To be submitted to
Phys. Letters B

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-79/71(P)
2 Novembre 1979

G. Parisi: SUMMING LARGE PERTURBATIVE CORRECTIONS IN QCD.

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-79/71(P)
2 Novembre 1979

G. Parisi: SUMMING LARGE PERTURBATIVE CORRECTIONS IN QCD.

ABSTRACT

The problem of summing up the large perturbative corrections in QCD near the kinematical boundaries is studied in few particular cases. Euristic arguments have been used to conjecture the solution of the problem, using methods due to the Russian school.

In the parton model the cross section for the production of a massive pair in hadronic collisions can be computed by using as an input the quark distributions inside the hadrons in the infinite momentum frame. In QCD, if effective q^2 dependent quark distributions are introduced, the parton model relations among the Drell-Yan cross section and the deep inelastic structure functions is asymptotically satisfied; however computable corrections of order $\alpha(q^2)$ are present(1). Neglecting the contributions of gluons in the initial state (it is often a good approximation), the final predictions are:

$$\sigma^n_{DY} = \sigma^n_{PM} R(\alpha, n) ,$$
$$R(\alpha, n) = 1 + \frac{\alpha(Q^2)}{2\pi} f(n) + O(\alpha^2(Q^2)) , \qquad (1)$$

where σ^n_{DY} are the moments in $\tau = Q^2/s$ at fixed Q^2 of the total Drell-Yan cross section and σ^n_{PM} are the predictions of the naive parton model, written in terms of the structure functions at $q^2 = -Q^2$.

The function f(n) has been computed[2, 3] : for large n one finds :

$$f(n) \simeq \frac{4}{3}\left[\frac{4}{3}\pi^2 + 2\ln^2 n\right] . \tag{2}$$

The factors π^2 and $\ln^2 n$ are rather large and the corrections to the parton model are of order 1 at present energies.

It has been noticed[3] that the large corrections are due to the emission of soft gluons and that it should be possible to resum all the large corrections. One can work in the approximation in which only terms proportional to $\alpha\pi^2$ or $\alpha\ln^2 n$ are retained, while other higher order terms are neglected.

We conjecture that in this approximation the following result holds:

$$R(\alpha, n) = \exp\left\{\frac{32\pi^2}{75\,t} + \frac{16}{25}\left[(t-2L)\ln(t-2L) + \right.\right.$$
$$\left.\left. + t\ln t - 2(t-L)\ln(t-L)\right]\right\} \tag{3}$$

where :

$$\alpha(Q^2) = 12\pi/(25\ln(Q^2/\Lambda^2)) , \qquad t = \ln^{-1}(Q^2/\Lambda^2) ,$$
$$L = \ln n . \tag{4}$$

Eq. (3) can be applied only if $Q^2 \gg \Lambda^2 n^2$.

Let us show the arguments leading to eq. (3). The factor π^2 in eq. (2) arises from the comparison of the elastic quark form factor at spacelike and timelike momenta. Using the well known conjectured asymptotic formula for the quark form factor[4] :

$$F(q^2) \simeq \exp\left\{- \frac{8}{25}\ln\left(-\frac{q^2}{\Lambda^2}\right)\ln\left[\ln\left(-\frac{q^2}{\Lambda^2}\right)\right]\right\} \tag{5}$$

one finds :

$$\lim_{Q^2\to\infty} \left|F(Q^2)\right|F(-Q^2)\left|^2 = \exp\left\{\frac{4}{3}\,\frac{\alpha(Q^2)}{2\pi}\,\pi^2\right\}. \tag{6}$$

Eq. (6) is a part of the first term in eq. (3). Indeed the Drell-Yan cross section is proportional to the modulus squared of the form factor.

The origine of the factor $\ln^2 n$ is different. In the dimensional regolarization the factor $\ln Q^2$ in one loop diagrams must be written as $\ln(p_\perp^M)^2$ where $p_\perp^M$ is the maximum allowed value of $p_\perp$ in the reaction(5, 6).

Now, for simple kinematical reasons $(p_\perp^M)^2$ is proportional to $q^2(1-x)$ in deep inelastic scattering and to $Q^2(1-\tau)^2$ in Drell-Yan processes. The difference in the kinematics is the origine of the factor $\ln^2 n$ $(1-x$ is of order $1/n)$. In other words, in the dimensional regolarization the corrections in the large n region to deep inelastic scattering (C_{DIS}) and to the Drell-Yan (C_{DY}) process are respectively(3):

$$C_{DIS}(q^2, n) = 1 + \frac{\alpha(q^2)}{2\pi}\left[\frac{4}{3}\ln^2 n - \frac{4}{9}\pi^2\right] + O(\alpha^2),$$

$$C_{DY}(Q^2, n) = 1 + \frac{\alpha(Q^2)}{2\pi}\left[\frac{16}{3}\ln^2 n - \frac{4}{9}\pi^2\right] + O(\alpha^2)\ . \tag{7}$$

The factor $\ln^2 n$ is due to the emission of soft gluons, according to the standard folklore it should exponentiate if the dependence of α on $p_\perp^M$ (and consequently on n) is neglected(7). In this approximation we obtain:

$$C_{DIS}(q^2, n) \simeq \exp\left\{\frac{\alpha(q^2)}{2\pi}\left[\frac{4}{3}\ln^2 n - \frac{4}{9}\pi^2\right]\right\},$$

$$C_{DY}(Q^2, n) \simeq \exp\left\{\frac{\alpha(Q^2)}{2\pi}\left[\frac{16}{3}\ln^2 n - \frac{4}{9}\pi^2\right]\right\}. \tag{8}$$

The exponentiation of the factor $\frac{4}{9}\pi^2$ is rather doubtfull, however it is a small factor and practically does not change the final result. The correct formulae can be obtained by a straightforward application of the results of refs. (5, 6, 8) to this particular case:

$$C_{DIS}(q^2, n) \simeq \exp\left\{\frac{16}{25}\left[L - (t-L)\ln(t-L)\right]\right\},$$

$$C_{DY}(Q^2, n) \simeq \exp\left\{\frac{32}{25}\left[L - \frac{1}{2}(t-2L)\ln(t-2L)\right]\right\}, \tag{9}$$

where the factor proportional to π^2 has been neglected.

The final result is obtained by substituting eqs. (6-9) in:

$$R(\alpha, n) = \left| \frac{F(Q^2)}{F(-Q^2)} \right|^2 C_{DY}(Q^2, n)/C_{DIS}(-Q^2, n) \ . \tag{10}$$

Let us comment on the limit of validity of eq. (10).

The results of refs. (5-8) imply that in the quasi elastic region in deep inelastic scattering the following formula holds:

$$\frac{dF(q^2, x)}{d \ln q^2} = -\frac{16}{25} \left[\ln \ln(q^2/\Lambda^2) - \ln \ln(W^2/\Lambda^2) \right] , \tag{11}$$

where W^2 is the final hadronic mass.

Eq. (11) implies that the function $F(q^2, x)$ at fixed W^2 decreases faster than any powers of q^2. This Sudakov-type result concerns only the contribution of twist 2 operator. Indeed the transition form factors decrease like $1/Q^4$[9] and they give a contribution in the threshold region which is dominant in the limit $Q^2 \to \infty$: this contribution is connected to the presence of higher twist operators[10].

In the nucleon case in the threshold region sending Q^2 at infinity at fixed $W^2 \gg m_N^2$ one gets, neglecting logaritms[10]:

$$F(q^2, x) \sim W^2/q^6 \sim (1 - x)/q^4 \ . \tag{12}$$

In other words eq. (11) breakes down when the r.h.s. becomes too small (i.e. less than -1).

The author is grateful to C. Altarelli for having stressed to him the importance of controlling the large corrections at high orders, to M. Greco, to G. Martinelli and to R. Petronzio for having pointed to him an error in the first draft of this note. He also thanks L. Baulieu, R.K. Ellis, C. Kounnas and N. Sourlas for useful discussions and suggestions.

REFERENCES.

(1) - A. V. Radyushkin, Phys. Letters 69B, 245 (1977); H. D. Politzer, Nuclear Phys. B129, 301 (1977); C. T. Sachrajda, Phys. Letters 73B, 185 (1978); G. Altarelli, G. Parisi and R. Petronzio, Phys. Letters 76B, 351 (1978); K. Kajantie and R. Ratio, Nuclear Phys. B139, 72 (1978).

(2) - G. Altarelli, R. K. Ellis and G. Martinelli, Nuclear Phys. B143, 521 (1978); Erratum, Nuclear Phys. B146, 544 (1978); J. Kubar-André and F. E. Paige, Phys. Rev. D19, 221 (1979); K. Harada, T. Kaneko and N. Sakai, CERN Preprint TH 2619 (1979).

(3) - G. Altarelli, R. K. Ellis and G. Martinelli, MIT Preprint CTP. 776 (1979).

(4) - C. P. Korthals-Altes and E. De Rafael, Nuclear Phys. B125, 275 (1977); A. Mueller, Columbia Univ. Preprint CU-TP-158 (1979).

(5) - V. N. Gribov and L. N. Lipatov, Sov. Journ. Nuclear Phys. 15, 446 (1972).

(6) - Y. L. Dokshitzer, D.I. D'yakonov and S. I. Troyan, Phys. Letters 79B, 356 (1978).

(7) - E. Etim, G. Pancheri and B. Touschek, Nuovo Cimento 51B, 276 (1967).

(8) - Y. L. Dokshitzer, Sov. Phys. -JEPT 46, 641 (1977).

(9) - D. R. Jackson, Thesis, Caltech, Pasadena (1977); A. V. Efremov and A. V. Radyushkin, Dubna Preprint JINR E 2-11983 (1978).

(10) - G. Parisi, Phys. Letters 84B, 225 (1979).