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G. Martinelli and G. Parisi: TESTABLE QCD PREDICTIONS
FOR SPHERICITY-LIKE DISTRIBUTIONS IN e^+e^-
ANNIHILATION.

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ABSTRACT.

We give a procedure to define sphericity type variables which are infrared finite and not very sensitive to jet pionization.

QCD predictions for the distribution of these variables can be computed in perturbation theory. We give numerical results in some examples.

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In the framework of perturbative QCD it is possible to make predictions for the asymptotic behavior of some inclusive distributions in high energy e^+e^- annihilation. Only differential cross sections which are free from infrared collinear divergencies can be safely computed; as an example the sphericity[1] which is "linear" in the momenta of the particles satisfies this criterium, while the sphericity, being "quadratic" in the momenta, is infrared divergent at the order α_s^2 and cannot be computed perturbatively.

At present energies strong non perturbative corrections are present, due to the emission of many soft pions.

This effects decrease like $1/Q^2$, where Q^2 is the mass of the virtual photon. In order to test better QCD predictions it is useful to choose variables less sensitive to these non perturbative effects.

One possibility, proposed in refs. (2, 3) is to divide the solid angle in N sectors (S_i; i = 1, N) and to define the four-momentum of each sector as:

$$p_\mu^{(i)} = \sum_{k \in S_i} p_\mu^k \qquad (1)$$

where the index k labels the final state particles going in the sector S_i. The advantage of introducing this definition is that the differential cross--section $\dfrac{d\sigma}{dp_\mu^{(i_1)} \ldots dp_\varrho^{(i_J)}}$ remains finite at all orders in perturbation theory : this happens because no divergence is present in the distribution of momenta inside a given solid angle[4].

Consequently the distribution of any quantity which is a smooth function of the $p^{(i)}$ is well defined.

For example one can predict the cross-section for $\dfrac{d\sigma}{dx_i}$ ($x_i = \dfrac{2E^{(i)}}{Q}$), i. e. the distribution for the energy deposed in the sector S_i (3, 5). In the same way we can compute the cross-section as a function of a new sphericity starting from the momenta $p^{(i)}$. More generally we can introduce for any n the tensor :

$$T_{ab}^n = 2 \frac{\sum_i \left| \vec{p}^{(i)} \right|^n (\delta_{ab} - n_a^{(i)} n_b^{(i)})}{\sum_i \left| \vec{p}^{(i)} \right|^n} \qquad (2)$$

$$n_a^{(i)} = \frac{p_a^{(i)}}{\left| \vec{p}^{(i)} \right|}$$

and define t^n as the smallest eigenvalue of T_{ab}^n ($0 \leqslant t^n \leqslant \frac{4}{3}$).

If $t^n = 0$ the event is a two jet event; if $t^n = 4/3$ the event is perfectly spherical and for a three particle event $t^n \leqslant 1$[6]. For n = 2 we recover the usual definition of sphericity.

Without introducing the sum over particles momenta of the same

sector only the distribution of t^1 can be predicted. With our definition we can predict the distributions of all the t^n. Experimentally, at increasing n, the noise due to pionization is decreased.

Of course both the theoretical and the experimental results will depend on the choice of the sectors. We choose our sectors in such a way that a whole jet could be absorbed in one sector, i.e. the size of the sector is wider than the angular opening of the jet, which is a function of the energy.

Theoretical predictions can be obtained in the following way. One divides into sectors the visible solid angle (we have considered below the case where the visible solid angle is 4π but one can give predictions also if only a portion of total solid angle is accesible). In our case we have divided the surface of the sphere in 6 approximatively equal sector defined as :

$$S_{i, \, i=5, 6} \quad : \quad 0 \leqslant \varphi \leqslant 2\pi ; \quad \cos\theta > \frac{\sqrt{2}}{2} \qquad \text{if } i = 5 ,$$

$$\cos\theta < -\frac{\sqrt{2}}{2} \qquad \text{if } i = 6 , \qquad (3)$$

$$S_{i, \, i=1, 4} \quad : \quad (i-1)90^\circ \leqslant \varphi_i \leqslant i \, 90^\circ ; \quad -\frac{\sqrt{2}}{2} \leqslant \cos\theta \leqslant \frac{\sqrt{2}}{2} .$$

The mean angular opening of the sectors is about 90° which is greater than the extimated opening of a jet at $Q \sim 20$ GeV. We have choosen a low value of N in order to decrease the effects of soft pions on experimental results. The theoretical predictions at order α_s can be obtained by integrating the differential cross-section for the process $e^+ e^- \rightarrow q\bar{q}G$. This cross section is given by :

$$\frac{d\sigma}{dx_q \, dx_{\bar{q}} \, d(\cos\theta_q)} =$$

$$= \frac{3}{8} \, \sigma_0 \left[D_T(x_q, x_{\bar{q}}) \, (1 + \cos^2\theta_q) + D_L(x_q, x_{\bar{q}}) \sin^2\theta_q \right] \qquad (4)$$

where $\sigma_0 = 3 \, (\sum_F e_f^2) \, \dfrac{4\pi\alpha_{e.m.}^2}{3Q^2}$ is the total cross-section for producing

$q\bar{q}$ pairs, e_f is the fractional electric charge of the quark with flavour f.

$$D_T(x_q, x_{\bar{q}}) = \frac{4}{3} \frac{\alpha_s(Q^2)}{2\pi} \left[\frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})} - 2(x_q + x_{\bar{q}} - 1) \right]$$

(5)

$$D_L(x_q, x_{\bar{q}}) = \frac{16}{3} \frac{\alpha_s(Q^2)}{2\pi} (x_q + x_{\bar{q}} - 1)$$

$x_q = \frac{2E_q}{Q}$ and $x_{\bar{q}} = \frac{2E_{\bar{q}}}{Q}$ are respectively the fraction of energy carried by the quark (antiquark).

$\alpha_s(Q^2) = \frac{12\pi}{25 \ln(\frac{Q^2}{\Lambda^2})}$ is the running coupling constant in the case of four flavours.

Notice that eq. (4) give the same cross-section as for the process $e^+e^- \rightarrow \mu^+\mu^-\gamma$ apart for a scale factor $\frac{4}{3}(\frac{\alpha_s(Q^2)}{\alpha_{e.m.}})$ and with the γ coming only from the muon legs.

An analytic computation being quite hard, we have done a numerical calculation. We have extracted 10^5 events randomly and we have computed their cross-section according to eq. (4). We first consider the quantity:

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx} = \frac{1}{\sigma_0} \sum_i (\frac{d\sigma}{dx_i})_{x_i = x}$$

(6)

which is the sum over all the sectors of the differential cross-section for fixed energy flowing in each sector. An analytic extimate for this quantity in the case of small angular volume of the sectors (many sectors) has been given in ref. (3). In the present case the result is shown in Fig. 1.

Similarly we give in Fig. 2 the cross-sections $\frac{d\sigma}{dt^n}$ for n = 1, 2, 4.

We consider also the case in which particles are always considered to belong to different sectors (an equivalent procedure would be to introduce an in
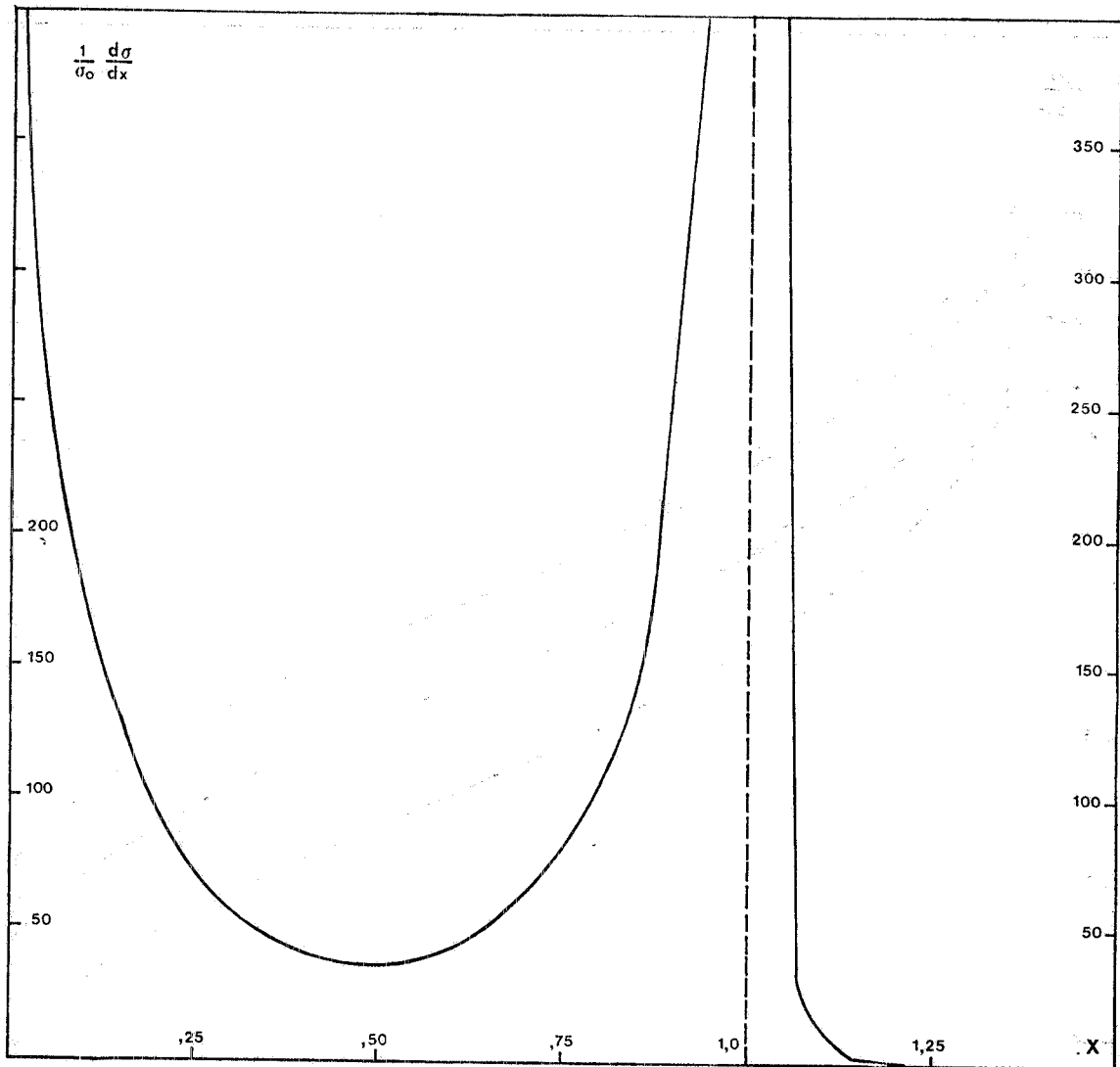
FIG. 1 - The quantity $\dfrac{1}{\sigma_O} \dfrac{d\sigma}{dx} = \dfrac{1}{\sigma_O} \sum_i \left(\dfrac{d\sigma}{dx_i} \right)_{x_i = x}$ $\quad \left(x_i = \dfrac{2E^{(i)}}{Q} \right)$

with $\dfrac{\alpha_s(Q^2)}{2\pi} = 1$ and with the solid angle divided into

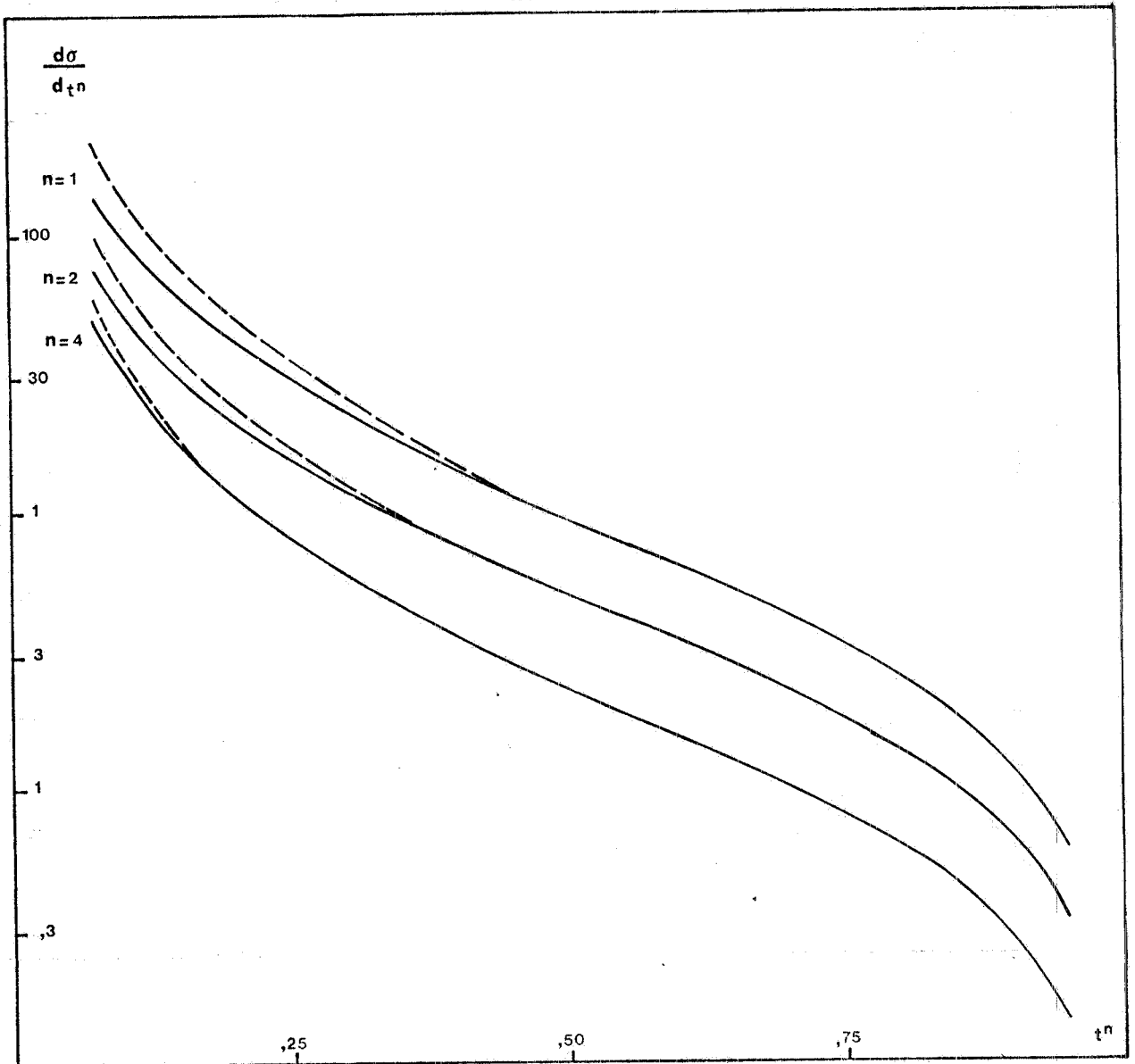six sectors, as defined in the text, is presented.

FIG. 2 - The cross-sections $\dfrac{1}{\sigma_o} \dfrac{d\sigma}{dt^n}$ with $\dfrac{\alpha_s(Q^2)}{2\pi} = 1$ (solid lines)

for n = 1, 2, 4 and with the solid angle divided into six sectors are presented. The dashed lines represent the same cross-sections in the limit of an infinite number of sectors.

finite number of sectors). At first order in α_s this can be done becau se the differential cross-sections $\frac{d\sigma}{dt^n}$ (n > 1) are finite. This last cross-sections are also shown in Fig. 2 (dashed lines). For $t^n \gtrsim 0.25$ the difference with the previous case is pratically negligible because for t enough large, if only three particles are present is quite likely that the particles will belong allways to different sectors.

We have computed only the first order cross-section in the case of N sectors (N = 6).

Second order corrections are estimated to be proportional to $\ln(N)/\ln\frac{Q^2}{\Lambda^2}$ non perturbative effects are proportional to $\frac{N}{Q^2}$. Our prediction cannot be extended at too large N. For reasonable values of N the choice of the number of sectors is irrelevant (as shown in Fig. 2).

As allways, our predictions are no more reliable at the limits of phase space (i. e. t = 0, 1 and x = 1). In these regions high order cor rections are quite strong and cannot be neglected. It would be interest ing to sum at all orders the leading corrections near the boundary of the phase space. The methods of ref. (8) may be useful.

It is evident that similar variables to the $p^{(i)}$ can be used in any jet-like analysis as for example deep inelastic scattering or proton- -proton collisions.

In these cases theoretical predictions depend on the proton struc- ture functions.

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