$ for SU(N_c) colour, $\bar{\alpha}(k_T)$ is the running coupling constant

$$\bar{\alpha}(k_T) = \frac{12\pi}{(33-2N_f)}\,\frac{1}{\ln(k_T^2/\Lambda^2)},$$

and $d\sigma_0$ is the zeroth-order point-like cross section. The lower limit k_{1T} in the k_T integral in eq. (1) defines the maximum transverse momentum relative to the jet axis, namely, $k_{1T} \equiv E\delta$, where δ is the half-angle of the jet cone *, while $k_{2T} = \frac{1}{2}Q = E$. $(\Delta\omega/E)$ represents the fraction of the quark energy which is emitted outside the jet cone. Then the ratio $(d\sigma_{\text{super}}/d\sigma_0)$ in eq. (1) gives the probability of finding a fraction $\epsilon \equiv \Delta\omega/2E$ of the total energy $2E$ outside a pair of opposite directed cones of half-angle δ.

First-order expansion of eq. (1) directly gives the result of Sterman and Weinberg, up to terms of order $\epsilon \ln \delta$. A more accurate kinematical analysis shows that the upper limit in x should be restricted to $1 - \Delta\omega/E = 1 - 2\epsilon$. This gives

$$I_q(\epsilon) \equiv \int_{2\epsilon}^{1-2\epsilon} dx\,\mathcal{P}_{gq}(x) = -C_F\{2\ln 2\epsilon + \tfrac{3}{2} - 2\epsilon(1-2\epsilon) + O(\epsilon^3)\}, \tag{3}$$

which agrees with the earlier calculation of Stevenson [15]. We mention in passing that this modification restores the complete symmetry between the quark and gluon x distributions, as expected to hold generally for massless quarks and gluons. In fact the corresponding quark distribution is [11]

$$\mathcal{P}_{qq}(x) = C_F\left\{\frac{1+x^2}{1-x}\right\}, \tag{4}$$

and thus

$$\int_{2\epsilon}^{1-2\epsilon} \mathcal{P}_{qq}(x)\,dx = \int_{2\epsilon}^{1-2\epsilon} \mathcal{P}_{gq}(x)\,dx. \tag{5}$$

The generalization of eq. (1) to gluon jets is obtained as follows. The probability

* To conform to the general notation [4,5] we have changed the definition of the angle δ which is a half of that used in ref. [7].

$P_{gq}(x)$ in eq. (2) has to be replaced by [11]

$$\mathcal{P}_{gg}(x) + N_f\,\mathcal{P}_{qg}(x) = N_c \left\{\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right\} + N_f \frac{x^2 + (1-x)^2}{2}, \qquad (6)$$

corresponding to the sum of the probabilities that a gluon radiates gluons or N_f ($q\bar{q}$) pairs. Just as in eq. (3) we have

$$I_g(\epsilon) \equiv \int_{2\epsilon}^{1-2\epsilon} [\mathcal{P}_{gg}(x) + N_f\,\mathcal{P}_{qg}(x)]\,dx$$

$$= -N_c \left\{2 \ln 2\epsilon + \left(\tfrac{11}{6} - \tfrac{1}{3}\frac{N_f}{N_c}\right) - 4\epsilon\left(1 - \frac{N_f}{2N_c}\right)(1-2\epsilon) + O(\epsilon^3)\right\}. \qquad (7)$$

The logarithmic and the constant terms in eq. (7) agree with those found by explicit perturbative calculations in refs. [12]. Finite terms in ϵ have not as yet been explicitly calculated in perturbation theory.

Notice that our distributions in ϵ for $I_q(\epsilon)$ and $I_g(\epsilon)$ have been obtained without any resource to singular distributions, unlike in the Altarelli-Parisi procedure [11]. The difference arises due to the different range of integration in x.

Using the above results the super-inclusive cross sections for n_q and n_g quark and gluon jets, respectively, are thus given by

$$d\sigma_{\text{super}}^{(n_q, n_g)} = d\sigma_0 \exp\left\{-\frac{1}{\pi}[n_q I_q(\epsilon) + n_g I_g(\epsilon)] \int_{k_{1T}}^{k_{2T}} \frac{d^2k}{k_T}\,\bar{\alpha}(k_T)\right\}, \qquad (8)$$

where, for the sake of simplicity, we have assumed the same kinematical limits k_{1T} and k_{2T} for all jets. The jet cross section (8) gives the leading terms in $\ln\delta$. All non-leading terms in δ are lumped in $d\sigma_0$ and have to be calculated for each process separately in perturbation theory. First-order expansion of eq. (8) agrees with the result of Smilga and Visotsky [5].

3. Transverse-momentum distributions

Now let us discuss the transverse-momentum distributions for $e^+e^- \to q\bar{q}$. A similar analysis can be performed in the more general case. From eq. (8), the factor

$$F_q(\epsilon, K_{1T}) \equiv \exp\left\{-\frac{2}{\pi} I_q(\epsilon) \int_{k_{1T}}^{Q/2} \frac{dk_T}{k_T}\,\bar{\alpha}(k_T)\right\}, \qquad (9)$$

gives the probability that a fraction ϵ of the total energy Q is emitted outside the oppositely directed cones of maximum transverse momentum $K_{1T} = E\delta$. We have, therefore,

$$\int_0^{K_{1\mathrm{T}}} \left(\frac{\mathrm{d}P}{\mathrm{d}K_\mathrm{T}}\right) \mathrm{d}K_\mathrm{T} = F(\epsilon, K_{1\mathrm{T}})\,, \tag{10}$$

where $(\mathrm{d}P/\mathrm{d}K_\mathrm{T})$ defines the K_T distribution, and for simplicity, we have omitted the quark index q. From eqs. (9) and (10) we have

$$\frac{\mathrm{d}P}{\mathrm{d}K_\mathrm{T}} \approx \frac{2}{\pi} I(\epsilon) \frac{\bar{\alpha}(K_\mathrm{T})}{K_\mathrm{T}} \left\{\frac{\bar{\alpha}(\frac{1}{2}Q)}{\bar{\alpha}(K_\mathrm{T})}\right\}^{I(\epsilon)\pi b}\,, \tag{11}$$

where $b = (33 - 2N_\mathrm{f})/12\pi$ and we have used the asymptotic freedom formula for $\bar{\alpha}$. Thus eq. (11) is valid only for $K_\mathrm{T} \gg \Lambda$. Of course, for our formula (8) to be applicable, we must also require $K_\mathrm{T} \ll \frac{1}{2}Q$.

So far, any correlation in the transverse momentum has been neglected, since it was assumed that the individual emission was statistically independent. To impose transverse-momentum conservation we introduce a factor

$$L(K_{1\mathrm{T}}) = \begin{cases} 1\,, & \text{for } \sum_n \boldsymbol{k}_{n\mathrm{T}} \leqslant \boldsymbol{K}_{1\mathrm{T}}\,, \\ 0\,, & \text{otherwise}\,, \end{cases}$$

$$= \frac{1}{4\pi^2} \int_0^{K_{1\mathrm{T}}} \mathrm{d}^2K_\mathrm{T} \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{x}\, \exp\{i(\sum_n \boldsymbol{k}_{n\mathrm{T}} - \boldsymbol{K}_\mathrm{T})\cdot \boldsymbol{x}\}\,, \tag{12}$$

in the nth-order expansion of the cross section which includes only the hard radiation, defined in ref. [7]. This leads to

$$\mathrm{d}\sigma_\text{hard} = \frac{\mathrm{d}\sigma_0}{4\pi^2} \int_0^{K_{1\mathrm{T}}} \mathrm{d}^2K_\mathrm{T} \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{x}\, \mathrm{e}^{-i\boldsymbol{x}\cdot \boldsymbol{K}_\mathrm{T}}\,.$$

$$\times \exp\left\{\frac{1}{\pi^2} I(\epsilon) \int_{m^2}^{Q^2/4} \frac{\mathrm{d}^2k_\mathrm{T}}{k_\mathrm{T}^2} \bar{\alpha}(k_\mathrm{T})\, \mathrm{e}^{i\boldsymbol{x}\cdot \boldsymbol{k}_\mathrm{T}}\right\}\,. \tag{13}$$

This has to be multiplied by the inclusive cross sections (virtual + real soft radiation) to obtain the super-inclusive one, which is free of the mass singularities:

$$\mathrm{d}\sigma_\text{super} = \frac{\mathrm{d}\sigma_0}{4\pi^2} \int_0^{K_{1\mathrm{T}}} \mathrm{d}^2K_\mathrm{T} \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{x}\, \mathrm{e}^{-i\boldsymbol{x}\cdot \boldsymbol{K}_\mathrm{T}}$$

$$\times \exp\left\{-\frac{1}{\pi^2} I(\epsilon) \int_0^{Q^2/4} \frac{\mathrm{d}^2k_\mathrm{T}}{k_\mathrm{T}^2} \bar{\alpha}(k_\mathrm{T})[1 - \mathrm{e}^{i\boldsymbol{x}\cdot \boldsymbol{k}_\mathrm{T}}]\right\}\,. \tag{14}$$

Thus eq. (14) is our replacement of eq. (8) for the two quark jets. We then obtain

$$\frac{d^2P}{d^2K_T} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d^2\boldsymbol{x}\, e^{-i\boldsymbol{x}\cdot\boldsymbol{K}_T} \exp\left\{-\frac{1}{\pi^2}I(\epsilon) \int_0^{Q^2/4} \frac{d^2k_T}{k_T^2}\, \bar{\alpha}(k_T)[1 - e^{i\boldsymbol{x}\cdot\boldsymbol{k}_T}]\right\} \tag{15a}$$

$$= \frac{1}{2\pi} \int_0^{\infty} x\, dx\, J_0(x\cdot K_T) \exp\left\{-\frac{2}{\pi}I(\epsilon) \int_0^{Q/2} \frac{dk_T}{k_T}\, \bar{\alpha}(k_T)[1 - J_0(x\cdot k_T]\right\}, \tag{15b}$$

which is properly normalized, for any fixed ϵ, to

$$\int_0^{Q/2} \left(\frac{d^2P}{d^2K_T}\right) d^2K_T = 1\,. \tag{16}$$

Integrating over K_T up to some K_{1T} we find for the probability of finding total transverse momentum K_{1T} and missing fractional energy ϵ

$$P(\epsilon, K_{1T}) = K_{1T} \int_0^{\infty} dx J_1(x\cdot K_T) \exp\left\{-\frac{2I(\epsilon)}{\pi} \int_0^{Q/2} \frac{dk_T}{k_T} \bar{\alpha}(k_T)[1 - J_0(x\cdot k_T)]\right\}. \tag{17}$$

This equation replaces the earlier expression for $F(\epsilon, K_{1T})$ given in eq. (9) and is the exact analogue of the fractional probability $f(\epsilon, \delta)$ defined by Sterman and Weinberg.

We postpone to sect. 4 a discussion of the range of validity of our eqs. (15) and (17). For completeness, we now give the expressions of the K_T moments of the distribution (15).

Intergrating eq. (15a) by parts, one simply gets

$$K_j \frac{d^2P}{d^2K_T} = \frac{i}{4\pi^2}\int d^2\boldsymbol{x}\, e^{-i(\boldsymbol{x}\cdot\boldsymbol{K}_T)}[\partial_j h(\boldsymbol{x})]\, e^{-h(\boldsymbol{x})}\,, \tag{18}$$

with

$$h(\boldsymbol{x}) = \frac{1}{\pi^2} I(\epsilon) \int_0^{Q/2} \frac{d^2k_T}{k_T^2} \bar{\alpha}(k_T)[1 - e^{i(\boldsymbol{k}_T\cdot\boldsymbol{x})}]\,. \tag{19}$$

Similarly,

$$K_l K_j \frac{d^2P}{d^2k_T} = \frac{1}{4\pi^2}\int d^2\boldsymbol{x}\, e^{-i(\boldsymbol{x}\cdot\boldsymbol{K}_T)}[\partial_l\partial_j h(\boldsymbol{x}) - \partial_j h(\boldsymbol{x})\cdot\partial_l h(\boldsymbol{x})]\, e^{-h(\boldsymbol{x})}\,. \tag{20}$$

Therefore, we have

$$\langle K_j K_l\rangle \equiv \int d^2K_T\, \frac{d^2P}{d^2K_T}\, K_j K_l$$

$$= \int d^2\boldsymbol{x}\,\delta(\boldsymbol{x})[\partial_i\partial_j h(\boldsymbol{x}) - \partial_j h(\boldsymbol{x}) \cdot \partial_i h(\boldsymbol{x})]\, e^{-h(\boldsymbol{x})}\,, \tag{21}$$

and finally

$$\langle K_T^2 \rangle = \frac{I(\epsilon)}{\pi^2} \int_0^{Q/2} d^2k_T\, \bar{\alpha}(k_T)\,. \tag{22}$$

This result gives a simple connection between the average K_T^2 of a jet and the effective coupling constant $\bar{\alpha}$. Using the asymptotic freedom expression for $\bar{\alpha}$ in the perturbative region, namely $k_T > c\Lambda$, we obtain

$$\langle K_T^2 \rangle = \frac{I(\epsilon)}{\pi} \int_0^{c\Lambda} dk_T^2 \bar{\alpha}(k_T) + \frac{I(\epsilon)\cdot\Lambda^2}{\pi b}\left\{\mathrm{Ei}\left(\ln\frac{Q^2}{4\Lambda^2}\right) - \mathrm{Ei}(\ln c^2)\right\},$$

which for large Q^2 leads to

$$\langle K_T^2 \rangle = \mathrm{const} + \frac{I(\epsilon)}{4\pi}\bar{\alpha}(\tfrac{1}{2}Q)\, Q^2 \left[1 + \mathrm{O}\left(\frac{1}{\ln Q}\right)\right]. \tag{23}$$

The constant term could only be obtained by a knowledge of $\bar{\alpha}(k_T)$ in the non-perturbative region. With the parametrization discussed in sect. 4 we find it negligible for a wide range of c values ($c \gtrsim 1$).

It is interesting to notice that the Q^2 dependence of $\langle K_T^2 \rangle$ is simply obtained from eq. (22) as

$$\frac{d}{dQ^2}\langle K_T^2 \rangle = \frac{I(\epsilon)}{4\pi}\bar{\alpha}(\tfrac{1}{2}Q)\,, \tag{24}$$

and offers, in our approach, a very simple prediction of perturbative QCD.

As a last remark we observe that for fixed ϵ and Q^2 we obtain the ratio

$$\frac{\langle K_T^2 \rangle_{\mathrm{gjet}}}{\langle K_T^2 \rangle_{\mathrm{qjet}}} = \frac{I_g(\epsilon)}{I_q(\epsilon)} \underset{\epsilon\to 0}{\to} \frac{2N_c^2}{N_c^2 - 1}\,, \tag{25}$$

which predicts gluon jets broader by about a factor of two than quark jets. Similar results have been obtained in refs. [12] and [13].

4. Discussion and numerical results

Some remarks are in order concerning the validity of the equations derived in sect. 3. As is clear from eqs. (15) and (17) the range of k_T integrals extends over a non-perturbative region, $k_T \lesssim c$, where our leading logarithmic approximation is questionable. The influence of the non-perturbative region is negligible for $K_T \gg c\Lambda$. which is the region of interest for testing QCD. For a qualitative understanding we observe that approximating $J_0(z)$ by $\theta(1-z)$ and $J_1(z)$ by $\delta(1-z)$ in eqs. (15)

and (17) one obtains the naive expressions (9) and (11). Therefore, the effect of the correlations, imposed by momentum conservation, is only local.

However, it affects the actual distributions in the perturbative region in a sensible way, leading to departures from the uncorrelated formula (9) and (11) for $K_T >> c\Lambda$.

In the following, we will assume the above correlated distributions to be valid in the entire range of K_T. The problem therefore arises of extrapolating the effective coupling constant in the non-perturbative region. In sect. 5 we will discuss a phenomenological parametrization of $\bar{\alpha}(k_T)$ for $k_T \lesssim c\Lambda$ suggested by a previous analysis [16] which compares all data for the ratio $R = \sigma(e^+e^- \to \mu^+\mu^-)$ in the entire energy range ($4m_\pi^2 \leqslant s \leqslant 60$ GeV^3) with the QCD predictions. This amounts to parametrizing the effective coupling constant as

$$\bar{\alpha}(k_T) = \begin{cases} \dfrac{1}{2b \ln c}, & \text{for } k_T \leqslant c\Lambda, \\ \dfrac{1}{2b \ln(k_T/\Lambda)}, & \text{for } k_T > c\Lambda, \end{cases} \tag{26}$$

with $c \simeq 4$, as discussed in detail in sect. 5.

With the above caveats, our results are presented in figs. 1 and 2 for a two-quark jet and in figs. 3 and 4 for a two-gluon jet. Let us discuss them in detail.

In figs. 1a, b the K_T distributions dP/dK_T of eq. (15) are plotted for different values of $\Delta\omega/E \equiv 2\epsilon$, and Q/Λ, and compared with the naive expression (11). As is clear, a close agreement between the two distributuons is only found for a limited range of K_T/Λ, where the correlations from momentum conservation are less important. Furthermore, the ϵ dependence is rather important, particularly for small ϵ. The agreement is better for the integrated distributions, plotted in figs. 2a, b. They are also in good agreement with those calculated by Ellis and Petronzio [5]. Our results show explicitly the very small influence of the details of the non-perturbative region for a large range of K_T values.

The corresponding results for two-gluon jet distributions are shown in figs. 3 and 4 for the same values of ϵ and Q/Λ. We have used $N_f = 4$. Negligible differences have been found between the case $N_f = 4$ and $N_f = 6$. From an inspection of figs. 3a, b it is evident that momentum correlations have much larger effects on gluon jet distributions as compared to quark jets. In much larger effects on gluon jet distributions as compared to quark jets. In the integrated K_T probabilities, however, some compensations occur such that there is a closer agreement with the naive formulae (see fig. 4). Furthermore, as anticipated earlier (see eq. (25)), there is much more broadening of the momentum distributions in the case of gluon jets (compare figs. 1 and 2 to figs. 3 and 4).

We conclude this section by stressing the fact that our predictions concern global properties of the jets, independent of the properties of the hadronization of partons, so that an experimental verification of our results should represent a rather direct test

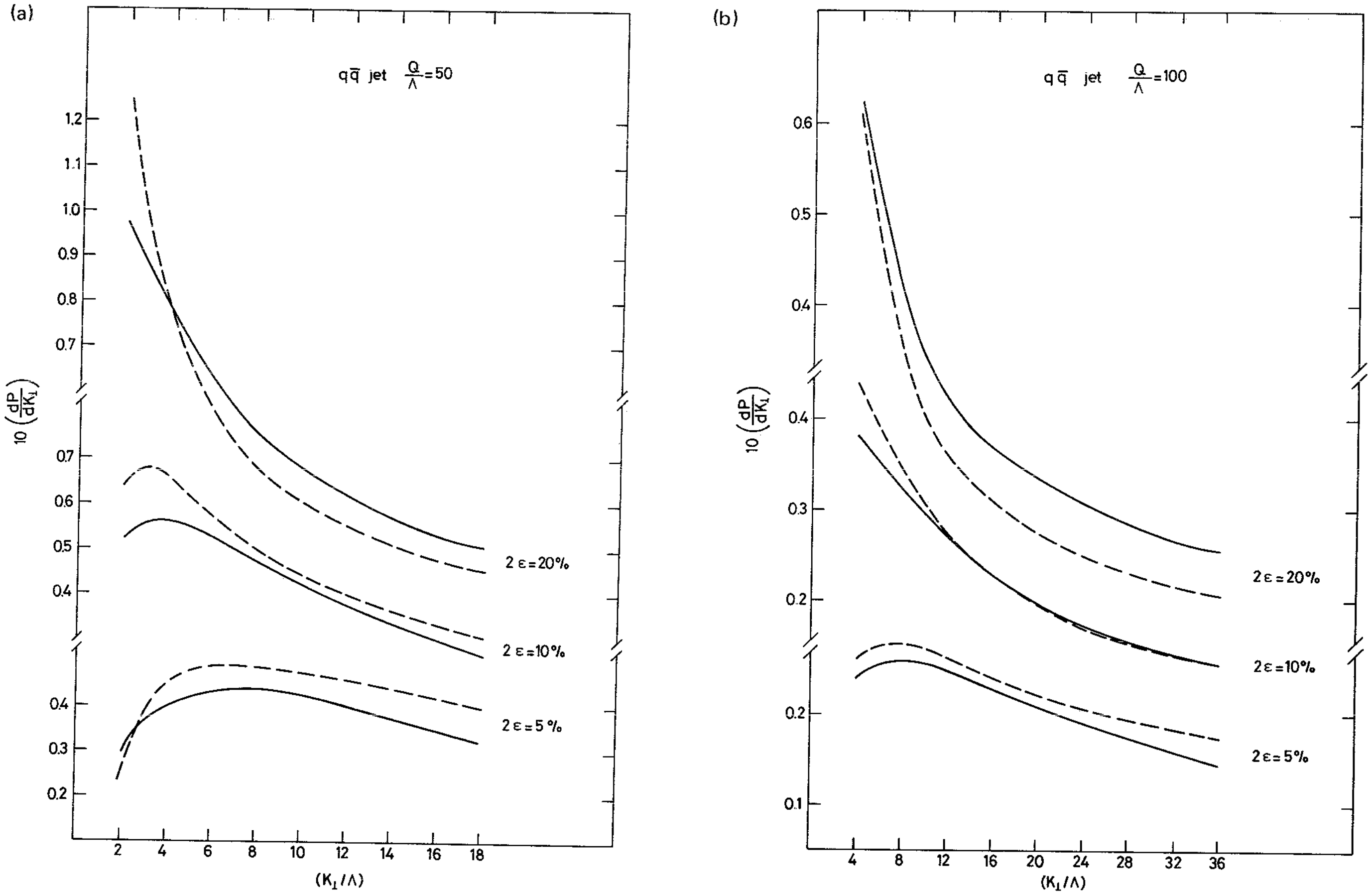

Fig. 1. (a) Total transverse momentum distributions for a q$\bar{q}$ jet at the total energy $Q = 50\ \Lambda$, for various ϵ. The full line refers to the case of transverse momentum conservation taken into account (eqs. (15)). The dashed line refers to the case of uncorrelated emission (eq. (11)). (b) The same, for $Q = 100\ \Lambda$.

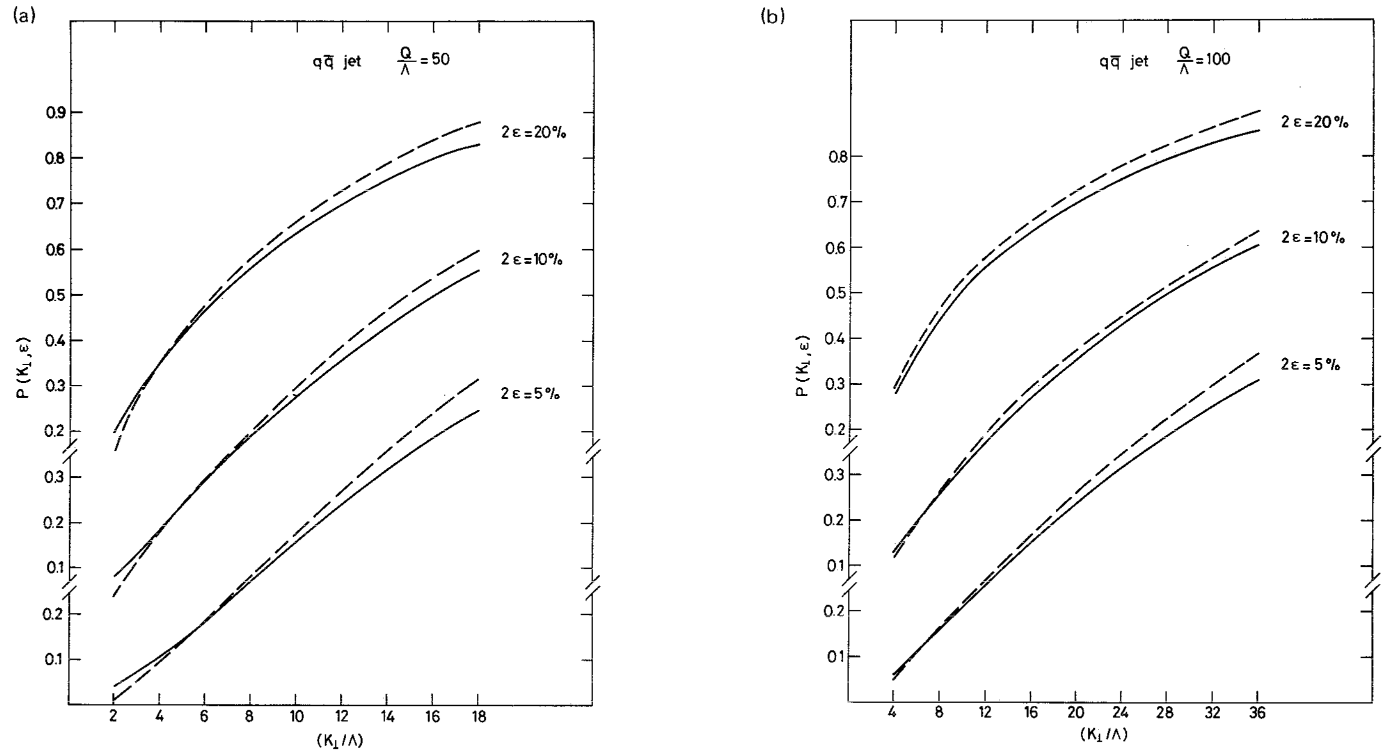

Fig. 2. (a) Probability functions $P(K_T, \epsilon)$ for a q$\bar{q}$ jet at the total energy $Q = 50\ \Lambda$, for various ϵ. Full and dashed lines as in figs. 1 (eqs. (17) and (9), respectively). (b) The same, for $Q = 100\ \Lambda$.

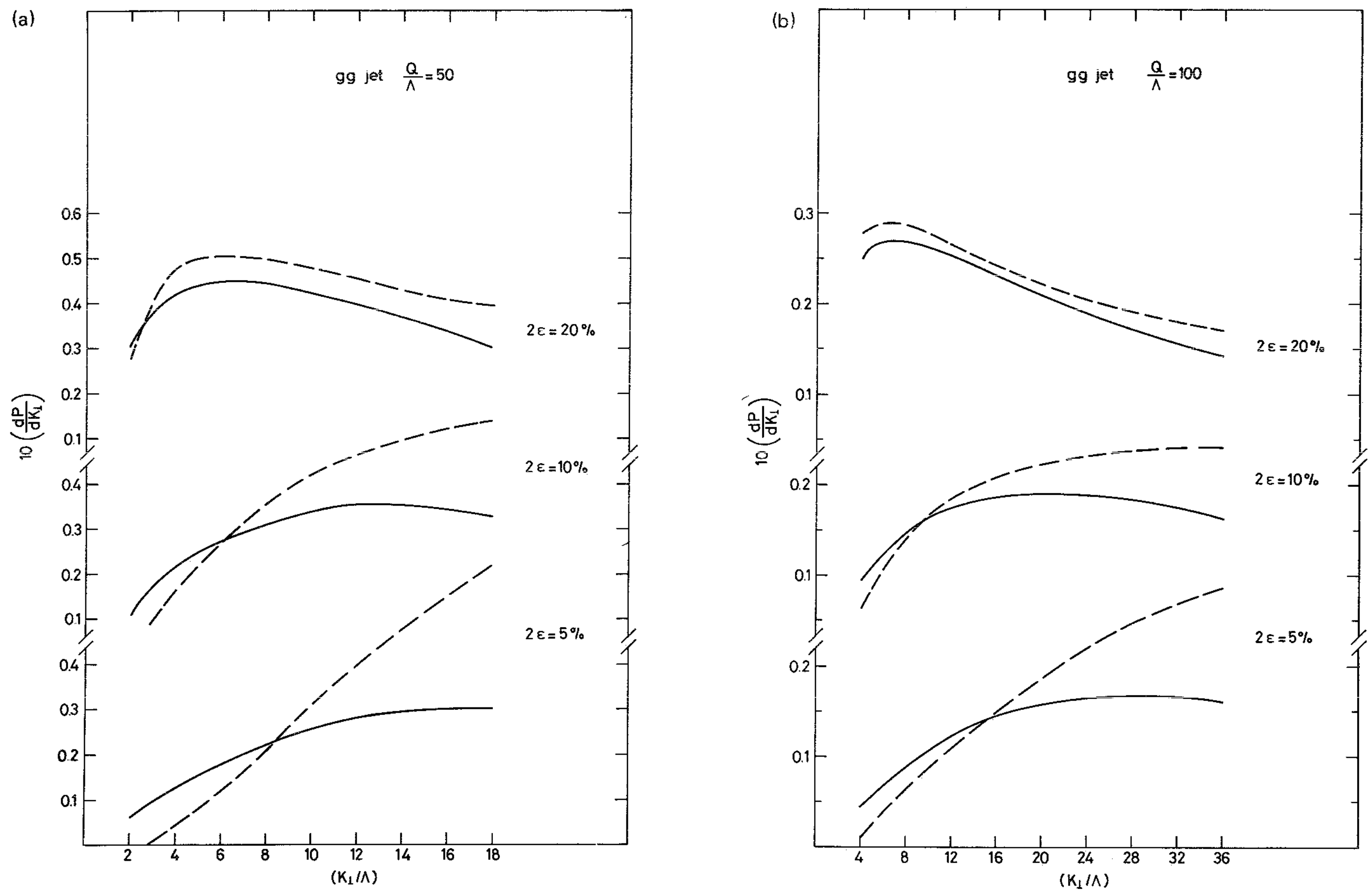

Fig. 3. (a) Total transverse momentum distribution for a gg jet at the total energy $Q = 50\ \Lambda$, for various ϵ. Full and dashed lines as in figs. 1 (eqs. (15) and (11), respectively). (b) The same, for $Q = 100\ \Lambda$.

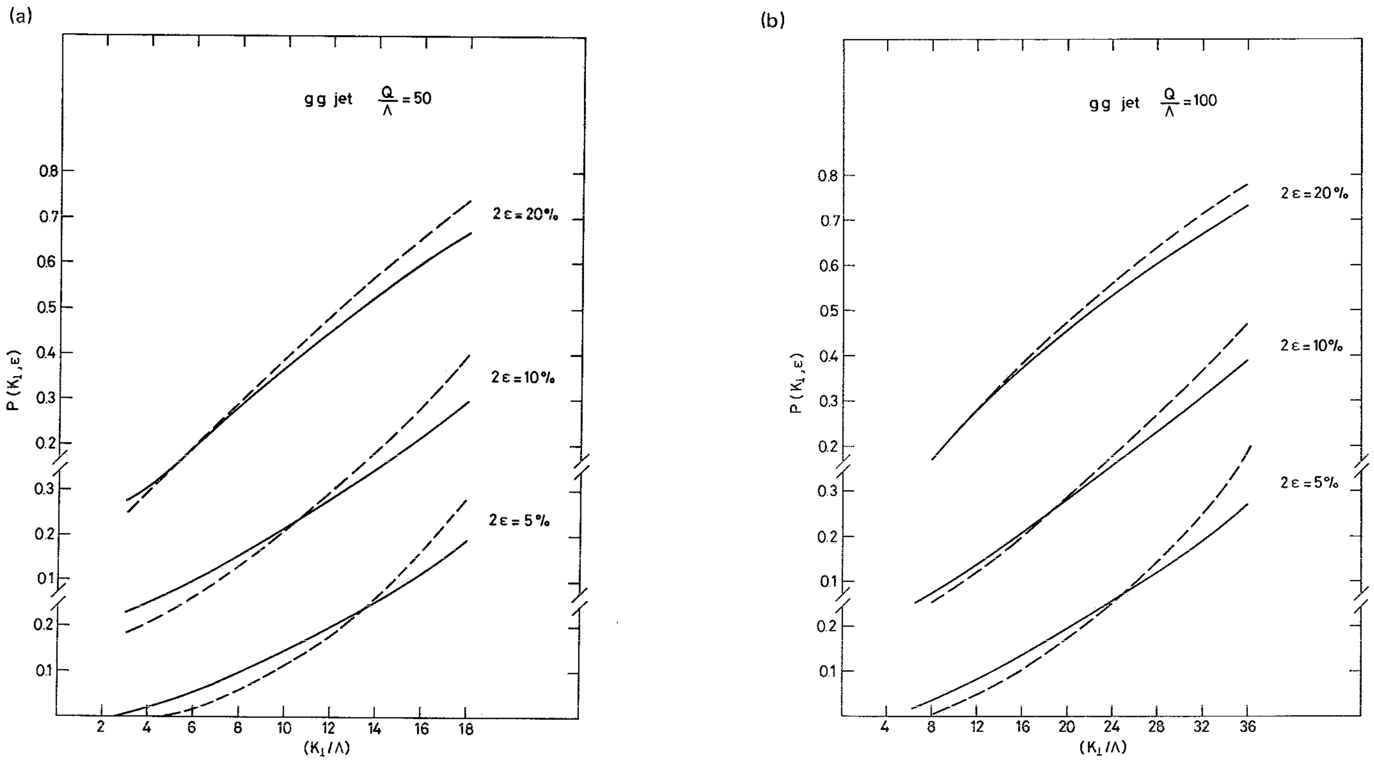

Fig. 4. (a) Probability functions $P(K_T, \epsilon)$ for a gg jet at the total energy $Q = 50\ \Lambda$, for various ϵ. Full and dashed lines as in figs. 1 (eqs. 17) and (9), respectively). (b) The same, for $Q = 100\ \Lambda$.

of QCD. This can be simply obtained by observing the behaviour of the total transverse momentum $\boldsymbol{K}_T$ relative to the jet axis defined as $\boldsymbol{K}_T = \Sigma_i \boldsymbol{k}_{Ti}$ where the sum over i extends to all particles produced in a half-plane orthogonal to the jet axis in a given cone.

5. Phenomenology of hadronic k_T distributions

In this section we discuss the transverse-momentum behaviour of a single hadron, by extrapolating our previous results on K_T distributions in the region $K_T \lesssim c\Lambda$. To achieve this goal, certain extra assumptions have to be made.

First we assume that the above $\mathrm{d}P(K_T)/\mathrm{d}K_T$ can also be used to describe the k_T distribution of an emitted hadron. For this case we limit the maximum transverse momentum allowed to a single hadron at a value $\sim\frac{1}{2}Q\langle n\rangle$, where $\langle n\rangle$ is the average number of particles produced in the e^+e^- annihilation at energy $\sqrt{s} = Q$. This request automatically restricts the K_T range to values $\lesssim c\Lambda$, for present and near-future e^+e^- beam energies. A stated earlier, in this region we use a constant value for $\overline{\alpha}(k_T)$ given by eq. (26). This parametrization is suggested by the observation [16] that, for e^+e^- energies less than about 3 GeV, the average observed value of R,

$$R_{\mathrm{av}} \equiv \frac{1}{\overline{s}} \int_{s_0}^{\overline{s}} \mathrm{d}s R_{\exp}(s) ,$$

for $\overline{s} \simeq 9\ \mathrm{GeV}^2$ agrees very well with the QCD prediction

$$R(s) = \sum_i Q_i^2 \left(1 + \frac{\alpha(s)}{\pi}\right) ,$$

provided one uses an effective constant value $\alpha(s) = \alpha(\overline{s})$ for $s < \overline{s}$. Using the best fitted value $\Lambda = 0.7$ GeV, which describes all R data for $s > \overline{s}$, and also agrees with a recent analysis [17] of scaling violation in neutrino experiments, we find $c \simeq 4$. Thus for $K_T < c\Lambda$, from eq. (15b) we obtain

$$\frac{\mathrm{d}^2 P}{\mathrm{d}^2 K_T} \simeq \frac{1}{2\pi} \int_0^\infty x\, \mathrm{d}x J_0(x \cdot K_T) \,.$$

$$\times \exp\left\{-\frac{2I(\epsilon)}{\pi} \overline{\alpha}(c\Lambda) \int_0^{Q\langle n\rangle/2} [1 - J_0(x \cdot k_T)] \frac{\mathrm{d}k_T}{k_T}\right\} . \qquad (27)$$

Although our final results, discussed below, are obtained by numerical evaluation of eq. (27), we give an approximate expression for eq. (27) which agrees rather well with the exact formula and could be rather useful for future phenomenological use.

With the approximation

$$\int_0^a \frac{dk_T}{k_T}[1 - J_0(xk_T)] \simeq \tfrac{1}{2}\ln(1 + \tfrac{1}{4}a^2x^2)\,, \tag{28}$$

which is obtained by interpolating between the small and large x behaviours of the left-hand side of eq. (28), we finally get

$$\frac{dP}{dk_T} \simeq \frac{2}{a}\frac{\beta}{\Gamma(1+\frac{1}{2}\beta)}\left(\frac{k_T}{a}\right)^{\beta/2} K_{1-\beta/2}\left(\frac{2k_T}{a}\right), \tag{29}$$

where $a = \frac{1}{2}Q\langle n\rangle$, $\beta = 2\bar{\alpha}(c\Lambda)\,I(\epsilon)/\pi$, and $K_{1-\beta/2}(x)$ is the modified Bessel function. In QED this result coincides with that directly obtained from the Bloch-Nordsieck theorem [18].

The distribution (29) exhibits the following limits. For $k_T \ll a$ one finds the power-law behaviour

$$\frac{dP}{dk_T} \simeq \begin{cases} \dfrac{\beta}{a}\dfrac{\Gamma(1-\frac{1}{2}\beta)}{\Gamma(1+\frac{1}{2}\beta)}\left(\dfrac{k_T}{a}\right)^{\beta-1}, & \text{for } \beta < 2\,, \\[2ex] \dfrac{4k_T}{a^2(\beta-2)}\,, & \text{for } \beta > 2\,. \end{cases} \tag{30}$$

On the other hand, in the limit $k_T \gg a$ we obtain

$$\frac{dP}{dk_T} \simeq \frac{\sqrt{\pi}}{a}\frac{\beta}{\Gamma(1+\frac{1}{2}\beta)}\left(\frac{k_T}{a}\right)^{\beta/2-1/2} e^{-(2k_T/a)}\,, \tag{31}$$

which provides an exponential cut-off to the large transverse momenta, independent of $\bar{\alpha}(c\Lambda)$ and ϵ.

Furthermore, the various moments of the distribution (29) are

$$\int\left(\frac{dP}{dk_T}\right)dk_T = 1\,, \tag{32}$$

$$\langle k_T\rangle = \int k_T\left(\frac{dP}{dk_T}\right)dk_T = \tfrac{1}{2}\sqrt{\pi}\tfrac{1}{2}\beta a\,\frac{\Gamma(\frac{1}{2}+\frac{1}{2}\beta)}{\Gamma(1+\frac{1}{2}\beta)}\,, \tag{33}$$

$$\langle k_T^2\rangle = \int k_T^2\left(\frac{dP}{dk_T}\right)dk_T = \tfrac{1}{2}\beta a^2\,, \tag{34}$$

which coincide with those calculated from the exact distribution (27), as can be seen directly, by comparison with eqs. (16) and (22) for $a = \frac{1}{2}Q$ and $\bar{\alpha}(k_T) = \text{const} = \bar{\alpha}(c\Lambda)$. This is a check of the accuracy of our approximate form (29).

Our predictions are compared with the experimental [1]

$$\frac{1}{\sigma}(d\sigma/dk_T)\,,$$

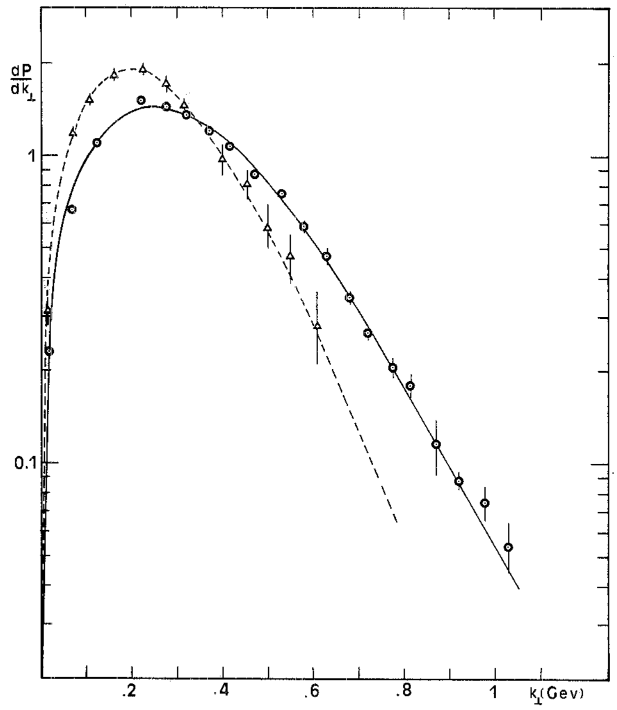

Fig. 5. Single inclusive transverse momentum distributions in $e^+e^- \to q\bar{q} \to$ hadrons at $Q = 3$ and 7.5 GeV. The experimental data are taken from Hanson [1]. The curves are the theoretical predictions (eq. (27)) for $2\epsilon = 0.025$.

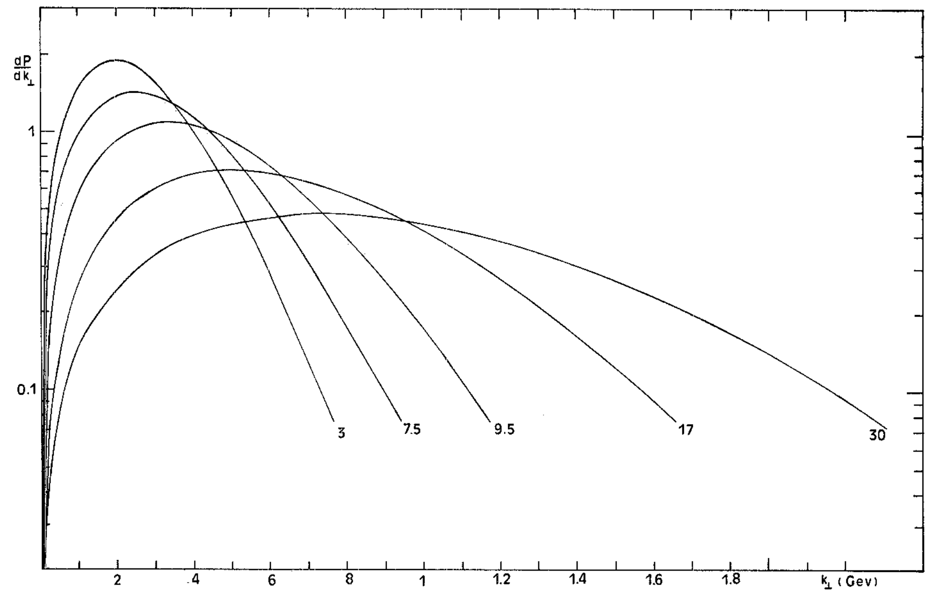

Fig. 6. The same as in fig. 5, for various c.m. total energies, and the same ϵ.

suitably normalized, in fig. 5. Also shown are the results for $Q \simeq 3$ GeV, with the same value of $I(\epsilon)$ and using $\langle n \rangle \sim 6$. The agreement is rather good, both in shape and the Q dependence. We also shown in fig. 6 our predictions for the k_T inclusive distributions at various energies accessible at PETRA and PEP for the same value $2\epsilon = 0.025$ and estimating $\langle n \rangle \sim 2\langle n \rangle_{ch} \sim 2(2.1 + 0.7 \ln s)$. In making comparisons with our formula it is important for fix ϵ for a given experiment. This can be done most straightforwardly by fixing $\langle k_T \rangle$ from the data (as eq. (33)). The effect of the braodening of the distributions (for the same ϵ) is quite evident. If confirmed, it should be considered as a decisive proof of the smoothness of extrapolating perturbative QCD in the infrared region.

Needless to say, a formula analogous to (27) for hadrons produced from gluon jets, for example in the decay $\Upsilon \to 3g$, is trivially obtained with the substitution $n_q I_q(\epsilon) \to n_g I_g(\epsilon)$, and the appropriate kinematical limits.

6. Conclusions

In this work we have analyzed e^+e^- jets in great detail, in the coherent-state formalism, developed for QCD. We have provided explicit expressions for general jet cross sections, which include infinite gluon and (massless) quark-antiquark excitations for each jet. Our approximation also includes finite terms in the energy loss ϵ, in addition to the usual $\ln \delta$ associated with the $\ln \epsilon$ terms. We have checked our results with explicit perturbative calculations, whenever, available.

Explicit expressions for the total K_T distribution, which takes into account the correlations induced by the momentum conservation in a jet, are presented. Predictions are also made for the Q^2 dependence of the mean K_T^2 for quark and gluon jets.

Our discussion has been motivated by the desire to deal with quantities which are accessible experimentally. We hope that such measurements will be forthcoming at PEP, PETRA and LEP energies, thus providing detailed tests of these ideas in QCD.

Finally, we have presented a phenomenological discussion of the single inclusive hadron k_T distributions measured at SPEAR ($W = 3$–7.8 GeV). Our results seem to indicate that perturbation theory can be extrapolated smoothly also in the small k_T region as also found for the total cross sections.

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