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Relativistic quantum field theory has a dualistic nature. The free quantum scalar field theory is a clear example of this statement: we can consider it as the quantum version of the classical field theory or as the quantum version of the free relativistic classical particles; also the ϕ^4 interaction can be considered as a non-linear term in the equations of motions for the field, or as a repulsion among particles.

In different regions, only one of the two classical pictures is appropriate to describe the properties of the quantum field theory. For example the meaning of the S-matrix can be easily explained in a language borrowed from the classical mechanics of particles, while it is natural to describe the Goldstone bosons as small oscillations around the equilibrium position of the classical field.

All that is well known for scalar theories. What happens to gauge theories? The language we use to describe gauge theories, is normally borrowed from classical field theory, e.g. the massless gauge bosons are clearly the equivalent of the Goldstone bosons; therefore we have the right to ask which is the equivalent in gauge theories of the particle like picture in scalar theories. We cannot answer to this question in the framework of the standard perturbative expansion because we would always remain in the phase where massless gauge particles are present. The answer can be obtained by defining gauge theories using the lattice regularization and by studying them by means of the high temperature expansion[1-3] (one recovers the continuum limit at the second order phase transition point, where the correlation lenght becomes infinite in lattice spacing units). Indeed the high temperature expansion is a strong coupling expansion and it is complementary to the standard small coupling perturbative expansion, which is a low temperature expansion.

Analysing the high temperature expansion, one finds that eucledian gauge field theories are equivalent to the statistical mechanics of closed self repulsing surfaces : the elements of the surface carry group indeces (more precisely they belong to representation of the gauge group)[4] and the details of the self repulsion depend on the gauge group. A similar analysis implies that scalar ϕ^4 field theories are equivalent to the statistical mechanics of closed self repulsing chains[5]. If we come back to Minkowski space, chains (1-dimensional manifolds) can be interpreted as the world-lines of particles, while surfaces can be interpreted as the world-lines of strings ; in other words scalar theories describe interacting particles and gauge theories describe interacting strings. However this last statement is correct only in the confined phase, in the unconfined phase the usual field theoretical description is more suited. At first sight it is not evident how to describe the massless gauge excitations of the unconfined phase in terms of strings, although it has been suggested[6] that the deconfining phase transition corresponds to the condensation of strings in the vacuum and that the usual gauge bosons arise as a natural generalization of Goldstone bosons.

If we want to stop to wave hands and to begin to compute something, we must find a good starting point. The natural zeroth order approximation consists in neglecting the self repulsion of the surfaces and in reducing the problem to the study a closed random surface[7], i. e. the free closed string; similarly the natural starting point to study an interacting scalar field theory, is the free field theory.

Who could have guessed ten years ago that dual models were the natural zeroth order approximation to gauge theories?

Unfortunately, if we try to implement this program, many problems arise. For historical reasons, let us consider the case of an open string. In the simplest approach the Green functions of the dual model are obtained by summing large fishnet planar diagrams[8,9]; in this construction one considers a region Ω (for simplicity a circle) of the two dimensional plane : the surface is described by a vector fiels $X_\mu(z)$ defined on Ω ($z \in \Omega$) and the Green functions are obtained by performing a Gaussian functional integral over $X_\mu(z)$. Unfortunately only the on mass shall Green functions are well defined ; the off mass shall Green functions (i. e. the Green functions in configuration space) are ultraviolet divergent : two nearby points of the surface (i. e. Δz small) are wildly separated and $\langle X^2 \rangle = \infty$, as an effect of the very strong oscillations on the small scale. The presence of a tachion, i. e. a particle with negative squared mass, is an other disturbing feature : it is unclear how to use such a pathological model as a zero order approximation to gauge theories.

Something is going wrong and a fresh start would be helpful.

In a different approach one studies gauge theories in the limit where the dimensions D of the space become infinite[10,11], and one tries to develope a systematic 1/D expansion. This can be done only if one works on the lattice in order to avoid the ultraviolet divergences which would be present starting directly in the continuum limit.

This program has been carried on long time ago in the case of the ϕ^4 theory(12) : one finds that, when D goes to infinity, the interacting theory becomes free. At higher orders in the 1/D expansion the theory is no more free, but one recovers a free field theory in the continuum limit, i. e. at the phase transition point. Indeed, if $D > 4$, the theory is non-renormalizable, for positive values of the coupling constant(13) the interacting theory does not exist (the only infrared stable fixed point is at zero coupling) and in the infinite cutoff limit only a free field theory can be obtained.

The same program can be carried on for gauge theories : we are not going to describe here the technical details, we only quote some of the main results of ref. (11).

In infinite dimensions the generical closed surface is the boundary of the generical connected set of three-dimensional cubes. With a probability going to 1 when $D \to \infty$, these clusters of cubes form biforcated tubes having thickness equal to the lattice spacing, in other words a polybranched polymer(14). Relative smooth surfaces are depressed in this limit and only these hydra-like configurations dominate the functional integral in the high temperature phase. The "intuitive" degrees of freedom of the surfaces are not excited and the final theory looks very similar to a conventional ϕ^3 theory. A closed expression for the free energy has been found and the 1/D corrections can be computed(11).

From the explicite solution one can see the existence of a second order phase transition ; going toward the transition the lenght of the polymers of cubes become increasingly larger, at the transition point ($T = T_0$) their lenght becomes infinite and below T_0 an infinite cluster is formed, as in the gelation transition(14). The complete collapse of the system (the "kinematical" ϕ^3 interaction is unstable) is forbidden by the presence of a repulsive interaction between cubes.

At the present stage it is unclear if the II order phase transition is an artefact of the approximation and if the ϕ^3 interaction induces a phase transition of the first order at $T_I > T_c$(15). If this happens, our results are valid for the free energy in the metastable phase $T_I > T > T_c$. This issue on the order of the phase transition can be clarified by a more careful analysis of the diagramatical rules (tunneling effects are proportional to $\exp(-1/D)$) and of the low temperature expansion.

The second order phase transition we have found is not the Wilson's deconfinement transition. Indeed the analysis of the spectrum of the theory shows that the only states, whose mass goes to zero at the transition point, are glueballs (the boxitons of ref. (16)), which interact via a ϕ^3 interaction(11). The mass of the string degrees of freedom remains finite (i. e. proportional to the inverse of the lattice spacing) also at the transition point. The phase transition can be described as the condensation of the boxitons in the vacuum.

A very intuitive picture of the situation can be obtained if we intro duce the surface tension σ, using the relation:

$$\langle \exp \oint_C A_\mu dx^\mu \rangle \longrightarrow \exp(-\sigma S) ,$$

where S is the minimal area enclosed by the large circuit C. The sur face tension σ goes to zero by definition at the deconfinement transition. An easy computation shows that σ remains different from zero also at the phase transition (when $D \to \infty$) and by continuity (the transition being of the second order) a similar behaviour holds also for $T < T_c$. As di- scussed in details in ref. (11), below T_c, the surface keeps its global ri gidity while it becomes locally infinitely deformable. This may happens in dimensions greater than 3, where the local difects of the surface may not interfere, being oriented in different transverse dimensions; this possibility is the favoured one in very large dimensions.

Below T_c, polymers of cubes, arriving up to infinity, start from any point of the surface pointing toward different directions; if X_μ are the cohordinates of a generical point of the surface, $\langle X^2 \rangle$ is infinite as an effect of these thread-like deformations of the surface. Moreover, if we forget to do the appropriate translations in the Boxiton field, a ta chion is present. As pointed out by Virasoro[17], these two pathologies are just the same of those found in the conventional treatment of the dual model. The Boxiton is the tachion of the dual models; the ultraviolet di- vergences in the off mass shall Green functions arise from the presence of this down of polymer of cubes around the surface. In other words at high temperature all the states of the dual model (also the future tachion) have a positive mass, at the critical temperature the mass of the tachion becomes zero, and below the critical temperature the tachion condensa- tes in the vacuum.

Below T_c (in the continuum limit) the standard treatment of the dual models is not correct because one has not taken into account the conden sation of the would be tachion in the vacuum. We need to construct a new dual model in which the tachion get a positive mass by translating its field and the dual Green functions are modified by this effect. This new dual model, not the old fashioned one, should be used as a starting point for constructing gauge theories.

At least three phases are possible for gauge theories: the low tem- perature unconfined phase where $\sigma = 0$, the high temperature confined phase where $\sigma \neq 0$ and the surface keeps its local rigidity, and an in- termediate temperature phase where $\sigma \neq 0$ and the surface has lost lo cal rigidity. In the standard SU(3) 4-dimensional gauge theory we be- lieve that the first phase is absent, however it is quite possible that a phase transition between the two confined phases is present at a finite temperature. If this happens, the high temperature expansion contains no informations on the behaviour of the theory in the physical relevant region (i.e. $T \to 0$).

It seems that dual models are the only starting point to study gauge theories ; however there are inveterated field theorist who strongly dis like dual models (maybe because they have a slightly small of bootstrap); for them I will describe a different approach which I hope will suit them better.

The strategy is the following: the non linear σ model is well under stood in 2 dimensions: it is asymptotically free and the mass is spontaneously generated. We must construct a field theory which interpolates between the non linear σ model in 2 dimensions and gauge field theory in 4 dimensions. This theory should always be asymptotically free and should describe particles of spin $\varepsilon/2$ in dimensions $D = 2+\varepsilon$; the associated geometrical objects should be manifolds of dimensions $1+\varepsilon/2$. By constructing the ε expansion for this theory, we would have in our hands a powerful tool to study gauge theories.

I fear that this proposal will convince everybody that dual models are by far the simplest and more efficient tool to understand 4 dimensional gauge theories.

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