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B. Baschiera, G. Basini, H. Bilokon, C. Castagnoli, B. D'Ettore
Piazzoli, G. Mannocchi and P. Picchi: INTEGRAL AND DIFFE
RENTIAL ABSOLUTE INTENSITY MEASUREMENTS OF COSMIC
RAY MUONS BELOW 1 GeV.

B. Baschiera^(x), G. Basini^(o), H. Bilokon^(o), C. Castagnoli⁽⁻⁾,
B. D'Ettorre-Piazzoli^(o), G. Mannocchi^(o) and P. Picchi⁽⁺⁾ :
INTEGRAL AND DIFFERENTIAL ABSOLUTE INTENSITY
MEASUREMENTS OF COSMIC RAY MUONS BELOW 1 GeV.

ABSTRACT.

Absolute differential and integral muon intensities below 1 GeV/c have been measured with a flash-tube range spectrograph in which stopping muons are identified by the decay sequence. The differential intensities at 0.314 GeV/c and 0.805 GeV/c are $(3.25 \pm 0.17) \times 10^{-3} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1} \text{ GeV/c}^{-1}$ and $(3.60 \pm 0.18) \times 10^{-3} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1} \text{ GeV/c}^{-1}$ respectively, i. e. 16% and 33% higher than that of Rossi. These measurements confirm the recent results of other authors according to which the Rossi reference point at 1 GeV/c is too low of about 25%. Our integral measurements at 0.457 GeV/c and 0.918 GeV/c support this evidence.

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- (x) Laboratorio di Cosmogeofisica del CNR, Torino.
(o) Laboratorio di Cosmogeofisica del CNR, Torino, and
INFN, Laboratori Nazionali di Frascati.
(-) Istituto di Fisica Generale dell'Università, Torino.
(+) Istituto di Fisica Generale dell'Università, Torino, and
INFN, Laboratori Nazionali di Frascati.

1. - INTRODUCTION.

The knowledge of the absolute cosmic muon spectrum is of great interest because the muon flux is related to the production spectrum of the parents mesons and is affected by the geomagnetic field and by the solar activity cycle. The shape of this spectrum has been extensively investigated and the experimental data have been traditionally normalized to the differential intensity at the momentum of 1 GeV/c given by Rossi⁽¹⁾.

This important reference point has come under criticism because various recent measurements at momenta around 1 GeV/c agree that the intensity given by Rossi is too low by 25%.

The experimental setups used for these measurements are magnetic^(2, 3) and range^(4, 5, 6) spectrographs and air Cerenkov light detectors⁽⁷⁾. In view of the importance of these results and of necessity to establish it by a different experimental technique we have performed absolute measurements of the differential muon intensity at two momenta below 1 GeV/c (0.314 and 0.805 GeV/c) using a flash-tube range spectrograph in which the muon stops in a liquid scintillation counter and is identified through its decay sequence. This technique enables the rejection of muon simulating electrons. The combination of visualization and simple geometry makes the evaluation of the corrections rather straightforward and accurate. In consequence the absolute differential intensities are measured with a good accuracy ($\sim 5\%$).

Besides we have carried out with the same apparatus the absolute measurement of the muon integral intensity at two muon momenta in the same momentum region (0.457 and 0.918 GeV/c).

2. - EXPERIMENTAL APPARATUS.

The experimental apparatus used at sea level for the study of muons with $E < 1$ GeV has been so designed to allow :

- a) an optical analysis of the event ;
- b) a good geometrical reconstruction of the event by using only a small and well defined acceptance over a larger area which is optically sensitive ;
- c) a measure of the differential spectrum by using both electronic and optical anti-coincidence criteria and the identification of the end-of-range muon through its decay .

The set up is shown in Fig. 1. S_1 and S_2 are two pairs of plastic scintillators^{each} watched by a single P. M., with useful areas of (123×123) mm² and (101×101) mm², positioned at a distance of 1262 ± 1 mm in order to accept vertical particles inside 4° . The acceptance weighted with a $\cos^2 \theta$ distribution is 97 ± 0.1 cm² sr. This value is a good compromise between an aperture independent of the spectral index, a small

total dead time and an acceptable counting rate. S_3 and S_4 are two layers each made of two NE 110 plastic scintillators of volume $140 \times 70 \times 3 \text{ cm}^3$.

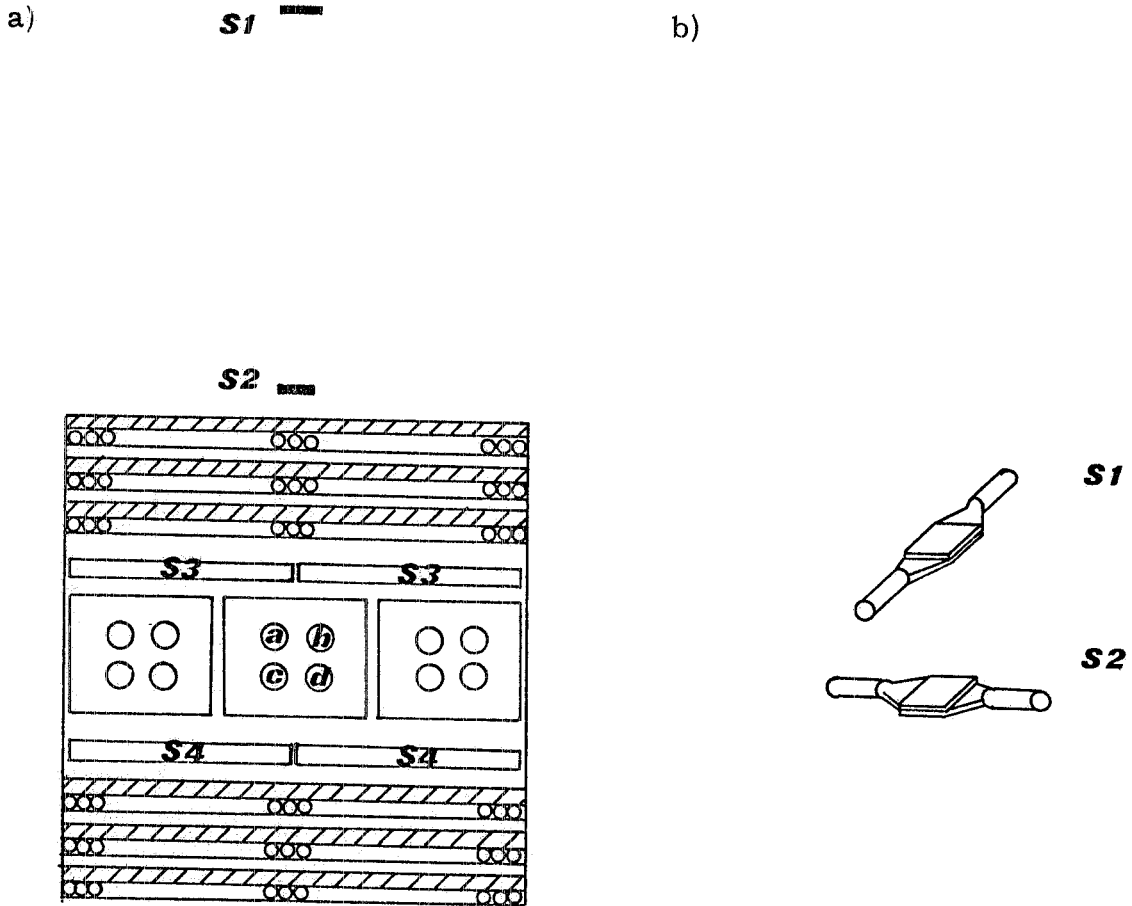


FIG. 1 - a) Front view of the apparatus. S_1 , S_2 , S_3 , S_4 : plastic scintillators; a, b, c, d: photomultipliers on one side of the liquid scintillation counter X. The two stacks of criss-crossed flash-tube chambers are sketched. Every stack has 3+3 chambers each made of two layers of staggered tubes. The anticoincidence counters R_1 and R_2 (see text) are not shown. b) The composition of the counters S_1 and S_2 is shown: each counter is constituted by two thin plastic scintillators put in coincidence.

Each scintillator is watched by 4 RCA 6655 A photomultiplier. S_3 has been used only for calibration while S_4 has been actually used as anti-coincidence during the measurement of the differential intensity. Two other similar scintillators R_1 and R_2 have been placed around the apparatus as a shield against EAS. Their distance from the apparatus (6 m) and their detection threshold (twice the ionization minimum) have been chosen so as to minimize the loss of those events in which muons are accompanied from knock-on electrons produced in the ceiling. The resulting correction is $< 1\%$. In this situation the shower

contamination is typically 30% and 40% for the integral and differential measurements respectively.

Two blocks of criss-crossed flash tube chambers are placed above and below three liquid scintillator tanks inside which stopping muons are detected through their decay. All tanks have been equally calibrated, but only the central one (X) has been actually used in these intensity measurements.

Each flash tube stack is made of six 150 x 150 cm² chambers (3 + 3 criss-crossed at 90°), and each chamber is made of two alternate layers of 70 flash tubes of thickness 0.8 cm, external diameter 2 cm and length 145 cm filled with a 70/30 mixture of helium-neon at a pressure of 375 tor. The total number of flash tubes is 1700, and their performance is described in ref. (8).

For test purpose each of the two 90° views of every stack has been subdivided by fiducials into 7 sectors. For every sector we measured the average layer efficiency $\eta_L = \frac{\sum_{k=1}^6 K n_k}{6 \sum n_k}$ where n_k is the number of times that k tubes have flashed, with the help of 4 spark-chambers for the track definition. Several tests on the flash-tube chambers have evidenced following features :

- a) The layer efficiency is approximately constant at $\eta \approx 0.82$ between 8 and 16 kV. As the tracks are quasi-vertical we obtain for the internal tube efficiency $\eta_{int} = (d_i + \delta) \eta_L / d_i = 0.925$, where d_i is the internal diameter and δ is the insensitive gap between tubes (typically $\delta = 2.5$ mm due to the thickness of the glass, the coating and the stacking pattern).
- b) The efficiency as a function of the delay between the particle passage and the H. V. pulse is constant up to 30 μs . During the differential intensity measurements this has allowed the firing of the flash tubes after $\sim 15 \mu s$ while waiting for the muon decay in the liquid scintillator and the storing of the relative information thus avoiding possible electromagnetic disturbances without any efficiency loss.
- c) The number n of flashed tubes for track follows a binomial distribution $P(n) = \binom{N_L}{n} \eta_L^n (1 - \eta_L)^{N_L - n}$, where N_L is the number of layers and η_L the mean layer efficiency. This result is practically the same for all layers. As a consequence it is possible in 85% of the cases to reconstruct for any stack an event with more than three tubes flashed on both views.

The level of spurious flashing at the working voltage of 12 KV has been measured to be about 3% for a 15 sec time interval between events. This dead time causes the working time to be reduced by 12.5% for the integral measurements and 5% for differential ones.

The read-out system of the 1700 flash-tubes which constitute the whole apparatus has been realized conveying the light of any tube by an optical fibre (CROFON, $\phi = 1$ mm) on a hodoscope board. This light is recorded on a film kept in contact to the board by means of an electromagnet. In such a way any optical deformation is avoided. The use of fiducial holes lighted by diodes and fibres allows the optical recognition of the event and its geometrical reconstruction.

Because of the length of the fibres ranging between 4 and 6 metres an excellent collected light uniformity is achieved. The efficiency of the flash-tubes as measured by this device resulted the same, during the tests, as those obtained by photographing with a camera at near 0° .

3. - THE LIQUID SCINTILLATION COUNTER.

Each liquid scintillator detector (NE 235 Nuclear Enterprise) is made by a stainless tank (thickness 2 mm) of $42 \times 42 \times 150$ cm³ closed at the ends by two windows of perspex (thickness 5 mm). Four photomultipliers (P. M.) RCA 6655 A face any window in optical contact with them. The inner walls are polished to have a mean reflectivity of about 60%.

The signals from any pair of crossed P. M. - a, d and b, c of Fig. 1 - are added, amplified (~ 40) and sent to a shaper. Thus a quadruple logic output is obtained. The working voltage of any P. M. pair is chosen to equalize their gain and adjusted so that a relativistic muon crossing the tank to the center (mean energy loss ~ 75 MeV) is detected with an efficiency > 0.95 when an attenuation of 16 db is applied before the amplifier.

The best arrangement which optimizes the efficiency and minimizes the noise is realized by a majority logic 2/4, that is by requiring a double coincidence between the four outputs of the tank. The resulting noise level is lower than 100 Hz.

The efficiency of throughgoing muons at the center of the tank as a function of the attenuation before the amplifiers and for different majority logics is shown in Fig. 2. Thus we obtain the rough indication that the efficiency of the detector decrease for energy losses lower than 15 MeV.

To obtain the efficiency to detect decay electrons the amplitude spectrum of the signals of ~ 5000 electrons from μ decay was recorded. For this measurement two little plastic counters (15×15 cm²) 20 cm away have been put on the tank together with suitable anticoincidence counters. A linear gate was open for about $7 \mu s$, 100 ns after the logic stop signal. The chance of accidental coincidence in this time interval is lower than 10^{-3} . The amplitude spectrum of decay electrons from a pair of P. M. is compared to the one obtained imposing to the same

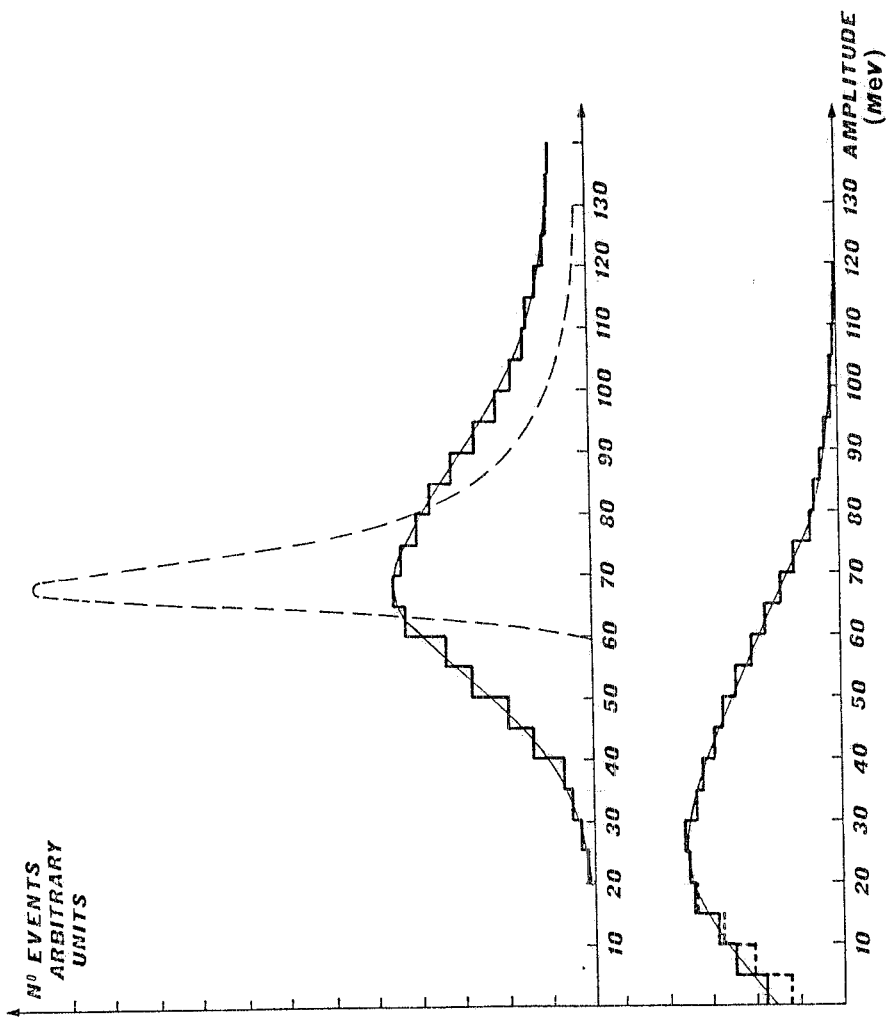
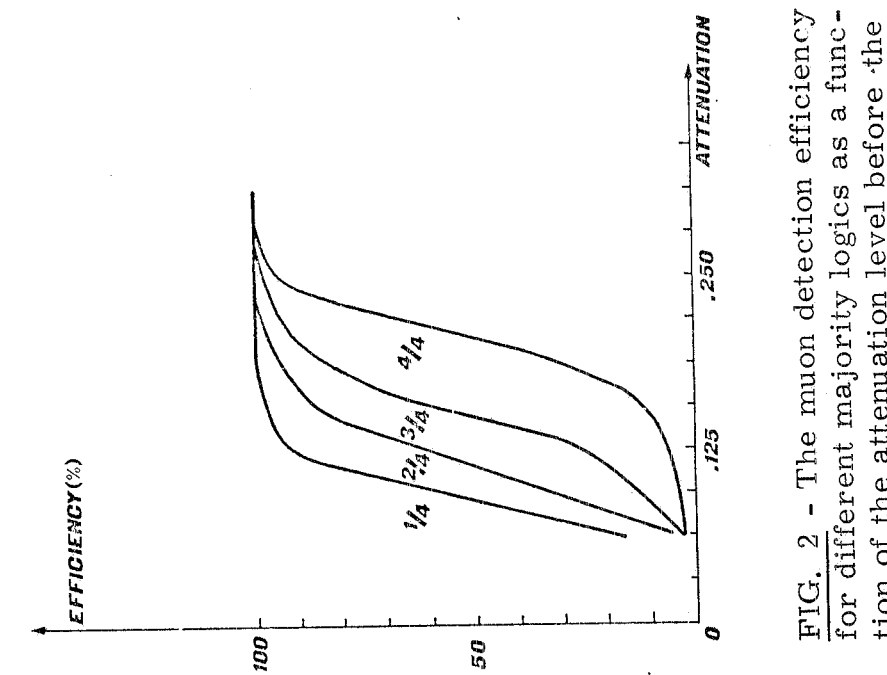


FIG. 2 - The muon detection efficiency for different majority logics as a function of the attenuation level before the amplifier. In this measurement the muons crossed the tank to the center.

threshold (histogram -----). The smooth curve of relativistic muon crossing the liquid scintillation counter (histogram -----). The smooth curve represent the result of the Montecarlo. The comparison with the energy loss distribution (Landau distribution, curve -----) shows the relevant weight of the light collection and electron multiplication statistical fluctuations.

FIG. 3 - Lower curve: pulse amplitude spectrum of decay electrons (histogram -----) and the same spectrum as modified by the condition that the pulse amplitude exceeds the discrimination



Upper curve: the pulse amplitude spectrum of the Montecarlo calculation. Upper curve: pulse amplitude spectrum of relativistic muon crossing the liquid scintillation counter (histogram -----). The smooth curve represent the result of the Montecarlo. The comparison with the energy loss distribution (Landau distribution, curve -----) shows the relevant weight of the light collection and electron multiplication statistical fluctuations.

signals to be higher than the discrimination threshold (Fig. 3). The two distributions differ of about 6% in the lower amplitudes range, the smooth behaviour being due to the different shape of signals with same total charge. Analogous measurements performed on other P. M. pairs give similar results.

With the same geometry the amplitude spectrum of throughgoing muons has been measured. The electron and muon distributions have been fitted with a Montecarlo (Appendix). The calculated curves reproduce fairly the experimental distributions allowing us to define a scale for the signal amplitude in terms of equivalent energy released into the liquid scintillator. We can see that by requiring the coincidence of a P. M. crossed pair some decay electrons with energy lower than ~ 20 MeV are lost. Moreover the Montecarlo shows that in 3.2% of the cases no photoelectrons reach the first dynode. In the previous measurement these events are in principle indistinguishable from spurious triggers. A similar calculation which takes into account the actual disposition of the counters S_1 and S_2 during the differential measure and of the majority logic 2/4 criterium gives a total efficiency $\epsilon = (95 \pm 2)\%$. It shows also that a small fraction ($1.0 \pm 0.5\%$) of the stopped muons in the tank is not detected due to the short path in the liquid scintillator, while a substantial fraction ($(18.0 \pm 0.5)\%$ of the total) of muons decays in the tank lower wall into an electron which does not enter the scintillator.

4. - TEST MEASUREMENTS.

The decay time of the muons is measured by sending the same logic output to both the START and STOP inputs of a time digitizer (conversion factor 36.46 ns/pulse). The START signal is 110 ns delayed. The decay electrons in this time interval are not detected, but that leaves out electronic background from pulse tail or reflections due to imperfect matching. The maximum time considered for the analysis is $7.767 \mu s$. A fraction $F = 0.925 \pm 0.005$ of all decays falls in the defined time range when we consider a muon mean life of $2.15 \mu s$ (this value results from the chemical composition of the liquid scintillator ($H/C \sim 2$) and from charge ratio $\mu^+/\mu^- = 1.25$ of the atmospheric muons).

The decay time spectrum of muons stopped into the whole tank is shown in Fig. 4. The experimental data are fitted by an exponential curve with $\tau = 2.13 \pm 0.03 \mu s$, in good agreement with the expectation. The differential linearity of the electronic chain was also checked by sending to the time digitizer random pulses from two uncorrelated scintillators. The measured non linearity does not exceed 0.5% (statistical limit).

In conclusion, all test measurements show that the liquid scintillator counter is able to detect, with known efficiency, electrons from muon decay without significant background.

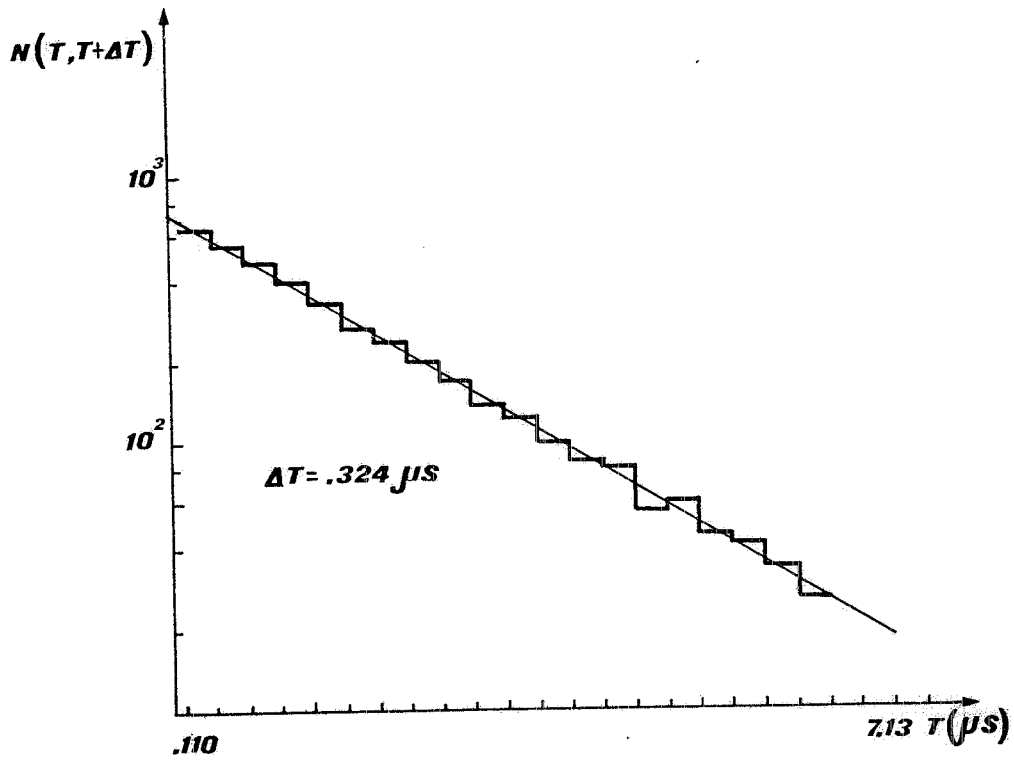


FIG. 4 - Muon decay curve as obtained during the test run. In ordinate counts per unit time interval Δt are reported. The solid line indicates a mean life $\tau = 2.13 \pm 0.03 \mu s$.

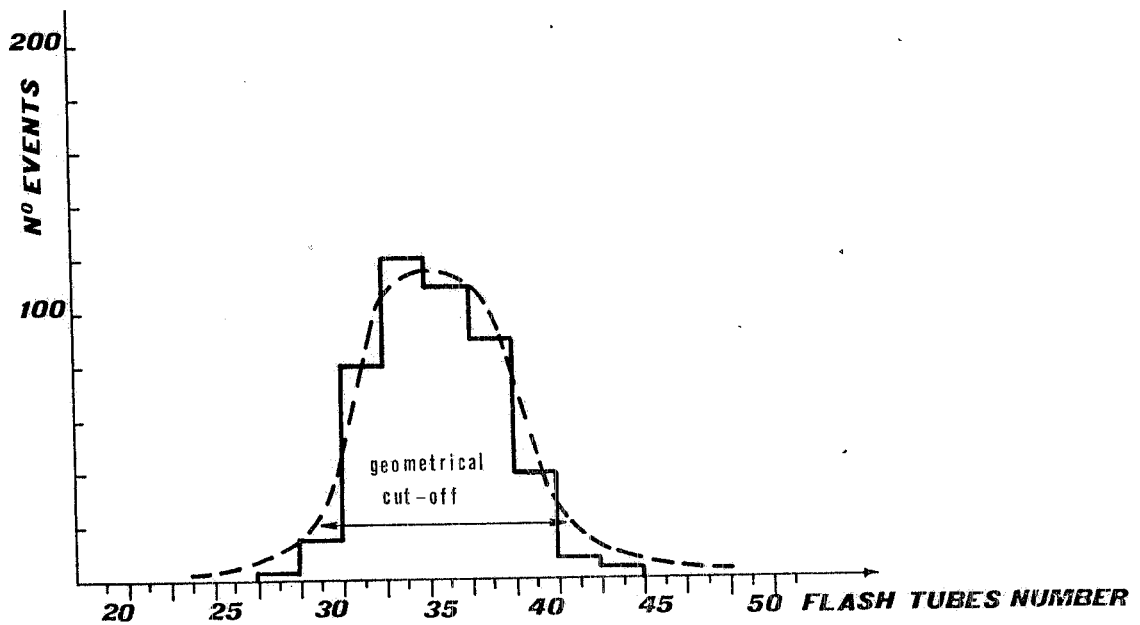


FIG. 5 - Firing frequency of the flash tubes of the upper stack lower chamber measured on a sample of muon stopping events (see text). The calculated distribution is shown for comparison. The geometrical cut-off corresponds to muon trajectories in the acceptance defined by S_1 and S_2 .

The time digitizer output converted in B-C-D format lights led diodes and, by optical fibres, is sent to the hodoscope board.

5. - CORRECTION FACTORS OF THE EXPERIMENTAL DATA.

Both integral and differential measurements have been performed at the National Laboratory of Frascati, 42° N latitude, $P_c = 4.5$ GeV/c. The triggering conditions were defined by the coincidences $S_1 \cdot S_2 \cdot X \cdot (R_1 + R_2)$ and $S_1 \cdot S_2 \cdot X \cdot (R_1 + R_2 + S_4)$ respectively. The optical requirements to select muon candidates were the crossing of the whole telescope (integral measurements) or track absence in the lower stack of flash-tube chambers (differential measurements).

By varying the iron absorber between the chambers of the upper stack the intensities were measured at two different momenta :

Integral measurements : cut-off momentum P_c (GeV/c)

$$0.457 \pm 0.10, \quad 0.918 \pm 0.10.$$

Differential measurements : momentum interval Δp (GeV/c) :

$$0.295 \div 0.388, \quad 0.758 \div 0.852.$$

These values - correct to within 1% - have been determined by using the data from the range-momentum tables of Serre⁽⁹⁾ taking into account the multiple scattering effects⁽¹⁰⁾.

Other corrections to be applied result from loss of particles by multiple scattering and contribution of spurious events in which knock-on electrons trigger S_1 and S_2 with an accompanying out-geometry muon which stop in the liquid scintillator. These events have an appreciable effect on the differential intensities only. To evaluate it we have compared the experimental distribution of struck flash-tubes of the last layer of the upper stack with the one expected from calculations done following the procedure of Eyges⁽¹¹⁾ and Sternheimer⁽¹²⁾ (Fig. 5). The selected events have more than 3 flashed tubes and a right signature of decay time. The agreement is good, the small difference near the tail of the distribution originates from poor efficiency of the tank for decay electrons from muons which stop near the walls. A fair agreement between measured and calculated distributions has been obtained also for muons crossing the whole apparatus. Only muons with reconstructed trajectories within the geometrical acceptance have been selected for the analysis, thus minimizing any edge or out-geometry effect. The overall correction from this cut-off is $(3.0 \pm 0.5)\%$ for integral measurements, $(12 \pm 1)\%$ and $(13 \pm 1)\%$ for differential ones. Other small corrections arise from the modulation by the 11-year cycle of solar activity and from normalization to the Kiev latitude. They have been applied following Allkofer and Jokisch⁽¹³⁾. No correction was applied to consider atmospheric effects.

All corrections are listed in Table I.

TABLE I

		Correction for	
		multiple scattering (%)	solar activity cycle+latitude effect (%)
Integral measurements	$p > 0.457 \text{ GeV}/c$	3.5 ± 0.5	- 0.5
	$p > 0.918 \text{ GeV}/c$	3.0 ± 0.5	- 0.5
Differential measurements	$\bar{p} = 0.314 \text{ GeV}/c$	13 ± 1	+ 0.5
	$\bar{p} = 0.805 \text{ GeV}/c$	12 ± 1	-

6. - DATA PROCESSING.

All stop events in the geometrical acceptance which are followed by a delayed coincidence, N_D , can be considered as stopping muons. Events (N_S) which are seen by optical recognition to stop in the liquid scintillator but are not accompanied by delayed signals in the interval $0.110 - 7.767 \mu s$ arise from stopping muons with undetected decay electrons as well as from electrons of very low energy whose shower is not identified in the chambers.

To evaluate the electron contamination, after rejection of shower events we have subdivided the in acceptance tracks in two categories

- a) clean events, with at least 3 aligned tubes flashed on both views and a total number lower than 10 of other random flashed tubes;
- b) dirty events for which one of the previous conditions is not satisfied.

The following are included in both categories with different percentages

- a) N_μ muons to end of range which stop in the tank and whose decay electron enter in the scintillator (N_μ) or does not leave the iron wall (N_{Fe});
- b) electrons which simulate a muon (N_e).

Events N_μ only can be followed by a delayed signal in the gate $0.110 - 7.767 \mu s$ while events N_{Fe} and N_e are not associated with delayed coincidence, the low limit of 110 ns eliminating possible accidental coincidences from knock on electrons. Therefore we have

$$N_D = N_\mu \cdot \varepsilon \cdot F,$$

$$N_S = N_\mu (1 - \varepsilon \cdot F) + N_{Fe} + N_e .$$

Then the total number of stopped muons $N = N_\mu + N_{Fe}$ is easily calculated from the known values for ε and F . Tables II and III show the event distribution. The electron contamination is higher in "dirty events" and in measurements at lower momenta.

TABLE II - Differential measurement

$\Delta p = 0.295 - 0.388 \text{ GeV}/c$				
	Exp. data		Muons	Electrons
	N_D	N_S		
clean events	661	275	879	57
dirty events	166	105	221	50
Effective working time $4.258 \times 10^6 \text{ sec}$	Total		1100	107

TABLE III - Differential measurement

$\Delta p = 0.758 - 852 \text{ GeV}/c$				
	Exp. data		Muons	Electrons
	N_D	N_S		
clean events	624	236	854	24
dirty events	90	55	119	26
Effective working time $3.394 \times 10^6 \text{ sec}$	Total		973	50

In Fig. 6 the decay curves for "clean" and "dirty" events recorded in the upper momentum range are shown. The agreement with the expected distribution is remarkable. Finally the corrections of Table I are applied.

The integral intensities have been obtained by considering muons in the acceptance defined by S_1 and S_2 and applying the corrections of Table I. Number of accepted muons and effective times of observation

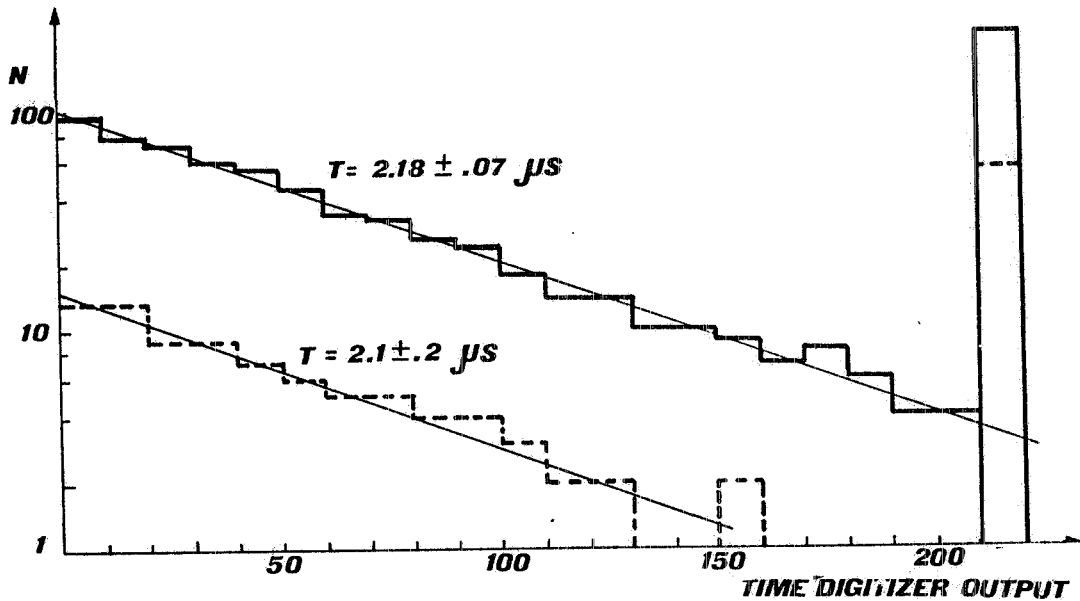


FIG. 6 - Differential measurement at mean momentum $p = 0.805$ GeV/c: the experimental decay curves for clean (———) and dirty (- - - - -) events are shown. N is the number of events recorded per unit time interval (one pulse from time digitizer = 36.46 ns). Events at a time later than $7.767 \mu\text{s}$ are added up. The straight lines result from a least square fit.

are shown in Table IV.

TABLE IV - Integral measurements

P (GeV/c)	N	T_{eff} (sec)
0.457	858	1.041×10^5
0.918	1040	1.512×10^5

7. - RESULTS AND DISCUSSION.

The absolute integral and differential intensities - $I (> p)$, $dI/dp(p)$ with p in GeV/c - measured by our experiment are given by

$$I (> 0.457) = (8.75 \pm 0.33) \times 10^{-3} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1},$$

$$I (> 0.918) = (7.27 \pm 0.26) \times 10^{-3} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1},$$

$$dI/dp (0.314) = (3.25 \pm 0.17) \times 10^{-3} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1} \text{ GeV/c}^{-1} ,$$

$$dI/dp (0.805) = (3.60 \pm 0.18) \times 10^{-3} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1} \text{ GeV/c}^{-1} .$$

The quoted errors arise from the uncertainties on the acceptance, effective working time, correction factors added to the statistical one. Other sources of error for differential measurements are the uncertainties on the identified number of stopping muons and on the momentum interval. As a result our absolute intensities have associated errors of about 3.7% (integral) and 5% (differential).

In Fig. 7 a summary of the more significant measurements are reported along with the form fit (FF) of De et al. (14) and the ACD fit of Allkofer et al. (2).

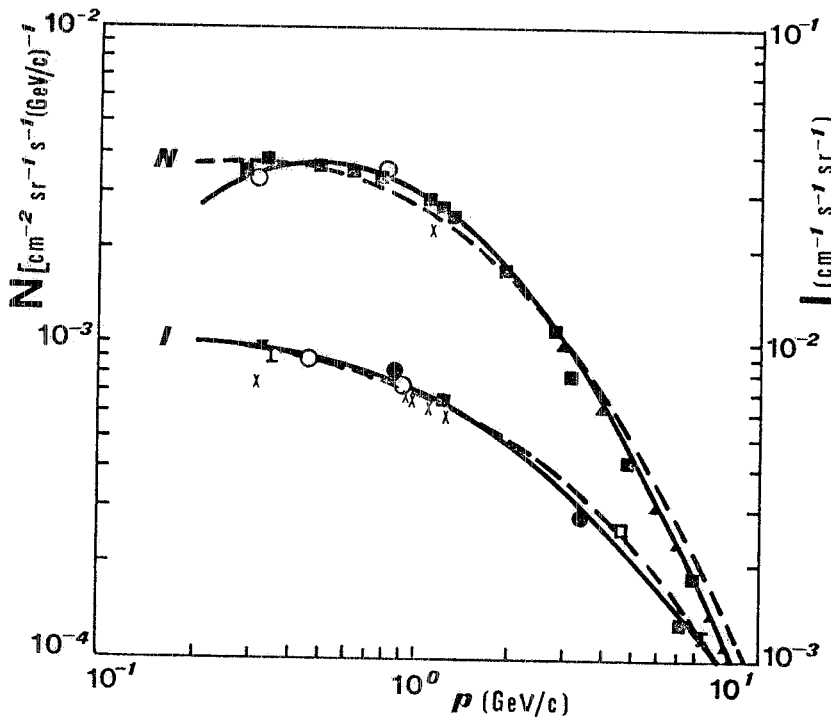


FIG. 7 - The differential (N) and integral (I) absolute vertical muon spectrum at sea level: o present work; ■ Kiel^(2, 13); ▲ College Station⁽¹⁶⁾; ▼ Ithaca⁽¹⁷⁾; x Calcutta^(14, 15); ● Durham⁽³⁾; I Nottingham⁽⁵⁾; □ Hong-Kong⁽⁷⁾. The Ithaca point has been corrected by Crookes and Rastin⁽⁵⁾. ——— form fit (FF), - - - - - ACO fit.

The differential intensities from the present work are somewhat greater than that of Rossi, 16% at 0.314 GeV/c and 33% at 0.805 GeV/c and agree within the error with the Kiel measurements^(2, 13) at similar momenta. Our data favour the FF spectrum eventually suggesting a less flat shape in this energy region. Besides our integral intensities at 0.457 and 0.918 GeV/c are in agreement with all recent measurements

in the same momentum range and suggest an integral intensity at 1 GeV/c of $7.0 \pm 0.3 \text{ cm}^{-2} \text{sr}^{-1} \text{s}^{-1}$.

In conclusion the present results confirm the earlier measurements obtained with different devices according to which the Rossi intensities below 1 GeV/c have to be raised of $25 \pm 8\%$.

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We mention the technical contribution of Mr. S. Viligiardi and Mr. E. Zante in constructing the flash-tubes.

APPENDIX.

In order to calculate the amplitude distribution of the muon and electron pulses a Montecarlo was done which make use of the following assumptions :

- a) The actual trajectories in the tank are considered and subdivided in a suitable number of small distances. The muon energy loss is accounted according to Paul⁽¹⁸⁾ and Serre⁽⁹⁾ calculations. Landau fluctuations are considered by using the N. S. B. ⁽¹⁹⁾ tables. Multiple scattering is calculated following Moliere's theory⁽²⁰⁾. For electrons we use the relation between energy and effective range and data on energy loss given in Ref. (20, 21).

- b) The angle and energy distribution of decay electrons is

$$\frac{d^2N}{dx d \cos \theta} = x^2 \left[3 - 2x + P \cos \theta (2x - 1) \right]$$

where :

$$x = E/E_{\max} ,$$

$$E = \text{electron energy} ,$$

$$E_{\max} = 52.8 \text{ MeV} ,$$

$$\theta = \text{zenith angle relative to muon direction} ,$$

$$P = \text{muon polarization} \approx 0.22 .$$

- c) The light production is isotropic with a Poisson distribution. Light is carried to the photomultiplier taking into account the internal geometry of the tank. The processes of light collection at the photocathode, production of photoelectrons and theirs collection on the first dynode follow together a binomial distribution. For the multiplication statistics we use the model developed by Prescott and Takhar⁽²²⁾ who have checked the behaviour of a photomultiplier like to our one.

Forcing the calculated electron and muon pulse distribution to fit the experimental ones we obtain the best estimate for the following parameters :

$$\bar{\beta}, \text{ mean reflection coefficient of the tank internal walls} = 54\% ,$$

$$\bar{n}, \text{ mean number of photoelectrons on the first dynode produced per unit of energy loss and of light emission solid angle} = 4.2 \times 10^2 \text{ (1/4}\pi\text{) ph. electrons/MeV sr} ,$$

$$\bar{G}, \text{ the photomultiplier mean gain} = 8 \times 10^5 ,$$

$$V_m, \text{ the photomultiplier relative variance} = 0.85. \text{ Thus, the single electron distribution approaches an exponential one}^{(22)} .$$

Errors on these estimates range between 20% and 30%. Using these data, the behaviour of the liquid scintillation counter in the differential measurements condition can be evaluated. In particular, we obtain the number of "blanks" - i. e. photomultiplier cascades that break for lack of any secondary electrons - and the number of muons which stop in the walls and whose decay electron does not enter the scintillator (§ 3).

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