

To be submitted to
Physics Letters

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-79/39(P)
27 Giugno 1979

G. Curci, M. Greco, Y. Srivastava and B. Stella:
QCD ANALYSIS OF JET LONGITUDINAL MOMENTA.

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QCD ANALYSIS OF JET LONGITUDINAL MOMENTA.

ABSTRACT.

Normalized probability distributions are presented for (massless) quark and gluon jet longitudinal momenta which take due account of correlations imposed by momentum conservation. A general algorithm is introduced to analyze the experimental data in terms of QCD parameters only.

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A general analysis of jets in QCD using the coherent state formalism has been presented recently^(1,2). In particular, the transverse momentum behaviour in a given quark or gluon jet has been obtained⁽²⁾ to all orders in the running coupling constant, in the leading logarithm approximation. In this letter, we discuss the problem of longitudinal momentum of the jets in the same approximation and obtain explicit formulae for the same. Also, we present a general algorithm to analyze the experimental data which should be particularly useful for revealing possible $q\bar{q}g$ or $3g$ jets. It is to be stressed that our final results depend only on QCD parameters and do not contain unknown quark or gluon fragmentation functions.

In reference (2), using the method of coherent state, the following formula has been derived for n_q and n_g quark and gluon jets respectively:

$$d\sigma_{\text{jet}}^{(n_q, n_g)} = d\sigma_0 \exp \left\{ -\frac{1}{\pi} \left[n_q I_q(\epsilon) + n_g I_g(\epsilon) \right] \int_{K_{1T}}^{K_{2T}} \frac{dk_{\perp}}{k_{\perp}} \bar{a}(k_{\perp}) \right\}, \quad (1)$$

where $d\sigma_0$ is the lowest order QCD cross-section and

$$I_q(\epsilon) = \int_{2\epsilon}^{1-2\epsilon} dx \mathcal{P}_{gq}(x), \quad (2a)$$

$$I_g(\epsilon) = \int_{2\epsilon}^{1-2\epsilon} dx \left[\mathcal{P}_{gg}(x) + N_f \mathcal{P}_{qg}(x) \right]. \quad (2b)$$

In eq. (2) $\mathcal{P}_{gq}(x)$, $\mathcal{P}_{gg}(x)$ and $\mathcal{P}_{qg}(x)$ are the Altarelli-Parisi⁽³⁾ probabilities. K_{1T} and K_{2T} are the appropriate limits for the cones (taken equal for all jets for simplicity).

In exact analogy with the transverse momentum case, we can define a longitudinal probability distribution which takes into account the conservation of longitudinal momentum in each jet. For the total hadronic longitudinal momentum $K_{||} = \sum_i p_{||}^i$ in a given jet, this normalized probability is given by

$$G(K_{\parallel} - K_0) = \frac{dP}{dK_{\parallel}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\sigma e^{-i\sigma(K_{\parallel} - K_0)} \cdot \exp \left\{ -\frac{\beta_{\delta}}{2} \int_0^{\frac{k_{\max}}{K_0}} \frac{dy}{y} P^{q/g}(y) [1 - \cos(y K_0 \sigma)] \right\}, \quad (3)$$

where $\beta_{\delta} = \frac{2}{\pi} \int_{K_{1T}}^{K_{2T}} \frac{dk_{\perp}}{k_{\perp}} \bar{\alpha}(k_{\perp})$, and $P^{q/g}$ are the appropriate probabilities defined in eq. (2).

This formula has the following simple physical interpretation. For the parent quark or gluon of momentum K_0 the above gives us the distribution in the total longitudinal momentum K_{\parallel} of the QCD radiation in a conical spread defined by β_{δ} . The parameter k_{\max} ($< K_0$) recalls to us that not all the momentum K_0 is residing in the cone. Otherwise said, some of the energy is "lost" through the transverse motion. When the cone opens completely, K_{1T} approaches K_{2T} , $\beta_{\delta} \rightarrow 0$ and $G(K_{\parallel} - K_0) \rightarrow \delta(K_{\parallel} - K_0)$, as it should.

The first few moments of the distribution (3) are given by

$$\langle K_{\parallel} - K_0 \rangle = 0, \quad (4)$$

$$\langle |K_{\parallel} - K_0| \rangle = K_0 \left(\frac{\beta_{\delta}}{2} \right) \int_0^{\bar{y}} y dy P^{q/g}(y), \quad (5)$$

$$\langle (K_{\parallel} - K_0)^2 \rangle = K_0^2 \left(\frac{\beta_{\delta}}{2} \right) \int_0^{\bar{y}} y^2 dy P^{q/g}(y), \quad (6)$$

where $\bar{y} = \frac{k_{\max}}{K_0}$.

It is interesting to observe that an analytic approximate form for eq. (3) is given by

$$\frac{d\tilde{P}}{d|K_{\parallel} - K_0|} = \frac{\sqrt{2}}{\sqrt{\pi} \Gamma(\beta/2)} \frac{1}{k_{\max}} \left(\frac{|K_{\parallel} - K_0|}{\sqrt{2} k_{\max}} \right)^{\frac{\beta-1}{2}} K_{\frac{\beta-1}{2}} \left(\frac{\sqrt{2} |K_{\parallel} - K_0|}{k_{\max}} \right), \quad (7)$$

where $\beta = C_F(N_c)\beta_\delta$ for quarks or gluons respectively, and $K_\nu(z)$ is a modified Bessel function. Eq. (7) is found to provide a good interpretation to the exact formula for small \bar{y} . This formula has been used successfully⁽⁴⁾ to analyze the single hadron distributions in various inclusive processes.

For illustration, we show in Fig. 1, the $K_{||}$ -distribution for a quark and a gluon jet, as well as that obtained from the approximate form. When appropriate data become available, an analysis can be

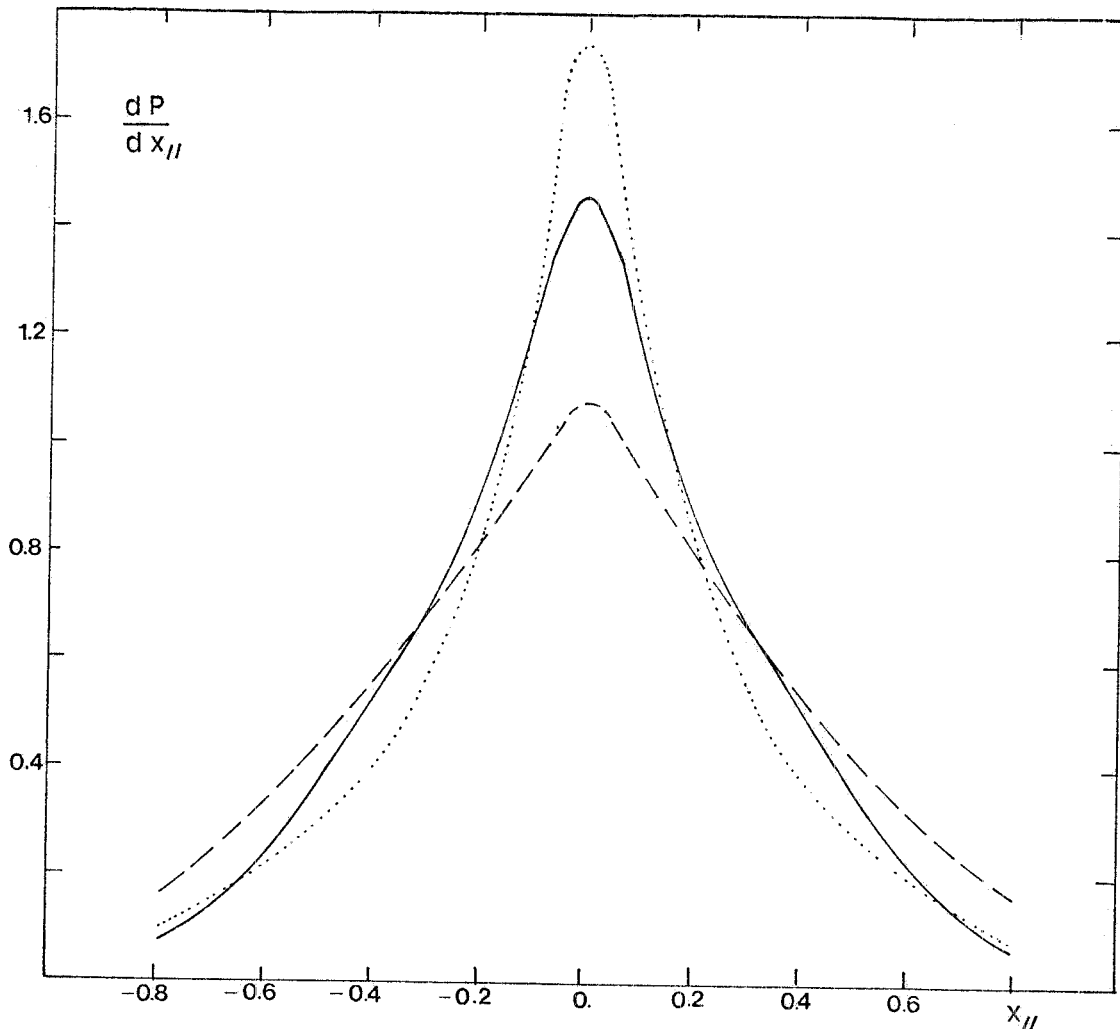


FIG. 1 - Longitudinal momentum distributions $dP/dx_{||}$, with $x_{||} = (K_{||} - K_0)/K_0$, for quark (full line) and gluon (dashed line) jets as given by eq. (3). The parameters β_δ and \bar{y} have been arbitrarily set to 1 and 0.5 respectively. Also shown (dotted line) is the approximate expression given by eq. (7) for the quark case.

performed using our formulae. For this purpose, eqs. (5, 6) would be useful in fixing the parameters β_δ and \bar{y} .

We now turn to a general discussion of the QCD jet cross-section. The above formalism suggests us to propose the following for an n-jet cross-section:

$$\frac{d\sigma^{\text{jet}}}{dK_{\parallel}^{(1)} \dots dK_{\parallel}^{(n)}} = \int (dk_{\parallel}^{(1)} \dots dk_{\parallel}^{(n)}) \left[\frac{d\sigma^{\text{Born}}}{dk_{\parallel}^{(1)} \dots dk_{\parallel}^{(n)}} \right] \prod_{i=1}^n G(K_{\parallel}^{(1)} - k_{\parallel}^{(i)}) \quad (8)$$

This is a direct consequence of eqs. (1) and (3) under the assumption that there are no correlations among different jets, which in turn requires a rather clean experimental separation of them.

It is understood that appropriate G's for quarks and/or gluons, as given by eq. (3), are to be inserted. As discussed earlier, G's are the analogues of the fragmentation functions since they give the appropriate spread of the parent particles. Thus, eq. (8) is the complete (longitudinal) jet cross-section corresponding to the basic (lowest order) Born cross-section (e. g. $e^+e^- \rightarrow q\bar{q}$, $q\bar{q}g$, ggg , etc.). In fact, eq. (8) reduces to its Born limit when G's approach δ -functions. Eq. (8) offers the possibility of analyzing the jet data in terms of QCD parameters only and therefore an experimental verification would be quite important. In particular, eq. (8) can be used to isolate the $(q\bar{q}g)$ component from the $(q\bar{q})$ one in the high energy data.

For jets at PETRA/PEP/LEP energies, K_{\parallel} analyses of the type suggested here, when coupled with K_T analyses suggested earlier⁽²⁾ should provide crucial tests of QCD ideas.

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