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General mass formula in broken supersymmetry

S. Ferrara
Laboratori Nazionali di Frascati, Istituto Nazionale di Fisica Nucleare, Frascati, Italy

L. Girardello*
Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts 02138

F. Palumbo
Laboratori Nazionali di Frascati, Istituto Nazionale di Fisica Nucleare, Frascati, Italy

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The mass formula $\sum_J(-1)^{2J}(2J+1)m_J{}^2=0$ is derived for a very general class of interactions with spontaneously broken supersymmetry. It shows the vanishing of the graded trace of the square of the mass operator, with m_J the mass associated with a (real) field of spin J. This mass relation is shown to be true even in the presence of explicit breaking, provided it fulfills suitable requirements.

I. INTRODUCTION

Supersymmetric theories have the possibility of being realistic if supersymmetry is broken either spontaneously or explicitly.[1] Such a mechanism removes the mass degeneracy among bosons and fermions in the same supermultiplet. Still, if supersymmetry (global or local) is broken spontaneously or even in certain explicit ways, a supersymmetric mass pattern survives.

We indeed show here[2] that in a very general class of supersymmetric theories with supersymmetry breaking under the above circumstances, the following mass formula holds at the tree level:

$$\sum_J(-1)^{2J}(2J+1)m_J{}^2=0\,, \qquad (1.1)$$

where m_J is the mass associated with the (real) field of spin J. This result is a remarkable example of a retained predictive power of supersymmetry. It says, for instance, that all the bosons cannot have large masses with respect to the fermions and vice versa. One can check that many phenomenological models,[3] constructed as an attempt to unify fundamental interactions, without U(1) axial gauge invariance, have mass patterns which indeed satisfy relation (1.1).

One can recognize in the left-hand side of (1.1) the graded trace of the square of the mass matrix for any supersymmetric theory. It is possible that our result could be derived in a pure group-theoretical way implemented with certain requirements on the breaking. This would have the advantage over our method of having a complete control of theories for which (1.1) is satisfied and, at the same time, a model-independent derivation. As a matter of fact, formula (1.1) is trivially satisfied for each supermultiplet in a theory with unbroken supersymmetry.

Even more remarkably, formula (1.1) holds for theories with broken local supersymmetry, i.e., supergravity theories in which a truly "super"-Higgs mechanism can occur,[4] namely, when the spin-$\frac{3}{2}$ gauge fields get massive at the expense of spin-$\frac{1}{2}$ Goldstone fermions. This situation is more delicate since (1.1) is meaningful only in the absence of a cosmological term.[5,6] This is not surprising, owing to the difficulties in interpreting the masses of the fields in space-time with constant curvature. On the other hand, when a cosmological term is present, the gauged "kinematic" group is not the graded Poincaré group but the graded de Sitter group. Interestingly enough, this follows from the observation that a vanishing cosmological term requires the scalar (pseudoscalar) auxiliary fields S, P of the supergravity multiplet[7] to have a vanishing expectation value. If, on the contrary, these fields have a nonvanishing expectation value, it follows immediately from the commutator algebra of local supersymmetry[7] that the commutator of two supersymmetry transformations acquires an extra Lorentz transformation which survives in the global limit of vanishing gauge fields.

At present, two kinds of models of supergravity are known in which spontaneous breaking of local supersymmetry can occur with a vanishing cosmological term. There are $N=1$ supergravity with a general interaction[6] with a scalar (chiral) matter multiplet of spin content $(0^{\pm},\frac{1}{2})$ and the recently constructed models[8] of spontaneously broken local extended supersymmetry up to $N=8$ spinorial charges ($N=8$ extended supergravity). In both cases relation (1.1) is confirmed.

Formula (1.1) will be proved here at first in a very general class of theories with global supersymmetry. In Sec. II we consider the most general self-interaction of a chiral multiplet with spontaneous breaking of supersymmetry.[9] The validity of (1.1) persists with an explicit breaking of supersymmetry via a term of the form $\mathrm{Re}f(z)$, $z=A+iB$, where A and B are the 0^+ scalar and 0^- pseudoscalar fields. We further give, in passing, an alternative derivation of the Goldstone theorem for supersymmetry.

Section III is devoted to supersymmetric (Abelian and non-Abelian) gauge theories. Here the Yang-Mills group is an arbitrary direct product of a semisimple Lie group with U(1) Abelian factors with vector gauge fields.

In Sec. IV we discuss the case of local supersymmetry (supergravity) for the particular class of models constructed in Ref. 6, with special emphasis on the question of vanishing cosmological constant.

While it is quite likely that a more direct (algebraic) proof of the mass formula (1.1) could be given in the general supersymmetric setting, we actually treat a large class of possible "physical" supersymmetric theories, and thereby definitely establish a "universal" validity. Models with axial gauge invariance[3] are exceptions to (1.1). However, these models are faced with axial anomalies, and renormalizability criteria seem to exclude them as candidates for grandunified theories.

The problem of quantum corrections to the mass formula is not discussed here and awaits attention. Note that in the Wess-Zumino model,[10] with soft explicit symmetry breaking, the formula is changed in higher orders by finite corrections only.[11] Formula (1.1) ensures that the induced cosmological constant, which is usually quartically divergent, is at most logarithmically divergent.[12]

II. CHIRAL MULTIPLETS

In this section we derive the mass formula for an arbitrary, parity-preserving interaction among multiplets of spin content $(0^{\pm},\frac{1}{2})$. We introduce a set of N chiral multiplets $\Sigma^a=(z^a,\chi_L^a,H^a)$ with flavor index $a=1,2,\ldots,N$. The complex components of Σ^a are related to the real components as follows:

$$z^a=A^a+iB^a,\quad \chi_L^a=\tfrac{1}{2}(1+\gamma_5)\chi^a,\quad H^a=F^a+iG^a. \tag{2.1}$$

The scalar multiplication of these multiplets is given by the rule

$$\Sigma^{a_1}\cdots\Sigma^{a_n}=\left(z^{a_1}\cdots z^{a_n},\ \sum_{k=1}^{n}z^{a_1}\cdots\hat{z}^{a_k}\cdots z^{a_n}\chi_L^{a_k},\ \sum_{k=1}^{n}z^{a_1}\cdots\hat{z}^{a_k}\cdots z^{a_n}H^{a_k}-\sum_{h\neq k=1}^{n}z^{a_1}\cdots\hat{z}^{a_h}\cdots\hat{z}^{a_k}\cdots z^{a_n}\chi_L^{a_h}\chi_L^{a_k}\right). \tag{2.2}$$

A flavor singlet function is given by

$$f(\Sigma)\equiv f(\Sigma^1,\Sigma^2,\ldots,\Sigma^N)=\sum_{n=0}^{\infty}c_{a_1\cdots a_n}\Sigma^{a_1}\cdots\Sigma^{a_n}, \tag{2.3}$$

in which $c_{a_1\cdots a_n}$ are real and symmetric coefficients. The supersymmetric Lagrangian is given by the kinetic part added to the F component of the multiplet $f(\Sigma^1,\ldots\Sigma^N)$:

$$\mathcal{L}_{ss}=-\tfrac{1}{2}\partial_\mu z^a\partial^\mu z^{a*}-\chi_L^a\not\partial\chi_R^a+\tfrac{1}{2}H^aH^a+f_{,a}(z)H^a+f_{,a}(z^*)H^{a*}-f_{,ab}(z)\chi_L^a\chi_L^b-f_{,ab}(z^*)\chi_R^a\chi_R^b. \tag{2.4}$$

We have used the conventions for the γ matrices given in Ref. 7. The bosonic potential, obtained by eliminating the auxiliary fields via their equations $H_a=-2f_{,a}(z^*)$, is

$$V(z,z^*)=2f_{,a}(z)f_{,a}(z^*). \tag{2.5}$$

The extremum conditions are

$$\frac{\partial V}{\partial z^b}=2f_{,ab}(z)f_{,a}(z^*)=0,\qquad \frac{\partial V}{\partial z^{b*}}=2f_{,a}(z)f_{,ab}(z^*)=0. \tag{2.6}$$

The trace of the square of the bosonic mass matrix is

$$\mathrm{Tr}\mathfrak{M}_{ab}^2=4\frac{\partial}{\partial z^b}\frac{\partial}{\partial z^{b*}}V=8f_{,ab}(z)f_{,ab}(z^*). \tag{2.7}$$

The fermionic mass matrix can be obtained from the expression

$$-\tfrac{1}{2}M_{ab}\bar{\chi}^a\chi^b+\tfrac{1}{2}N_{ab}i\bar{\chi}^a\gamma_5\chi^b, \tag{2.8}$$

where

$$M_{ab}=f_{,ab}(z)+f_{,ab}(z^*),\quad N_{ab}=i[f_{,ab}(z)-f_{,ab}(z^*)] \tag{2.9}$$

are real symmetric matrices.

By choosing a basis in which

$$\gamma^0=\begin{pmatrix}\tau_3 & 0\\ 0 & \tau_3\end{pmatrix}\quad\text{and}\quad -i\gamma^0\gamma_5=\begin{pmatrix}\tau_1 & 0\\ 0 & \tau_1\end{pmatrix},$$

the above fermionic mass matrix can be written in the form

$$\mathfrak{M}_{Fab}=\begin{bmatrix}M_{ab} & N_{ab} & & \\ N_{ab} & -M_{ab} & & 0 \\ & & M_{ab} & N_{ab}\\ 0 & & N_{ab} & -M_{ab}\end{bmatrix}. \tag{2.10}$$

The square of the fermionic mass matrix is

$$\mathfrak{M}_{Fab}{}^2 = \begin{bmatrix} M_{ab}{}^2+N_{ab}{}^2 & [M,N]_{ab} & & 0 \\ [N,M]_{ab} & M_{ab}{}^2+N_{ab}{}^2 & & \\ & & M_{ab}{}^2+N_{ab}{}^2 & [M,N]_{ab} \\ 0 & & [N,M]_{ab} & M_{ab}{}^2+N_{ab}{}^2 \end{bmatrix}, \qquad (2.11)$$

and its trace is

$$\tfrac{1}{4}\operatorname{Tr}\mathfrak{M}_{Fab}{}^2 = \operatorname{Tr}(M^2+N^2) = 4f_{,ab}(z)f_{,ab}(z^*)\,. \qquad (2.12)$$

Comparing (2.6) and (2.8), we obtain

$$\sum m_B{}^2 = 2\sum m_F{}^2\,, \qquad (2.13)$$

i.e., formula (1.1) when only scalar (pseudoscalar) and spin-$\frac{1}{2}$ fields appear in the theory. From the extremum condition (2.6) one can read immediately the Goldstone theorem. Equation (2.6) is, in fact,

$$\begin{pmatrix} M_{ab} & N_{ab} \\ N_{ab} & -M_{ab} \end{pmatrix} \begin{pmatrix} F_b \\ G_b \end{pmatrix} = 0\,. \qquad (2.14)$$

Spontaneous supersymmetry breaking implies a nontrivial solution for the system (2.14), which leads to the vanishing of the determinant of the fermionic mass matrix. This implies a massless fermion in the theory, the Goldstone mode.

Note that if $N=1$, spontaneous supersymmetry breaking is impossible. In fact, the extremum condition would imply that $f_{,zz}(z)=0$, i.e., $m_A{}^2 + m_B{}^2=0$. One of the scalar particles would be a tachyon, which would lead to an unstable solution. Needless to say, one could envisage the exceptional situation in which $f_{,zzz}=0$ also at the extremum, which would imply that $m_A=m_B=m_\chi=0$ while still breaking supersymmetry ($f_{,z}\neq 0$). Note that this results is true for any interaction and not only in the Wess-Zumino Lagrangian, where it was noticed long ago.[13]

We conclude this section by observing that the mass formula is also true if an explicit supersymmetry-breaking term $\mathcal{L}_{\text{break}}(z,z^*)$ is present, provided this satisfies the condition

$$\frac{\partial}{\partial z^a}\frac{\partial}{\partial z^{a*}}\mathcal{L}_{\text{break}}=0\,. \qquad (2.15)$$

Therefore, if $g(z)$ is a real analytic function, it follows that any Lagrangian of the form

$$\mathcal{L}_{ss}+\operatorname{Re}g(z) \qquad (2.16)$$

leads to (2.11). This is indeed the case in the explicitly broken model investigated by Iliopoulos and Zumino[11] in which $g(z)=z$ and the mass formula $m_A{}^2+m_B{}^2=2m_\chi{}^2$ holds.

III. GAUGE THEORIES

We extend our investigation to the supersymmetric interaction of scalar-spinor matter with massless multiplets of spin-particle content $(\frac{1}{2},1)$ gauging an internal-symmetry group G. We start by considering the Abelian case $G=\mathrm{U}(1)$.[14,15] The gauge U(1) transformation is realized on a doublet of chiral multiplets Σ_a as follows[14]:

$$\delta\Sigma_a=\Lambda\epsilon_{ab}\Sigma_b \quad (\epsilon_{12}=-\epsilon_{21}=1,\ \epsilon_{11}=\epsilon_{22}=0)\,. \qquad (3.1)$$

The vector potential V is a real (pseudoscalar) superfield transforming as

$$\delta V=(i/g)(\Lambda-\Lambda^*) \qquad (3.2)$$

under gauge transformation of (chiral) parameter Λ. The chiral combination $\Sigma_a\Sigma_a$ is gauge invariant, so that any function $f(\Sigma_a\Sigma_a)$ can describe a gauge-invariant supersymmetric interaction of the chiral doublet. The overall Lagrangian in superfield notation reads

$$[\mathcal{L}(V)+\Sigma_a\Sigma_a^*\cosh(gV)+i\epsilon_{ab}\Sigma_a^*\Sigma_b\sinh(gV)+\xi V]_D + f(\Sigma_a\Sigma_a)_F\,. \qquad (3.3)$$

In the Wess-Zumino gauge $\cosh(gV)=1+\frac{1}{2}g^2V^2$, $\sinh(gV)=gV$, and the Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{ss}=&-\tfrac{1}{2}(\partial_\mu z_a\partial^\mu z_a^*+2\chi_L^a\not{\partial}\chi_R^a+\bar{\lambda}\not{\partial}\lambda)-\tfrac{1}{4}F_{\mu\nu}{}^2+\tfrac{1}{2}(D^2+H_aH_a^*)-\tfrac{1}{2}igD\epsilon^{ab}z_az_b^*\\ &-\tfrac{1}{2}g^2v_\mu{}^2z_az_a^*+gv_\mu\epsilon_{ab}(z_a^*\overleftrightarrow{\partial}_\mu z_b-\chi_{aL}\gamma_\mu\chi_{bR})+g(\lambda_L\epsilon_{ab}\chi_{aL}z_b^*+\lambda_R\epsilon_{ab}\chi_{aR}z_b)-\xi D+[f(\Sigma_1{}^2+\Sigma_2{}^2)]_F\,.\end{aligned} \qquad (3.4)$$

The equations of motion of the auxiliary fields are

$$D=\xi+\tfrac{1}{2}ig\epsilon_{ab}z_az_b^*,\quad H_a=-2\frac{\partial f}{\partial z_a^*}\,. \qquad (3.5)$$

The overall scalar potential is therefore

$$\mathcal{V}=\tfrac{1}{2}(\xi+\tfrac{1}{2}g\epsilon^{ab}z_az_b^*)^2+2f_{,z_a}f_{,z_a^*}\,, \qquad (3.6)$$

while the vector and fermion mass terms are

$$-\tfrac{1}{2}g^2v_\mu{}^2z_az_a^*-f_{,ab}\chi_{aL}\chi_{bL}-f^*_{,ab}\chi_{aR}\chi_{bR} - g(\lambda_L\epsilon_{ab}\chi_{aL}z_b^*+\lambda_R\epsilon_{ab}\chi_{aR}z_b)\,. \qquad (3.7)$$

We have

$$\frac{\partial^2 \mathcal{V}}{\partial z_a \partial z_b^*} = (\xi + \tfrac{1}{2} i g \epsilon_{dc} z_d z_c^*) \tfrac{1}{2} i g \epsilon_{ab} + \tfrac{1}{4} g^2 (\delta_{ab} z_c z_c^* - z_a z_b^*) + 2 f_{,ac} f_{cb}^* , \quad (3.8)$$

and therefore the trace of the square of the (pseudo-) scalar mass matrix is

$$\mathrm{Tr}\mathfrak{M}_{ab}{}^2 = 4\,\mathrm{Tr}\frac{\partial \mathcal{V}}{\partial z_a \partial z_b^*} = g^2 z_a z_a^* + 8 f_{,ac} f_{,ac}^* . \quad (3.9)$$

The trace of the square of the bosonic mass matrix is

$$\mathrm{Tr}\mathfrak{M}_{ab}{}^2 + 3\mathfrak{M}_v{}^2 = 4 g^2 z_a z_a^* + 8 f_{,ac} f_{cb}^* , \quad (3.10)$$

where $\mathfrak{M}_v{}^2$ is the square of the mass of the vector field v_μ. Let us now compute the fermion mass matrix. We have

$$\mathfrak{M}_{F\alpha\beta} = \begin{bmatrix} \tilde{m}_{\alpha\beta} & \tilde{n}_{\alpha\beta} & & \\ \tilde{n}_{\alpha\beta} & -\tilde{m}_{\alpha\beta} & & 0 \\ & & \tilde{m}_{\alpha\beta} & \tilde{n}_{\alpha\beta} \\ 0 & & \tilde{n}_{\alpha\beta} & -\tilde{m}_{\alpha\beta} \end{bmatrix} , \quad (3.11)$$

where

$$\tilde{m}_{\alpha\beta} = \begin{pmatrix} M_{ab} & m_a \\ m_b & 0 \end{pmatrix} , \quad \tilde{n}_{\alpha\beta} = \begin{pmatrix} N_{ab} & n_a \\ n_b & 0 \end{pmatrix} , \quad (3.12)$$

with

$$m_a = -\tfrac{1}{2} g \epsilon^{ab} (z_b + z_b^*) , \quad n_a = \tfrac{1}{2} g \epsilon^{ab} i (z_b - z_b^*) . \quad (3.13)$$

The trace of the square of the fermionic mass matrix is

$$\tfrac{1}{4}\mathrm{Tr}\mathfrak{M}_{F\alpha\beta}{}^2 = \mathrm{Tr}(M^2 + N^2) + 2 g^2 z_a z_a^* = 4 |f_{,ab}|^2 + 2 g^2 |z_a|^2 . \quad (3.14)$$

From a comparison with (3.10) we finally obtain

$$\sum_J (-1)^{2J} (2J+1) m_J{}^2 = 0$$

or, more explicitly,

$$\sum m^2_{\text{(pseudo)scalar}}\ (J=0) + 3 m^2_{\text{vector}} = 2 \sum m^2_{\text{fermion}}\ (J = \tfrac{1}{2}) . \quad (3.15)$$

We can again derive the Goldstone theorem in a simple way. The extremum condition is

$$\frac{\partial \mathcal{V}}{\partial z_a} = \frac{\partial \mathcal{V}}{\partial z_a^*} = 0 , \quad (3.16)$$

which can be rewritten

$$-n_a D - M_{ab} F_b - N_{ab} G_b = 0 , \qquad -m_a D + N_{ab} F_b - M_{ab} G_b = 0 . \quad (3.17)$$

If we add to these two equations the conditions

$$m_a F_a + n_a G_a = 0 , \quad n_a F_a - m_a G_a = 0 , \quad (3.18)$$

which express the U(1) invariance of the interaction $\epsilon^{ab} z_b^* H_a = 0$ [because $H_a \sim z_a^* f'(Z^*)$, $Z = z_1{}^2 + z_2{}^2$], then the system (3.17) and (3.18) can be written as

$$\begin{pmatrix} \tilde{m} & \tilde{n} \\ \tilde{n} & -\tilde{m} \end{pmatrix} \begin{pmatrix} \tilde{F} \\ \tilde{G} \end{pmatrix} = 0 , \quad \tilde{F} = \begin{pmatrix} F \\ 0 \end{pmatrix} , \quad \tilde{G} = \begin{pmatrix} G \\ D \end{pmatrix} . \quad (3.19)$$

Spontaneous symmetry breaking implies

$$\det \begin{pmatrix} \tilde{m} & \tilde{n} \\ \tilde{n} & -\tilde{m} \end{pmatrix} = 0 ; \quad (3.20)$$

i.e., the fermionic mass matrix has at least one vanishing eigenvalue, corresponding to a Goldstone mode.

The previous analysis extends immediately to the case in which a flavor index is added. We now have a set of scalar fields z_i^α, $i = 1, 2$ (color index), $\alpha = 1, 2, \ldots, n$ (flavor index). The most general function of z_i^α which is a flavor and a color singlet has the form

$$f(Z) + \Omega_{\rho\sigma} g_{\rho\sigma}(Z) , \quad g_{\rho\sigma} = -g_{\sigma\rho} \quad (3.21)$$

with $Z^{\alpha\beta} = z_i^\alpha z_i^\beta$, $Z = z_i^\alpha z_i^\alpha$, and $\Omega^{\alpha\beta} = \epsilon^{ij} z_i^\alpha z_j^\beta$. Again it follows that U(1) invariance implies

$$H_i^\alpha z_j^{\alpha *} \epsilon^{ij} = 0 , \quad (3.22)$$

since

$$H^i \sim \frac{\partial}{\partial z_i^{\alpha *}} [f(Z^*) + \Omega^*_{\rho\sigma} g_{\rho\sigma}(Z^*)] = \frac{\partial}{\partial z^{\alpha\delta *}} [f(Z^*) + \Omega^*_{\rho\sigma} g_{\rho\sigma}(Z^*)] 4 z_i^{\delta *} + 2 \epsilon^{im} z_m^{\sigma *} g_{\alpha\sigma}(Z^*) . \quad (3.23)$$

Then all the arguments carried out in the absence of flavor can be worked out immediately.

Finally, we investigate the non-Abelian situation.[16] This is a straightforward generalization of the Abelian case. The only changes are

$$\mathrm{Tr}\mathfrak{M}_{\text{scalar}}{}^2 = 8 f_{,ab} f_{,ab}^* + C(R) z_\alpha^a z_\alpha^{a*} ,$$
$$\mathrm{Tr}\mathfrak{M}_{\text{vector}}{}^2 = C(R) z_\alpha^a z_\alpha^{a*} , \quad (3.24)$$
$$\mathrm{Tr}\mathfrak{M}_{\text{fermion}}{}^2 = 4 f_{,ab} f_{,ab}^* + 2 C(R) z_\alpha^a z_\alpha^{a*} ,$$

where a is the color index, $C(R)$ is the eigenvalue of the quadratic Casimir operator labeling the representation R to which the chiral multiplets belong, and an extra index α $(=1,2)$ is needed if R is a complex representation. Again, one can easily observe that formula (1.1) is verified.

Owing to the additive structure of the auxiliary fields, any linear combination of the models so far considered leads to our mass formula. As in the chiral case, we can imagine adding gauge-invariant terms which break supersymmetry explicitly, but satisfy (2.15) without then affecting the mass relation.

We close this section by noting that there are models for which our formula does not hold. As anticipated in the Introduction, these are the models having a U(1) axial gauge invariance.[3,17] The simplest of these is the so-called super-Higgs model.[18] In this model chiral symmetry allows a mass term for the scalars, but prevents the fermions from getting massive.[19]

IV. SUPERGRAVITY

In the present section we extend our analysis to the case of local supersymmetry. This is the most interesting case because both the Higgs effect for spin-1 and the super-Higgs effect for spin-$\frac{3}{2}$ particles can take place. We will confine our analysis to the interaction of the matter chiral multiplet (z,χ_L,H) with particle-spin content $(0^{\pm},\frac{1}{2})$ and the supergravity multiplet $(e_{a\mu},\psi_\mu,A_\mu,U)$ with particle-spin content $(\frac{3}{2},2)$. The fields $H=F+iG$, $U=S-iP$, A_μ are auxiliary fields[7] and can be eliminated by means of their field equations. For the interaction under consideration, the mass formula (1.1) has already been derived in Ref. 6, under the assumption of vanishing cosmological term and canonical kinetic term for the scalar field $z=A+iB$. The formula reads

$$m_A^2+m_B^2=4m_\psi^2\,. \tag{4.1}$$

The spin-2 graviton stays massless, being Poincaré invariance unbroken, and the spin-$\frac{3}{2}$ fermion has a helicity factor 4 because it has become massive through the absorption of the spin-$\frac{1}{2}$ Goldstone fermion of the chiral multiplet. For details of the super-Higgs phenomenon and the structure of the interaction, we refer the reader to Ref. 6.

It is the purpose of this section to comment further on these results. Again, we discuss only the spontaneously broken situation in which the super-Higgs effect occurs. In the unbroken case, the spin-$\frac{3}{2}$ fermion stays massless and our mass relation is trivially satisfied.

The absence of a cosmological term at the extreme of the potential can be expressed most elegantly as follows:

$$\langle U\rangle=0 \quad \text{at the extreme}\,, \tag{4.2}$$

where U is the complex scalar (pseudoscalar) field of the supergravity multiplet. This condition is equivalent to the statement that the theory must admit a global Poincaré supersymmetry. Violation of (4.2) gives global limit with de Sitter supersymmetry, as can be seen, for instance, from the transformation laws of the supergravity multiplet, and therefore the concept of mass no longer makes sense. Equation (4.2) can be easily proved in the model under investigation by observing that the field equation for U is

$$U=\frac{9}{\phi}(-\tfrac{1}{2}+\tfrac{1}{3}|J_{,z}|^2)+\tfrac{1}{2}\bar{\chi}(1+\gamma_5)\chi(J_{,zz}-\tfrac{1}{3}J_{,z}^{\;2})\,, \tag{4.3}$$

where, without any loss of generality, we have chosen the chiral potential function $g(z)=1$ by redefining the vierbein field through a field-dependent Weyl rescaling. We have set $J_{,zz^*}=-\frac{1}{2}$ (canonical kinetic terms) and followed the notations and conventions of Ref. 6, viz.,

$$\phi=\phi(z,z^*)\,,\quad J=3\ln(-\phi/3)\,. \tag{4.4}$$

It is seen that the vanishing of the right-hand side of (4.3),

$$\tfrac{1}{3}|J_{,z}|^2-\tfrac{1}{2}=0,\quad J_{,zz}-\tfrac{1}{3}J_{,z}^{\;2}=0\,, \tag{4.5}$$

corresponds precisely to the conditions

$$\mathcal{V}=\mathcal{V}_{,z}=0\,, \tag{4.6}$$

i.e., the vanishing of the cosmological term at the extremum of the potential $\mathcal{V}$. It is interesting to note that if $\langle U\rangle\neq 0$, we have the super-Higgs effect in de Sitter space.

The trace of the square of the bosonic mass matrix reads

$$\begin{aligned}4\mathcal{V}_{,zz^*}&=4e^{-\mathcal{G}}[2(\mathcal{G}_{,z}^{\;2}-\mathcal{G}_{,zz})(\mathcal{G}_{,z^*}^{\;2}-\mathcal{G}_{,z^*z^*})-1]\\&=4e^{-\mathcal{G}}\end{aligned} \tag{4.7}$$

because $|\mathcal{G}_{,z}^{\;2}-\mathcal{G}_{,zz}|^2=1$ owing to the extremum condition

$$-\frac{\partial\mathcal{V}}{\partial z}=[e^{-\mathcal{G}}(3-2\mathcal{G}_{,z}\mathcal{G}_{,z^*})]_{,z}=0\,, \tag{4.8}$$

where $\mathcal{G}=J+\ln 4$ since $g(z)=1$ here. We still use the expression "mass term" although, in the presence of a cosmological term, masslike terms do not have the physical meaning of mass. For the fermion mass terms we have

$$\exp(-\tfrac{1}{2}\mathcal{G})[\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu-\bar{\psi}\cdot\gamma\hat{\mathcal{G}}_{,z}\chi-\bar{\chi}(\hat{\mathcal{G}}_{,z}^{\;2}-\hat{\mathcal{G}}_{,zz})\chi]\,. \tag{4.9}$$

If we call m_ψ the "mass" of the spin-$\frac{3}{2}$ field, (4.1) is still satisfied. It is easy to see that a necessary and sufficient condition for the super-Higgs effect to occur is that the coefficient of the term $\bar{\psi}\cdot\gamma\chi$ be nonzero. If $\mathcal{G}_{,z}=0$, with $\mathcal{G}$ finite, (4.9) describes "massless" spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ particles[5] and we have unbroken supersymmetry in de Sitter space.

We conclude this section by commenting on more general interactions in supergravity. Owing to the additive structure of the auxiliary fields, it is likely that our formula will remain valid in any interaction with canonical kinetic terms and a vanishing cosmological term. This should also be true for gauge interactions with "vector" multiplets.

However, in supergravity a complete classification of interactions for which our mass formula is valid is missing at present. It is remarkable that the mass relation (1.1) is verified in the spontaneously broken version of SO(8) extended supergravity, recently constructed by Scherk and Schwarz[8] via dimensional reduction. In this theory all of the supersymmetry generators are broken and the corresponding eight spin-$\frac{3}{2}$ fermions acquire a mass via the super-Higgs mechanism. Also the usual Higgs mechanism for most of the 28 vector bosons takes place. The resulting theory describes the interaction of gravity with massive spin-$\frac{3}{2}$ and spin-1 particles together with lower spin states. It is crucial that this theory fulfill our necessary criterion, i.e., the absence of a cosmological constant. All masses are calculated[8,20] in terms of four arbitrary parameters m_1, m_2, m_3, m_4. Again, the relation

$$\sum_{J=0,1}(2J+1)m_J^2(\text{bosons})=\sum_{J=\frac{1}{2},\frac{3}{2}}(2J+1)m_J^2(\text{fermions}) \tag{4.10}$$

is confirmed.

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*On leave from Istituto di Fisica, Università degli Studi, Milano, and INFN, Sezione di Milano, Italy.

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