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SMALL TRANSVERSE MOMENTUM DISTRIBUTIONS IN HARD PROCESSES

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The effects of soft gluon brehmsstrahlung on the $k_\perp$ distributions of μ pairs produced in hadron–hadron collisions are studied using the Block–Nordsiek method. At moderate energies we obtain a good fit to present experimental data by adjusting the values of two phenomenological parameters. At quite large energies the predictions are independent of specific values assigned to the parameters and the whole $p_\perp$ distribution, including $p_\perp \simeq 0$, turns out to be computable.

1. Introduction

Much work has recently been devoted to the study in the framework of QCD of transverse momenta distributions in processes which involve a highly virtual photon of mass Q. In particular, the transverse momenta distributions of muon pairs produced in hadron–hadron collisions have been computed perturbatively for large values of $k_\perp$, i.e., in the region $k_\perp \simeq \mathrm{O}(Q)$ [1, 2]. In the region where $k_\perp \ll Q$ perturbation theory breaks down and one must use explicit resummation formulae. This task has been accomplished by Dokshitzer, D'Yakonov and Troyan (DDT) [3] who gave an expression valid in the double logarithmic (bilogarithmic) approximation. It is well-known in QED [4] that double logarithms can be simply resummed by using a Block–Nordsiek [5] approach in order to control the emission soft photons; we propose to apply this technique to soft gluons for sufficiently small values of $k_\perp$ where, as we shall see, the leading bilogarithmic approximation breaks down**.

In this paper we concentrate our attention on the $k_\perp$ distribution of μ pairs of mass Q produced in hadron–hadron collisions. We will argue that for large values of the total c.m. energy S, the whole $k_\perp$ distribution, including $k_\perp \simeq 0$, is computable from first principles. At first sight this result is surprising: one is inclined to

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** In QCD this approach has been followed in ref. [6].

believe that at $k_\perp=0$ one feels the effects of the original $k_\perp$ distribution of partons inside the hadrons. However, the probability of having a parton–antiparton annihilation in $\mu^+\mu^-$ with no emission of gluons having transverse momenta greater than a fixed value decreases asymptotically faster than any power of Q^2. Events at $k_\perp \simeq 0$ may be obtained asymptotically only by the emission of at least two gluons whose transverse momenta are not small and add to zero. This process is clearly computable and the effects of original $k_\perp$ distribution of partons inside the hadrons are washed out.

The plan of the paper is the following: in sect. 2 we present the method in the case of QED, we discuss the main features of $k_\perp$ distributions and we compare them with the DDT formula. In sect. 3 we try to extend the results to QCD and we discuss which are the main effects of a running coupling constant: asymptotic analytic expressions for the $k_\perp$ distributions are presented in the appendix. In sect. 4 we compare our result with existing data, we give predictions for the mass range of the weak neutral bosons and we make some concluding remarks.

2. Summing soft photons

Let us consider the process $e^+e^- \to \mu^+\mu^- + n\gamma$ *via* one-photon exchange and let us compute the $k_\perp$ distribution of the μ pair due to the emission of photons from the e^+e^- pair. We denote by s the mass squared of the e^+e^- system and by Q^2 that of the $\mu^+\mu^-$ system. We consider the $k_\perp$ distribution in the region $s(1-\delta) \leq Q^2 \leq s$, δ being a not too small but fixed number. The cross section is given by [7]

$$\frac{d\sigma}{dk_\perp^2} = \sigma_0(s)\left[\delta(k_\perp^2) + \frac{\alpha}{\pi}\left(\frac{\ln(s/k_\perp^2) + O(1)}{k_\perp^2}\right)\right], \tag{2.1}$$

where $\sigma_0(s)$ is the total cross section for the process $e^+e^- \to \mu^+\mu^-$. If we include the effects of virtual photon emissions we find for the $k_\perp$ inclusive cross section

$$\sigma_0(s)\Sigma(k_\perp^2) \equiv \int_0^{k_\perp^2} dp_\perp^2 \frac{d\sigma}{dp_\perp^2}$$
$$= \sigma_0(s)\left[1 - \frac{\alpha}{2\pi}\left(\ln^2\frac{s}{k_\perp^2} + O\left(\ln\frac{s}{k_\perp^2}\right)\right)\right]. \tag{2.2}$$

Neglecting simple logarithms and differentiating eq. (2.2), we find for small $k_\perp$

$$\frac{d\sigma}{dk_\perp^2} = \sigma_0(s)\left[\delta(k_\perp^2) + \frac{\alpha}{\pi}\left(\frac{\ln(s/k_\perp^2)}{k_\perp^2}\right)_+\right] \tag{2.3}$$
$$\equiv \sigma_0(s)[\delta(k_\perp^2) + \alpha\nu(k_\perp)],$$

where the cross section is a distribution [2] (it makes sense only after a smoothing in $k_\perp$) and the plus sign is defined by

$$\int_0^s \mathrm{d}k_\perp^2 [H(k_\perp)]_+ f(k_\perp) \equiv \int_0^s \mathrm{d}k_\perp^2 H(k_\perp)[f(k_\perp) - f(0)] . \tag{2.4}$$

With this definition, integrating eq. (2.3) we get back eq. (2.2). The sign plus takes care as usual of virtual photon exchange.

We want to find an expression for $\Sigma(k_\perp^2)$ (or $\mathrm{d}\sigma/\mathrm{d}k_\perp^2$), which is valid in the region $(\alpha/\pi)\ln^2(s/k_\perp^2) \sim \mathrm{O}(1)$. It is clear that the double logarithms arise from the emission of photons having momenta much smaller than Q. In this soft region the cross section for the production of n photons factorizes into independent distributions [5]

$$\frac{1}{\sigma_0}\mathrm{d}\sigma = \frac{\alpha^n}{n!}\mathrm{d}k_\perp^1 \cdots \mathrm{d}k_\perp^n \nu(k_\perp^1) \cdots \nu(k_\perp^n) , \tag{2.5}$$

where $\nu(k_\perp)$ is the probability distribution for one photon. If we call $p_\perp$ the sum of transverse momenta of all emitted photons, i.e., the transverse momentum of the recoiling μ pair, we find by a repeated use of the convolution theorem the eikonal result [8]

$$\begin{aligned} \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}^2 p_\perp} &= \frac{1}{4\pi^2}\int \mathrm{d}^2 b\, \mathrm{e}^{-ib\cdot p_\perp}\tilde{\sigma}(b) , \\ \tilde{\sigma}(b) &\equiv \exp[\alpha\tilde{\nu}(b)] , \\ \tilde{\nu}(b) &= \frac{1}{\pi}\int \mathrm{d}^2 k_\perp\, \mathrm{e}^{ik_\perp\cdot b}\nu(k_\perp) . \end{aligned} \tag{2.6}$$

In the large b region

$$\tilde{\nu}(b) \propto -\alpha \ln^2(1 + b^2 s) . \tag{2.7}$$

The learned reader may be surprised by the fact that we speak of independent photon emissions; it is known that the damping in the virtual mass of the electron implies in the single leading log approximation an ordering in the photon emission. However, this is not the case for inclusive soft photon emission. The matrix element for the soft bremsstrahlung from a charged fermion is proportional to $\alpha(p\cdot\epsilon/p\cdot k)$, where ϵ is the photon polarization, p is the particle's momentum and k is that of the photon, and α the fine structure constant. The matrix element for n photon emissions in the soft limit is

$$\begin{aligned} &\alpha^n \frac{p\cdot\epsilon_1}{k_1\cdot p}\frac{p\cdot\epsilon_2}{(k_1+k_2)\cdot p}\cdots\frac{p\cdot\epsilon_n}{(k_1+k_2\cdots+k_n)\cdot p} + (n!-1)\text{ permutations} \\ &= \alpha^n \frac{p\cdot\epsilon_1}{k_1\cdot p}\frac{p\cdot\epsilon_2}{k_2\cdot p}\cdots\frac{p\cdot\epsilon_n}{k_n\cdot p} , \end{aligned} \tag{2.8}$$

i.e., it factorizes in the product of independent matrix elements. We have neglected in eq. (2.8) terms proportional to k_ik_j in respect to $p \cdot k_i$, $p \cdot k_j$: this is true only in the soft region, where the electron energy is much greater than that of the photons*.

If we do the Fourier transform in eq. (2.7) in the leading bilogarithmic approximation, we find

$$\Sigma_{ee}(p_\perp^2) = \exp\left[-\frac{\alpha}{2\pi}\ln^2\frac{s}{p_\perp^2}\right], \tag{2.9}$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_\perp^2} = \frac{1}{p_\perp^2}\frac{\alpha}{\pi}\ln\frac{s}{p_\perp^2}\exp\left[-\frac{\alpha}{2\pi}\ln^2\frac{s}{p_\perp^2}\right]. \tag{2.10}$$

The result is slightly different from that of DDT in the same approximation

$$\Sigma_{\rm DDT}(p_\perp^2) = \left\{\int_1^2 dz\,\exp\left[-\frac{\alpha}{2\pi z^2}\ln^2\frac{s}{p_\perp^2}\right]\right\}^2, \tag{2.11}$$

although it coincides with it at the first non-trivial order in α. We are not able to pin down the origin of the discrepancy. We feel that DDT have strongly used the ordering in $k_\perp$'s of emitted photons, which breaks down in the soft region: this difference may be crucial. Note however, that both expressions have the same qualitative dependence on $p_\perp$. Eq. (2.10) is not a very accurate expression for the Fourier transform in the region where $p_\perp$ is small: it predicts that the differential cross section at fixed p goes to zero faster than any power of s; in particular, one obtains $d\sigma/dp_\perp^2|_{p_\perp^2=0} = 0$.

If we relax the leading bilog approximation for the Fourier transform of eq. (2.7) we find that, for small $p_\perp$ and when $s/m_e \gtrsim e^{1/\alpha}$** ($m_e$ being the electron mass)

$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_\perp^2}\bigg|_{p_\perp\approx 0} \propto \int_0^\infty db^2\,\exp\left[-\frac{\alpha}{\pi}\ln^2(1+b^2 s)\right]$$

$$\simeq \frac{1}{s}\int_0^\infty dt\,\exp[t-\alpha t^2] \sim \frac{1}{s}\exp\frac{\pi}{4\alpha}. \tag{2.12}$$

As one can see also for small $p_\perp$ the relevant region for the Fourier transform is the small b region. The physical interpretation is clear: $\Sigma_{ee}(p_\perp^2)$ plays the role of an elastic form factor similar to that of a virtual electron of mass $p_\perp^2$ and it gives the

* This approximation is known to give correct results for QED form factors to double log accuracy [4].

** This condition, when applied to QCD, does not imply fantastically high energies as in QED.

probability of muon pair production without emission of photons having $k_\perp$ greater than $p_\perp$. This probability decreases faster than any power of s and at small $p_\perp$ the leading contribution is given by multiple photon emission corresponding to small b. The formulation in impact space automatically takes care of this kind of effect. The general prescription that one obtains is the following: derive the results in the leading bilog approximation in impact space and do the Fourier transform to momentum space only at the end. This procedure allows the computation of $p_\perp$ distributions also at $p_\perp \simeq 0$.

Let us apply this approch to the full DDT formula for the process $A+B \to \mu^+\mu^- +$ anything, where A and B are particles made of electrons and positrons. We denote by $e_A(x, Q^2)$ the effective Q^2 dependent distribution of electrons (positrons) in the colliding particles (x is the fraction of momentum) and by y the c.m. rapidity of the μ pair. DDT found in the region where $p_\perp \ll Q$

$$\Sigma(p_\perp^2, Q^2, s) \equiv \int_0^{p_\perp^2} \left.\frac{d\sigma}{dk_\perp^2\, dQ\, dy}\right|_{y=0} dk_\perp^2$$

$$= C[e_A(x, p_\perp^2)\bar{e}_B(x, p_\perp^2) + (A \leftrightarrow B)]F(p_\perp^2/Q^2)\,, \tag{2.13}$$

$$C = \frac{8\pi\alpha^2\tau}{3Q^3}\,, \qquad \tau = \frac{Q^2}{s}\,, \qquad x = \sqrt{\tau}\,,$$

where the form factor F is given by

$$F\left(\frac{p_\perp^2}{Q^2}\right) = 1 - \frac{\alpha}{2\pi}\ln^2\frac{Q^2}{p_\perp^2} + O(\alpha^2)\,. \tag{2.14}$$

If we define the cross section in impact space, we obtain in the region $bQ \gg 1$

$$\frac{d\sigma}{dp_\perp^2}(p_\perp^2, Q^2, s) = \frac{1}{4\pi^2}\int d^2b\, e^{-ib\cdot p_\perp}\tilde{\sigma}(b, Q^2, s)\,, \tag{2.15}$$

$$\tilde{\sigma}(b, Q^2, s) = C[\tilde{e}_A(x, b^2)\tilde{\bar{e}}_B(x, b^2) + (A \leftrightarrow B)]\tilde{F}(Q^2b^2)\,,$$

where

$$\tilde{e}_A(x, b^2) \equiv e_A\left(x, \frac{1}{b^2}\right)^\star,$$

$$\tilde{F}(Q^2b^2) = 1 - \frac{\alpha}{\pi}[\ln^2(1+Q^2b^2)] \equiv 1 - \alpha\Delta(b)\,.$$

The expression we propose is

$$\tilde{\sigma}(b, Q^2, s) = C[\tilde{e}_A(x, b^2)\tilde{\bar{e}}_B(x, b^2) + (A \leftrightarrow B)]G(Qb)\,, \tag{2.16}$$

★ This definition follows from simple dimensional analysis: to double log accuracy, this is a sensible choice for the singly logarithmic dependence of electron distribution functions.

where $G(Q^2b^2) \equiv \exp - \alpha\Delta(b)$, which is valid in the region $1/Q^2 \leq b^2 \leq 1/m_e^2$, m_e being the typical mass scale. At distances $b > 1/m_e$ the dependence of $\tilde{e}_A(x, b^2)$ on b is connected to the details of the target: however, if Q^2 is large enough, $\exp(-\alpha\Delta(Q/m_e))$ is a very small number and this region is never relevant for the Fourier transform of eq. (2.12).

3. Summing soft gluons

In this section we propose to apply our summation to QCD. In this case, we cannot directly use eq. (2.16) because of the sizeable dependence of strong coupling constant on Q^2 which might modify the exponentiation of double logs. The simplest suggestion would be to use independent gluon emissions with a probability proportional to $\alpha(k_\perp^2)$

$$\frac{1}{\sigma_0}\frac{d\sigma^1}{dk_\perp^2}\bigg|_{k_\perp^2 \neq 0} \frac{4}{3\pi}\frac{\alpha(k_\perp^2)\ln(Q^2/k_\perp^2)}{k_\perp^2} + O[\alpha^2(k_\perp^2)], \qquad (3.1)$$

where $\alpha(k_\perp^2) \equiv 12\pi/25 \ln(k_\perp^2/\Lambda^2)$ with four active flavours.

The leading log result for the form factor $F(Q^2/p_\perp^2)$ of eq. (2.13) is then obtained by exponentiating the first order as in the case of QED

$$F\left(\frac{Q^2}{p_\perp^2}\right) = \exp\left[-\int_{p_\perp^2}^{Q^2} \frac{1}{\sigma_0}\frac{d\sigma^{(1)}}{dk_\perp^2} dk_\perp^2\right]. \qquad (3.2)$$

This result may be questioned for being too naive. A more learned approach would be to study the form factor $F(Q^2/p_\perp^2)$ with the same resummation techniques used to find the Soudakov form factor in QCD. In this case a renormalization group equation has been derived by De Rafael, Coquereaux and Korthals-Altes [9], which allows one to control the resummation of double logs. Using their formalism, which amounts to the introduction of Q^2 dependent anomalous dimensions, we find that

$$F(Q^2, p_\perp^2) = \exp\left\{-\frac{16}{25}\ln\frac{Q^2}{\Lambda^2}\ln\frac{\ln(Q^2/\Lambda^2)}{\ln(p_\perp^2/\Lambda^2)} + \frac{16}{25}\ln\frac{Q^2}{p_\perp^2}\right\}. \qquad (3.3)$$

Eqs. (3.2) and (3.3) coincide and the naive approach seems to be justified, although a more careful investigation of this point would be needed. The corresponding exponentiation in impact parameter space reads

$$\tilde{F}(Q^2, b) = \exp\Delta(b),$$

$$\Delta(b) \equiv \frac{1}{\pi}\int d^2k_\perp \left[\frac{4\alpha(k_\perp^2)\ln(Q^2/k_\perp^2)}{3\pi k_\perp^2}\right]_+ \exp ib\cdot k_\perp$$

$$= \frac{1}{\pi}\int d^2k_\perp \frac{4}{3\pi}\frac{\alpha(k_\perp^2)\ln(Q^2/k_\perp^2)}{k_\perp^2}[\exp(ib\cdot k_\perp) - 1].$$

Some care must be used in order to define the integral in eq. (3.4) as far as the coupling constant has a singularity at $k_\perp^2 = \Lambda^2$. It is useful to perform the split

$$\Delta(b) = \frac{1}{2\pi} \int_{M^2}^{Q^2} \mathrm{d}k_\perp^2\, \nu(k_\perp) \int \mathrm{d}\varphi\, [\exp(i\bar{k}_\perp \cdot \bar{b}) - 1]$$
$$+ \frac{1}{2\pi} \int_0^{M^2} \mathrm{d}k_\perp^2 \nu(k_\perp) \int \mathrm{d}\varphi\, [\exp(i\bar{k}_\perp \cdot \bar{b}) - 1], \tag{3.5}$$

where M^2 is a typical mass of order Λ^2 which acts as an infrared cutoff and

$$\nu(k_\perp) \equiv \frac{4}{3\pi} \alpha(k_\perp^2) \frac{\ln(Q^2/k_\perp^2)}{k_\perp^2}.$$

Whilst the first integral is well-defined, the second is not because the perturbative expression for $\nu(k_\perp)$ cannot be safely used. However, if we assume that the complete theory will not show up a physical singularity in the mass region from zero up to M^2, the contribution of the second integral is proportional to b^2M^2 and is negligible in the small b region which is, as we will see, the only relevant integration region.

We finally obtain the approximated expression for $\tilde{F}(Q^2, b^2)$ in the region $1/Q \ll b \ll 1/\Lambda$ at leading log level in b space

$$\tilde{F}(b_2, Q^2) \simeq \left[\frac{\ln(1/b^2\Lambda^2)}{\ln(Q^2/\Lambda^2)}\right]^{(16/25)\ln(Q^2/\Lambda^2)}. \tag{3.6}$$

The final expression for the $p_\perp$ distribution relative to the process $\mathrm{A}+\mathrm{B} \to \mu^+\mu^-$ + anything at zero rapidity in the c.m. frame is (the generalization for arbitrary rapidity is trivial):

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp^2\, \mathrm{d}Q\, \mathrm{d}y}\right|_{y=0} = \tfrac{1}{2} \int b\, \mathrm{d}b\, J_0(|b||p_\perp|)\tilde{\sigma}(b, Q, s),$$
$$\tilde{\sigma}(b, Q, S) = \tfrac{1}{3}C\left\{\sum_i e_i^2 q_i^{\mathrm{A}}\left(x, \frac{1}{b^2}\right)\bar{q}_i^{\mathrm{B}}\left(x, \frac{1}{b^2}\right) + (\mathrm{A} \leftrightarrow \mathrm{B})\right\}\tilde{F}(Q^2, b^2), \tag{3.7}$$

and $J_0(Z)$ is the Bessel function of first kind. The general notation is the same as in eq. (2.16): e_i are the quark electric charges and q_i^{A} their effective distribution functions. This equation breaks down at $b \sim 1/\Lambda$ where the effects of hadronic size get important and the effective quark distribution cannot anymore be computed. Summarizing, we can distinguish three regions: in the first, $0 \leqslant b \leqslant 1/Q$, lowest-order perturbation theory can be used; in the second, $1/Q \leqslant b \leqslant 1/\Lambda$, resummed perturbation theory can be used and in the third $b \geqslant 1/\Lambda$ one is sensitive to the non-perturbative structure of the hadron. However, for $b \geqslant 1/\Lambda$, the b integral is already depressed by a form factor which goes asymptotically as*

$$\left(\frac{\Lambda^2}{Q^2}\right)^{(16/25)\ln\ln(Q^2/\Lambda^2)}.$$

It tends to zero faster than any power of Q^2, implying that the cross section is dominated by the first two regions.

4. Comparison with experimental data

Let us apply the formulae derived in sect. 3 to muon pair production in p-matter collisions. We will use slightly simpler formulae: at leading bilogarithmic approximation neglect the b dependence of quark densities. We express the lowest-order perturbative cross section as

$$\left.\frac{d\sigma}{dk_\perp^2\,dQ\,dy}\right|_{y=0}=\sigma_0[\delta(k_\perp^2)+\alpha(k_\perp^2)\nu(k_\perp^2)]\,, \tag{4.1}$$

where $\nu(k_\perp)$ can be computed using the formulae in ref. [2] and

$$\sigma_0\equiv\left.\frac{d\sigma}{dQ\,dy}\right|_{y=0}.$$

In the leading log approximation in the $k_\perp$ space one would find

$$\left.\frac{d\sigma}{dk_\perp^2\,dQ\,dy}\right|_{y=0}=\sigma_0\frac{d}{dk_\perp^2}\exp\left(-\int_{k_\perp^2}^{s}dp_\perp^2\,\alpha(p_\perp^2)\nu(p_\perp^2)\right). \tag{4.2}$$

We exponentiate in b space and using the arguments developed in previous sections we obtain

$$\left.\frac{1}{\sigma_0}\frac{d\sigma}{dk_\perp^2\,dQ\,dy}\right|_{y=0}=\frac{1}{4\pi}\int d^2b\,e^{-ib\cdot k_\perp}\sigma(b)\,,$$

$$\sigma(b)=[\exp\Delta(b)]\rho(b)\,, \tag{4.3}$$

where

$$\Delta(b)=\frac{1}{\pi}\int d^2k_\perp(e^{ik_\perp\cdot b}-1)\alpha(k_\perp^2)\nu(k_\perp^2)\,,$$

and $\rho(b)$ is a smearing function which has a characteristic scale of the order of the hadronic size. It reproduces phenomenologically the effects of an intrinsic transverse momentum distribution of partons.

* We assume that the neglected terms coming from the integration region from 0 to M^2 do not have such a singular dependence to compensate the damping of the perturbative part of the form factor.

The exponentiation of the complete lowest-order result is correct at leading bilog approximation and presents the advantage of being automatically in agreement with the lowest-order calculation when $k_\perp \simeq O(Q)$.

In order to calculate $\Delta(b)$ in eq. (4.3) we have to regularize the pole of $\alpha(k_\perp^2)$. The choice of the regularization prescription does not influence the asymptotic predictions: its effects are in fact relevant only for large b where, however, the integrals are damped by the form factor in impact space $\tilde{F}(Q^2, b_2)$ in eq. (3.7).

In fig. 1, we plot the quantity $\Delta(b)$ as a function of b expressed in Fermi for $s \simeq 7.5 \times 10^5$ and $Q^2 \sim 7500\ \mathrm{GeV}^2$. Notice that at $b \simeq 0.2$ Fermi, i.e., at a distance still small compared to hadronic size, the form factor damps the b integral by a factor 50.

Our procedure is the following: at moderate energies we will show that experimental data can be fitted with a reasonable choice of the regularization procedure and of the function $\rho(b)$: indeed, the results depend on this choice. At high energy the results are practically independent of the values of phenomenological parameters.

We use the following explicit procedure: we replace $\alpha(k_\perp^2)$ with $\alpha[k^2+M^2]$: if $M^2 > \Lambda^2$ the pole is not in the physical region. Notice that in the large $p_\perp$ region

$$\alpha(k_\perp^2 + M^2) \underset{k_\perp \to \infty}{\longrightarrow} \alpha(k_\perp^2)\left[1 - \exp\left(-\frac{12\pi}{25\alpha(k_\perp^2)}\right)\frac{25\alpha(k_\perp^2)}{12\pi}\frac{M^2}{\Lambda^2}\right],$$

which obviously does not modify the perturbative result.

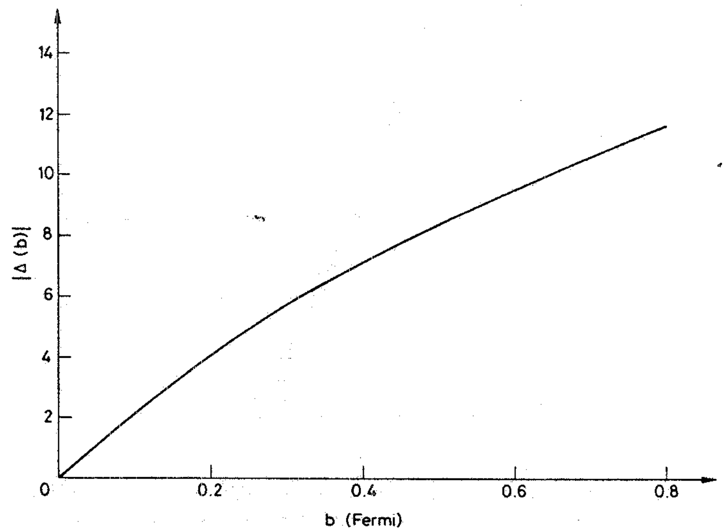

Fig. 1. The absolute value of $\Delta(b)$ (see text) for $s = 7.5 \times 10^5\ \mathrm{GeV}^2$ and $Q^2 = 7500\ \mathrm{GeV}^2$ as a function of b.

A simple choice for the function $\rho(b)$ is

$$\rho(b) = \exp\left(-\frac{b^2}{4A}\right),$$

corresponding to a transverse momentum distribution

$$\exp(-p_\perp^2 A),$$

and to an average value $\langle p_\perp^2 \rangle_{\text{intrinsic}} = 1/A$.

A good fit to the data is obtained with different choices of A and M^2; in fig. 2 we show a fit to experimental data using $M^2 = 1.25\ \text{GeV}^2$ and $A = 2.5\ \text{GeV}^{-2}$ corresponding to $\langle p_\perp^2 \rangle_{\text{intrinsic}} \simeq 0.4\ \text{GeV}^2$. In the same figure the dashed line corresponds to the choice $A = \infty(\langle p_\perp^2 \rangle_{\text{intrinsic}} = 0)$ and $M^2 = 1.75$, which, in the small $p_\perp$ region, differs from the data by a factor 2. This shows that at present energies, the low $p_\perp$ distribution is sensitive to the choice of parameters. Notice that by

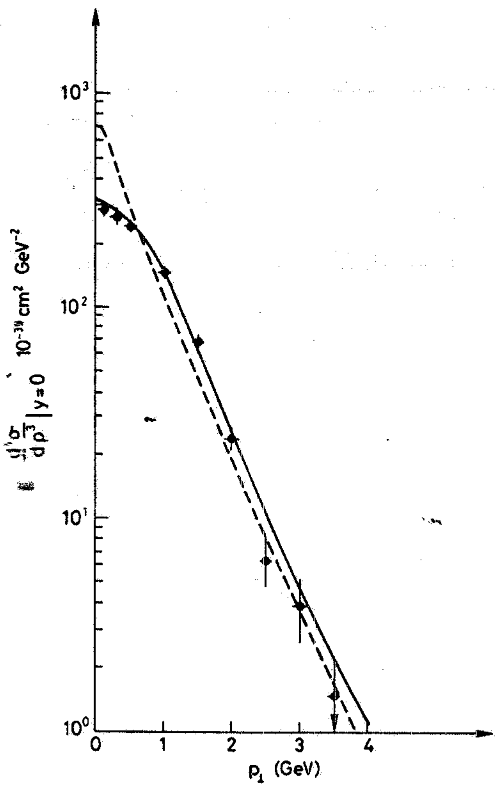

Fig. 2. Theoretical predictions are compared with the experimental data of ref. [10] at $s = 750\ \text{GeV}^2$ and $Q^2 = 56\ \text{GeV}^2$. We report the results obtained with two different choices of parameters $\langle p_\perp^2 \rangle_{\text{intrinsic}}$ and M: (i) first choice: $\langle p_\perp^2 \rangle_{\text{intrinsic}} = 400\ \text{MeV}^2$, $M = 1.25\ \text{GeV}^2$ (full line) (ii) second choice: $\langle p_\perp^2 \rangle_{\text{intrinsic}} = 0$, $M = 1.75\ \text{GeV}^2$ (dashed line).

taking into account the $k_\perp^2$ dependence of α we obtain a sizeable decrease of the intrinsic $\langle p_\perp^2 \rangle_{\text{intrinsic}}$ needed to fit the data. The main difference with respect to ref. [2] is the replacement of $\alpha(Q^2)$ with $\alpha(k_\perp^2)$; as far as $k_\perp^2$ is $O(Q^2)$ the two results asymptotically coincide but large differences are possible in the low Q^2 region.

In fig. 3 we present the shape of the $k_\perp$ distributions in arbitrary units for quite large energies: $s = 7.5 \times 10^5$ and $Q^2 = 7500\ \text{GeV}^2$. On the top of the figure we report the ratio R of predictions made with the two sets of parameters ($\{A = 2.5\ \text{GeV}^{-2}, M^2 = 1.25\ \text{GeV}^2\}$ and $\{A = \infty, M^2 = 1.75\ \text{GeV}^2\}$) which gave at moderate energies large differences in the low $p_\perp$ region. At high energies the difference between the predictions obtained with the two sets of parameters is reduced to about 10% at $p_\perp \approx 0.3$ GeV showing the independence of the result of the

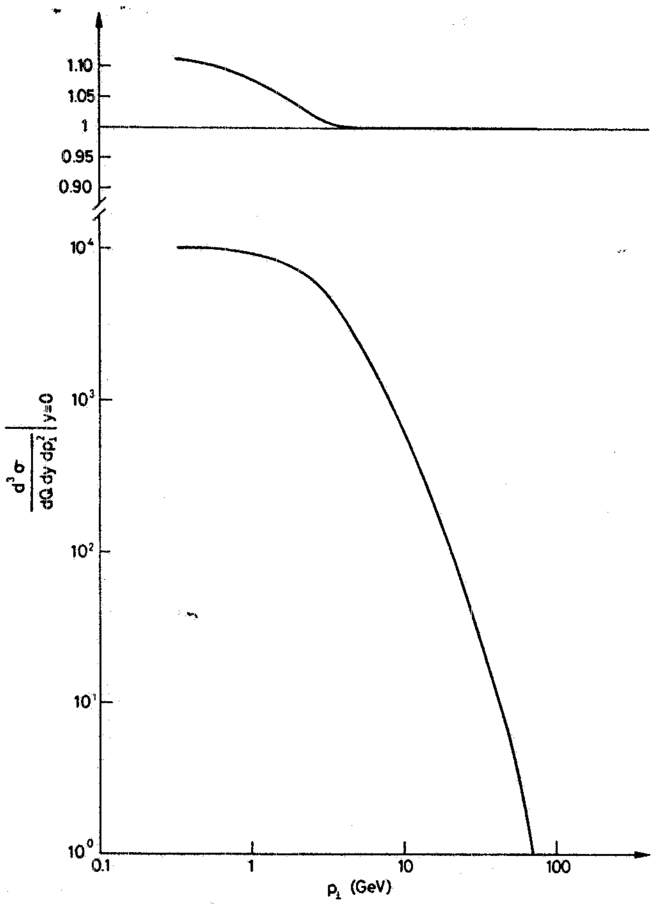

Fig. 3. The $p_\perp$ distribution in arbitrary units for $s = 7.5 \times 10^5\ \text{GeV}^2$ and $Q^2 = 7500\ \text{GeV}^2$, using the first choice of parameters of fig. 2. On top we show the ratio between the predictions obtained with the second choice and the first choice of parameters.

particular values assigned to parameters. In fig. 4 we report the same quantities as in fig. 2 for $s=7.5\times10^5$ and $Q^2=120$. The general feature is that the peak at $p_\perp=0$ flattens when Q^2 increases according to the increase of $\langle p_\perp^2\rangle$ with Q^2 at s fixed in this region of Q^2/s [1].

The calculation we have presented is based on the conjecture with the soft limit of QCD can be treated in a full analogy with that of QED with the minor technical change of α into $\alpha(k_\perp)$. The possibility for gluons to emit gluons may affect the independent bremsstrahlung picture of the Block–Nordsiek method. The existing results on elastic form factors in QCD however seems to confirm our simple guess, but further investigations of this point are definitely quite important. The approach we have discussed can be applied to any hard process which is damped in the

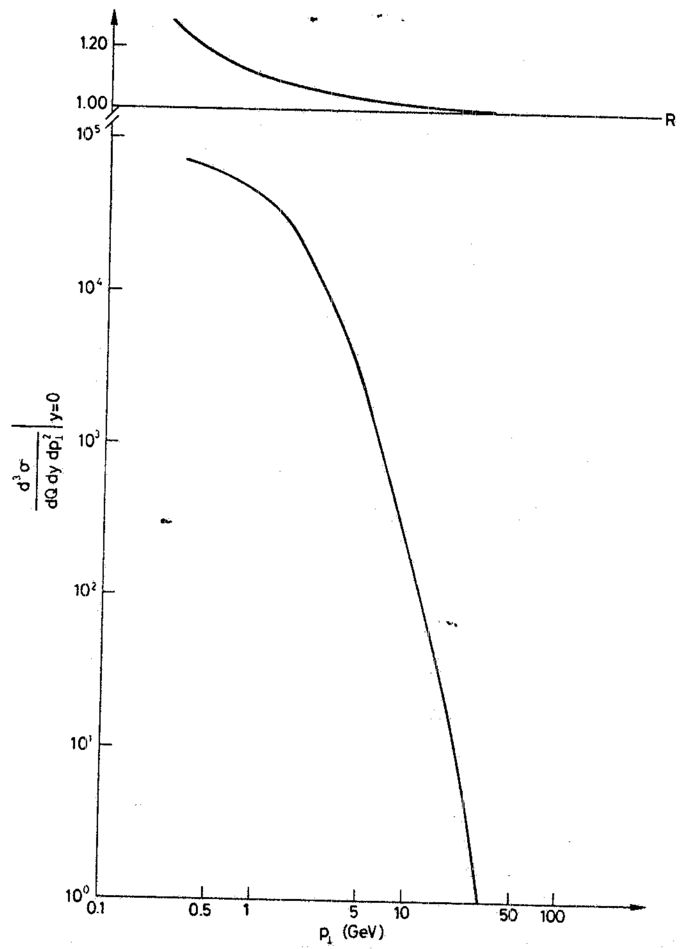

Fig. 4. We plot the same quantities as in fig. 3 for $s=7.5\times10^5$ GeV^2 and $Q^2=120$ GeV^2.

double logarithmic approximation, as it is the case for transverse momenta distributions. Owing to the damping, small $p_\perp$ distributions are obtained asymptotically as a compensating sequence of semi-hard processes and turn out to be computable by means of suitable resummation methods.

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Appendix

We want to present some analytic expressions for the asymptotic behaviour of $p_\perp$ distributions in the $p_\perp \sim 0$ region.

We write down the cross section as follows:

$$\left.\frac{d\sigma}{dp_\perp^2}\right|_{p_\perp\sim 0} \simeq \left(\frac{d\sigma}{dp_\perp^2}\right)_{p_\perp=0} + \left[\frac{d}{dp_\perp^2}\left(\frac{d\sigma}{dp_\perp^2}\right)\right]_{p_\perp=0} p_\perp^2$$
$$\equiv \sigma_0 R_0\{1 - R_1 p_\perp^2\}, \tag{A.1}$$

where

$$R_0 = \left(\left.\frac{d\sigma}{dp_\perp^2}\right|_{p_\perp=0}\right) \Big/ \int \frac{d\sigma}{dp_\perp^2} dp_\perp^2 ,$$
$$R_1 = -\left[\frac{d}{dp_\perp^2}\left(\frac{d\sigma}{dp_\perp^2}\right)\right]_{p_\perp=0} \Big/ \left(\frac{d\sigma}{dp_\perp^2}\right)_{p_\perp=0} .$$

Using eq. (3.3) we find, in the asymptotic region

$$R_0(Q^2) \sim \int_{1/Q^2}^{1/\Lambda^2} db^2 \left\{ (Q^2 b^2) \left[\frac{\ln(1/b^2\Lambda^2)}{\ln(Q^2/\Lambda^2)}\right]^{\ln(Q^2/\Lambda^2)} \right\}^{16/25}$$
$$\simeq \frac{1}{Q^2} \int_0^1 dz \exp\{T[(1+A)z + A\ln(1-z)]\} ,$$
$$T = \ln(Q^2/\Lambda^2) , \qquad A = 16/25 . \tag{A.2}$$

We evaluate the integral with the saddle point method and we find for large Q^2

$$R_0(Q^2) \simeq \left(\frac{\Lambda^2}{Q^2}\right)^{\eta_0} , \qquad \eta_0 = A \ln\left[1 + \frac{1}{A}\right] \simeq 0.6 .$$

In a similar way we find

$$R_1(Q^2) \simeq \left(\frac{\Lambda^2}{Q^2}\right)^{[\eta_1 - \eta_0]} , \qquad \eta_1 = A \ln\left[1 + \frac{2}{A}\right] \simeq 0.91 .$$

One therefore obtains that the peak at $p_\perp = 0$ flattens with a width proportional to $(Q^2/\Lambda^2)^{0.31}$.

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