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G. Bologna, F. Celani, A. Codino, B. D'Ettorre Piazzoli, F. L. Fabbri, P. Laurelli, G. Mannocchi, P. Picchi, G. Rivellini, L. Satta, P. Spillantini and A. Zallo :
ELECTROSTATIC FIELD IN A CYLINDRICAL PROPORTIONAL CHAMBER.

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Cylindrical wire proportional chambers (CWPC) are now operating around beam pipe of the storage rings as well as around thin targets in conventional accelerators experiments.

In view of the construction of CPWC with small radius a problem arises about the field distribution into the gaps, since in principle we expect some field asymmetry due to the geometry. We have obtained the electrostatic field in a CWPC by a conformal transformation from the well know formulae valid for a plane MWPC.

Let us consider an infinite grid of cylindrical wires of diameter d with spacing s situated at the complex coordinate $z_K = iy_o + x_K = iy_o + K \cdot s$, $K = 0, \pm 1, \ldots$ between two parallel plane electrodes, as shown in Fig. 1 a. The electrodes are grounded while the wires are maintained at a positive potential V_o and acquire a charge q per unit length. The complex potential is given by[1]:

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$$\Psi(z) = \frac{q}{4\pi\varepsilon}\left[i\,\frac{2\pi y_o z}{Ls} - 2\ln\left\{\frac{\vartheta_1\left[(\pi/s)(z - iy_o)/i\,4Ls\right]}{\vartheta_4\left[(\pi/s)(z+iy_o)/i\,4Ls\right]}\right\}\right] \simeq$$

$$\simeq \frac{q}{4\pi\varepsilon}\left[i\,\frac{2\pi y_o z}{Ls} + \frac{2\pi L}{s} - 2\ln\left\{2\sin\left[\frac{\pi}{s}(z - iy_o)\right]\right\}\right]$$

where we have used the series expansion of the theta functions in the variable $p = e^{-4\pi Ls}$.

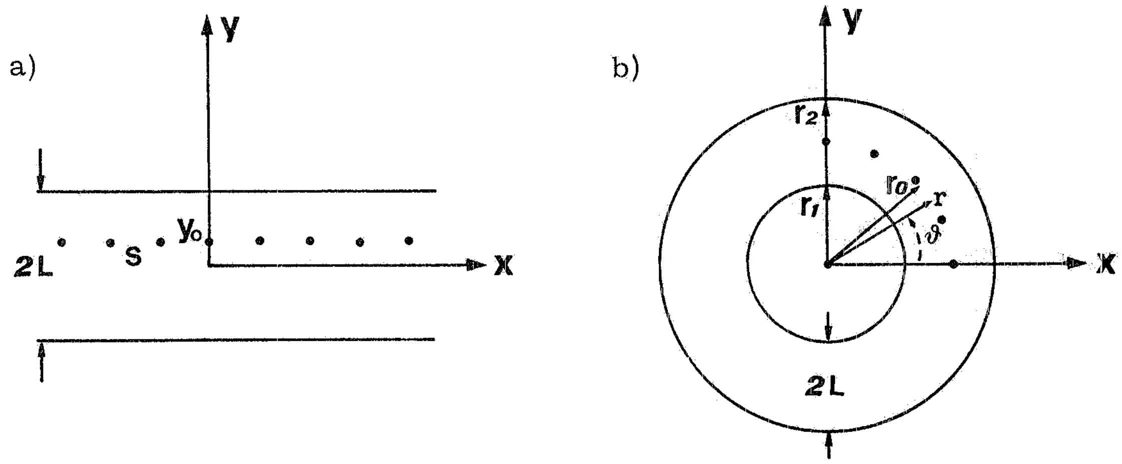

FIG. 1 - Coordinate system for plane (a) and cylindrical (b) configurations.

The physical potential V is the real part of $\Psi(z)$

$$V(x,y) = \frac{q}{4\pi\varepsilon_o}\left[\frac{2\pi L}{s} - \frac{2\pi y y_o}{Ls} - \ln\left\{4\left[\sin^2\left(\frac{\pi x}{s}\right) + \sinh^2\left(\frac{\pi}{s}(y - y_o)\right)\right]\right\}\right]$$

The value of q is obtained by requiring that $V = V_o$ for $y = y_o \pm \frac{d}{2}$ and $x = Ks$, $K = 0, \pm 1 \ldots$

The resulting capacitance per unit length is:

$$C = \frac{2\pi\varepsilon_o}{\frac{\pi L}{s}\left(1 - \frac{y_o^2}{L^2}\right) - \ln\left(\frac{\pi d}{s}\right)} \qquad (\varepsilon_o = 8.85\ \text{pF/m})\,.$$

To obtain the potential for a grid of n wires on a cylindrical surface of radius r_o between two cylinders, the inner one of radius

r_1 and the outer one of radius r_2, we use the transformation[2]

$$w = -i \ln(z/\bar{r}) \qquad \bar{r} = \sqrt{r_1 r_2}$$

$$L = \ln(\sqrt{r_2/r_1})$$

$$y_o = \ln(\bar{r}/r_o)$$

$$s = 2\pi/n$$

which changes the distribution of Fig. 1 a (now called the w plane) in to the distribution of Fig. 1 b. The wires are separated by an arc of length $l = 2\pi r_o/n$. The resulting complex potential is given by

$$\psi(z) = \frac{q}{4\pi\varepsilon_o}\Bigg[n \ln(z/\bar{r})\, F(r_o, r_1, r_2) - 2 \ln \left\{ \frac{\vartheta_1 \left[-i \frac{n}{2} \ln(z/r_o) / i \frac{n}{\pi} \ln(r_2/r_1) \right]}{\vartheta_4 \left[-i \frac{n}{2} \ln(z r_o/\bar{r}^2) / i \frac{n}{\pi} \ln(r_2/r_1) \right]} \right\} \Bigg] \simeq$$

$$\simeq \frac{q}{4\pi\varepsilon_o}\left[n \ln(z/\bar{r})\, F(r_o, r_1, r_2) - 2 \ln\left\{ 2 \sin(-i \tfrac{n}{2} \ln(z/r_o)) \right\} \right]$$

having used the series expansion of the theta functions in the variable $p = (r_1/r_2)^n$. The function

$$F(r_o, r_1, r_2) = \frac{\ln(r_1 r_2/r_o^2)}{\ln(r_2/r_1)}$$

vanishes for $r_o = \bar{r}$, which is the condition for a symmetric cylindrical chamber.

Using polar coordinates (r, ϑ) we obtain for V and C the following expressions :

$$V(r,\vartheta) = \frac{q}{4\pi\varepsilon_o}\Bigg[n \ln(r/\bar{r})\, F(r_o, r_1, r_2) + \frac{n}{2} \ln(r_2/r_1) - \ln\left\{ 4 \sin^2\left(\frac{n\vartheta}{2}\right) + \left(\frac{r}{r_o}\right)^n + \left(\frac{r}{r_o}\right)^{-n} - 2 \right\} \Bigg]$$

$$C = \frac{4\pi\varepsilon_o}{n \ln(r_o/\bar{r})\, F(r_o, r_1, r_2) + \frac{n}{2}\ln(r_2/r_1) - 2\ln(\frac{nd}{2r_o})} . \quad (1)$$

The field intensity at any point (r, ϑ) is given by $\vec{E}(r,\vartheta) = \hat{u}_r E_r(r,\vartheta) + \hat{u}_\vartheta E_\vartheta(r,\vartheta)$ ($\hat{u}_r$, $\hat{u}_\vartheta$ unit vectors) with

$$E_r(r,\vartheta) = -\frac{\partial}{\partial r} V(r,\vartheta) = -\frac{CV_o}{2\pi\varepsilon_o s}\left(\frac{r_o}{r}\right) \cdot$$

$$\cdot \left[F(r_o, r_1, r_2) - \frac{(\frac{r}{r_o})^n - (\frac{r}{r_o})^{-n}}{4\sin^2(\frac{n\vartheta}{2}) + (\frac{r}{r_o})^n + (\frac{r}{r_o})^{-n} - 2} \right] ,$$

$$E_\vartheta(r,\vartheta) = -\frac{1}{r}\frac{\partial}{\partial\vartheta} V(r,\vartheta) = \frac{CV_o}{2\pi\varepsilon_o s}\left(\frac{r_o}{r}\right) \cdot \quad (2)$$

$$\cdot \left[\frac{4\sin(\frac{n\vartheta}{2})\cos(\frac{n\vartheta}{2})}{4\sin^2(\frac{n\vartheta}{2}) + (\frac{r}{r_o})^n + (\frac{r}{r_o})^{-n} - 2} \right] .$$

From (1) and (2) we calculate, practically, the same capacitance and field which can be obtained for a corresponding plane chamber with same wires spacing gap, and wire diameter.

The maximum percentual difference of the electric field around a wire at a fixed radial distance r is shown in Table I for a chamber with radius $r_o = 36.83$ mm, 100 wires, 2L = 10 mm, d = 20 μm. Results for an equivalent plane chamber are shown for comparison. The symmetric geometry ($r_o = \bar{r}$) gives the minimum field asymmetry, but also for a chamber with equal gaps the asymmetry appears small enough to do not affect a right operation[(3)].

TABLE I

r (μm)	Plane chamber	Cylindrical chamber with $r_o = \frac{r_1+r_2}{2}$	Cylindrical chamber with $r_o = \sqrt{r_1 r_2}$
10	0.02 %	0.44 %	0.20 %
50	0.41 %	1.01 %	0.48 %
100	1.65 %	2.73 %	1.80 %

$d = 20$ m , $r_o = 36.83$ mm , $2L = 10$ mm , $n = 100$.

REFERENCES.

(1) - G. A. Erskine, Electrostatic problems in multiwire proportional chambers, Nuclear Instr. and Meth. 105, 565 (1972).

(2) - A cylindrical chamber with small radius (r_o = 3 cm) and equal gaps (L = 7 mm), built at the Laboratori Nazionali di Frascati, were operated successfully, see: G. Bologna et al., Frascati Preprint LNF-79/19 (1979).

(3) - P. M. Morse and H. Feshbach, Methods of Theoretical Physics (McGraw-Hill, 1953), Part II, pag. 1242.