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A. Turrin: POLARIZATION EIGENVECTOR IN HIGH-ENERGY ACCELERATORS EQUIPPED WITH SIBERIAN SNAKES.

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A. Turrin: POLARIZATION EIGENVECTOR IN HIGH-ENERGY ACCELERATORS EQUIPPED WITH SIBERIAN SNAKES.

To be presented at the Rome Meeting of the ECFA-LEP Working Group, to the SSG on Polarization (Sept. 27 - Oct. 4, 1979).

INTRODUCTION

The purpose of this note is to give explicit formulae for the polarization eigenvector (n_x, n_y, n_z) in a high-energy accelerator equipped with spin-flipping devices (Siberian Snakes[1]), restricting ourselves to two specific (and reasonably typical) cases, as follows:

i) One Siberian Snake of the first kind[2], installed in a Proton Synchrotron.

ii) One Siberian Snake of the first kind[2] (with variable geometry), plus one of the second kind[3] with fixed geometry, placed symmetrically apart around an Electron Synchrotron.

The nomenclature and notations will follow the ones adopted in Refs. 3 and 4. Ref. 3 provides the explicit expressions for the elements of the 2 x 2 complex unitary matrices connecting the occupation numbers of the two states for traversing the snakes[2,3]; in Ref. 4, conversely, a general formula is given for (n_x, n_y, n_z) in terms of the 2 x 2 matrix elements for one revolution, with any snake-configuration inserted.

CASE i)

This is the case already considered in Refs. 5 and 6 (where the optimum rotation angle of the Steffen's arrangement[2] was found to be $\varphi = \pi/2$.

It must be emphasised, however, that an algebraic error has crept into the calculation of n_x and n_y leading to the (wrong) Eqs. (3 a, b) of Ref. 5. (Ref. 6 deals with the derivation of $\cos(\pi\nu)$ only, and calls for no comment).

The new, correct expressions for (n_x, n_y, n_z) in the straight section diametrically opposite to the snake insertion (for the Steffen's arrangement[2] rotated by $\varphi = \pi/2$) are found to be

$$n_x = B/R \ , \qquad n_y = C/R \ , \qquad n_z = A \sin(\chi/2)/R \ , \qquad (1\,a, b, c)$$

where $R = \sqrt{1 - A^2 \cos^2(\chi/2)}$, and where A, B and C are given by Eqs. (5 a, b, c) of Ref. 3. Here, $\chi = 2\pi GE/(mc^2)$ is the precession phase per revolution (G is the gyromagnetic anomaly).

The graphs of $|n_y|_{min}$ and $|n_y|_{max}$ are given in Fig. 1 (solid lines) together with the graphs of $|\cos(\pi\nu)|^{\varphi=\pi/2}_{extr}$ (Eq. (2) of Ref. 5 or Eq. (3) of Ref. 6, with $\varphi = \pi/2$) (dotted line), which has been re-plotted here.

CASE ii)

This is the case already considered in Ref. 3 (where, however, both the snakes are supposed to operate under fixed-geometry conditions).

Specializing the results of Ref. 3 to the case of interest (i. e. to the case where the snake of the first kind[2] is operating under variable-geometry conditions), we have $A = 0$, $B = 0$, $C = 1$. Thus one gets for (n_x, n_y, n_z) at the mid-point of one of the two main bending arcs

$$n_x = -\beta \cos(\chi/4) - \alpha \sin(\chi/4) \ , \qquad (2\,a)$$

$$n_y = \alpha \cos(\chi/4) - \beta \sin(\chi/4) \ , \qquad (2\,b)$$

$$n_z = \gamma \ , \qquad (2\,c)$$

where α, β and γ are given by Eqs. (2 a, b, c) of Ref. 3.

Note that $\cos(\pi\nu) = 0$ for any E/E_o, in this case.

The graph of $|n_z|$ is represented in Fig. 2.

To conclude, it is very important to point out that if $E_{inj} \simeq E_o$, where E_{inj} is the injection energy, we have at the mid-point of the

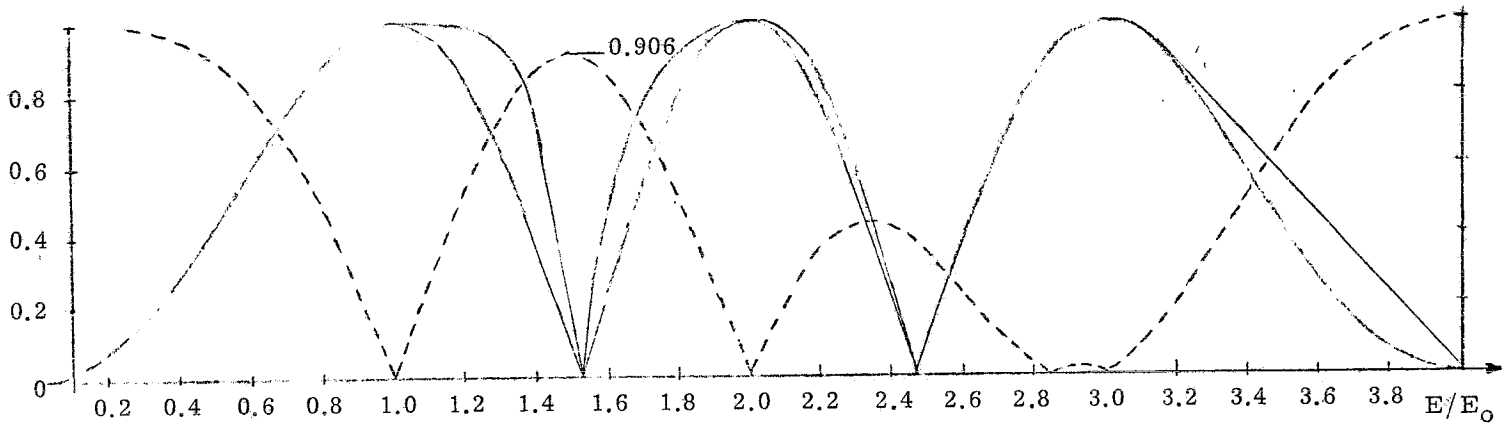

FIG. 1 - $\left|\cos(\pi\nu)\right|_{extr}^{\varphi=\pi/2}$ (dotted line); $\left|n_y\right|_{max}$ and $\left|n_y\right|_{min}$ (solid lines) at the straight section diametrically opposite to the snake insertion (case i)).

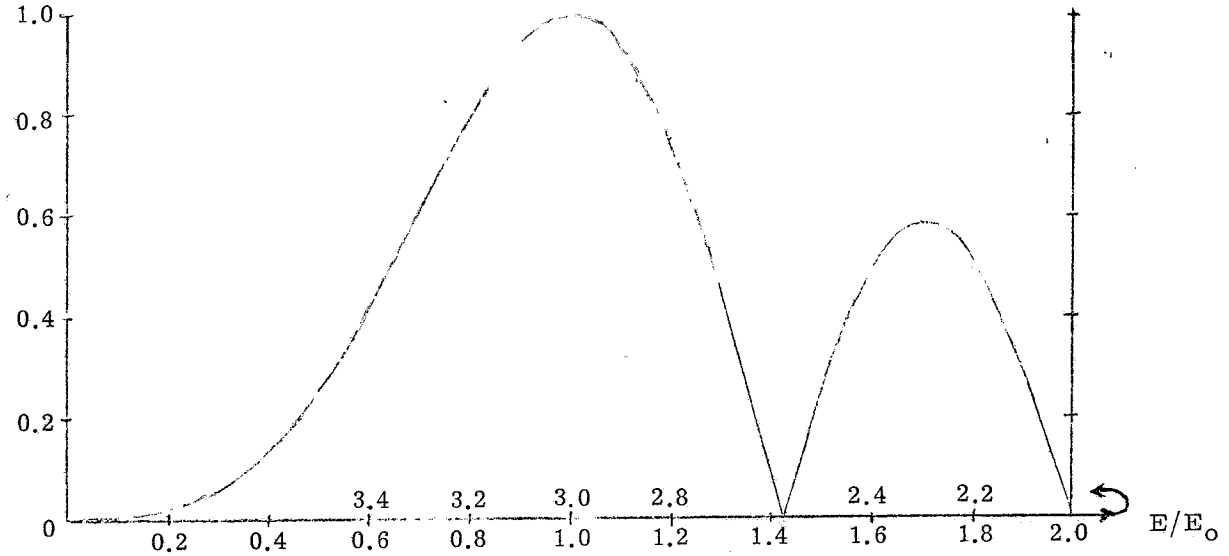

FIG. 2 - $\left|n_z\right|$ at the mid-point of either main bending arc (case ii)).

snake[3] of the second kind,

$$|n_y| \simeq 1 \qquad \text{for} \qquad E \simeq 3E_{inj}.$$

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4. - B. W. Montague, Ref. 1, p. 129. See Eqs. (7) and (10).
5. - A. Turrin, LNF-78/59 (1978).
6. - A. Turrin, LNF-79/8 (1979).