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ABSTRACT.

A magnet arrangement for a Siberian Snake of the 2-nd kind is devised. The performance of the Double Siberian Snake consisting in the Steffen's system plus such a configuration is analyzed. Variable-geometry operation seems to be unavoidable.

Recently, the use of spin flippers has been proposed by Derbenev and Kondratenko[1] to avoid depolarizing effects due to the chromaticity of the polarization direction in high energy electron storage rings. It is necessary to have the particle's spin parallel to the main field during one half revolution, rotate it by 180º before the next one half revolution starts, and then rotate it back by 180º at the beginning of the subsequent one half revolution. This can be done, in principle, by installing two[1] Siberian Snakes in two diametrically opposite straight sections of the machine (Double Siberian Snake). Thus the Double Siberian Snake consists[1] of a snake (of the 1-th kind[2]) which transforms the spin vector (P_x, P_y, P_z) into $(-P_x, P_y, -P_z)$, plus a snake (of the 2-nd kind[2]) that performs the transformation $(P_x, P_y, P_z) \longrightarrow (P_x, -P_y, -P_z)$. Here, P_x, P_y and P_z are the radial, longitudinal and vertical components of the spin vector . Of course, the transformations above are strictly valid for one energy E_0 (reference energy[3]) only.

To the best of the author's knowledge, the most feasible snake of the 1-th kind devised to date is that of Steffen[4]. On the other hand, there is work that remains to be done looking at suitable 2-nd kind snake configurations. Therefore, in this note, we consider a configuration for a

snake of the 2-nd kind and analyze in detail the performance of the Dou̲
ble Siberian Snake composed by the Steffen's system plus the one repre̲
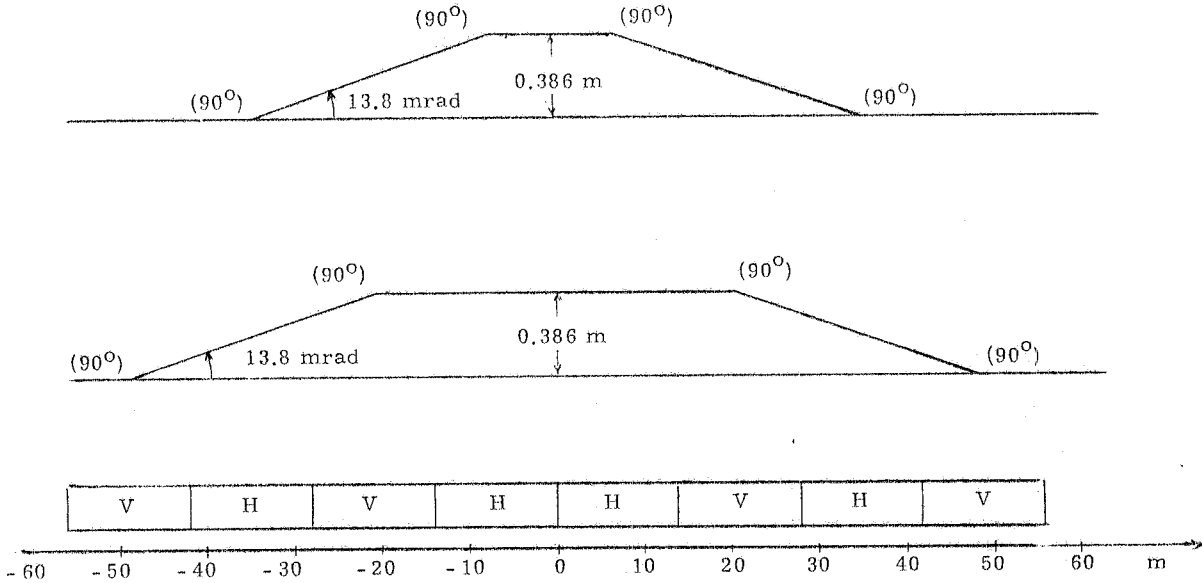sented in Fig. 1. Such a 2 x 56 m long series of magnets consists of

FIG. 1 - Siberian Snake of the 2-nd kind. V and H are the verti-
cal and horizontal bending magnets to produce the transformation
$(P_x, P_y, P_z) \longrightarrow (P_x, -P_y, -P_z)$. The upper (lower) curve indicates
horizontal (vertical) displacement of the beam. The angles of spin
rotation are indicated (in brackets) at various point along the
trajectory. Also indicated are the (same) bending angle of each
magnet and the maxima displacements of the orbit. See text for
some other detail.

8(x14 m long) individual magnets. The situation represented in Fig. 1
corresponds[3] to a reference energy $E_0 = 50$ GeV, namely to dipoles
having bending strenght 2.3 Tesla · m each.

The use of 2-component spinor algebra[3] enables one to express,
for the 2-nd kind snake represented in Fig. 1, the complete transforma̲
tion matrix connecting the occupation numbers of the states before and
after the system is passed through. One obtains

$$\| \Pi \| = \begin{vmatrix} \Pi_1 & \Pi_2 \\ -\Pi_2^* & \Pi_1^* \end{vmatrix} , \quad \text{where} \quad \Pi_1 = \alpha + i\beta , \quad \Pi_2 = i\gamma , \tag{1}$$

and
$$\alpha = c(2)\left[\, 2\, s^2(2) + c^3(2)\,\right] \,, \tag{2a}$$

$$\beta = -\, s^2(4)\, s\,(1), \tag{2b}$$

$$\gamma = s(2)\left[\, c(1) - c^3(2)\,\right] \tag{2c}$$

$$(\text{Det } \|\, \text{II}\,\| \equiv \alpha^2 + \beta^2 + \gamma^2 = 1).$$

Here,

$$c(k) = \cos(\not{p}/k), \qquad s(k) \equiv \sin(\not{p}/k),$$
$$k = 1;2;4;8, \tag{3a,b}$$

and
$$\not{p} = \pi\, E/E_0; \tag{3c}$$

E_0 is the reference energy and E is the particle's energy.

Analogously, collecting the formulas of our previous report[5], we have, for the Steffen's system rotated by any angle, φ, around the longitudinal (velocity) y axis,

$$\|\, \text{I}\,\| = \begin{vmatrix} I_1 & I_2 \\ -I_2^* & I_1^* \end{vmatrix}, \quad \text{where} \quad \begin{aligned} I_1 &= A + i\,B\cos\varphi \,, \\ I_2 &= C + i\,B\sin\varphi \,, \end{aligned} \tag{4}$$

and

$$A = c(4)c(2)\left[\, c(4) + s^2(4)\,\right]\left[\, 1 + 2s^2(4)\,\right] \,, \tag{5a}$$

$$B = (1/2)s^2(8)s(1)\left[\, 1 + 2s^2(4)\,\right] \,, \tag{5b}$$

$$C = s^2(4)\left[\, 3 - 4s^4(4)\,\right] \tag{5c}$$

$$(\text{Det } \|\, \text{I}\,\| \equiv A^2 + B^2 + C^2 \equiv 1) \,.$$

A short computation yields the following result for the extrema of the half trace $\cos(\pi\nu)$ of the transfer matrix, $\|\, a\,\|$, around one revolution:

$$\left|\, \cos(\pi\nu)\,\right|_{\substack{\text{extr} \\ \varphi}} = \left|\, B\gamma\sin\varphi\,\right| + (\, \alpha^2 + \beta^2\,)^{1/2}(A^2 + B^2\cos^2\varphi)^{1/2} \,. \tag{6}$$

Eq. (6) appears to be a very weakly dependent function of φ for $0 \leq \not{p} \leq 3\pi/2$ (consider, for simplicity, the point at $\not{p} = 3\pi/2$). This means that this Double Snake is quite insensitive to the optimization

procedure consisting in rotating the Steffen's system about the momentum axis. Thus, from here downwards, we will set $\varphi = 0$ in all our equations. The corresponding graph of $\left| \cos(\pi\nu) \right|_{extr}$ is represented in Fig. 2 (dashed line). One concludes that acceleration under
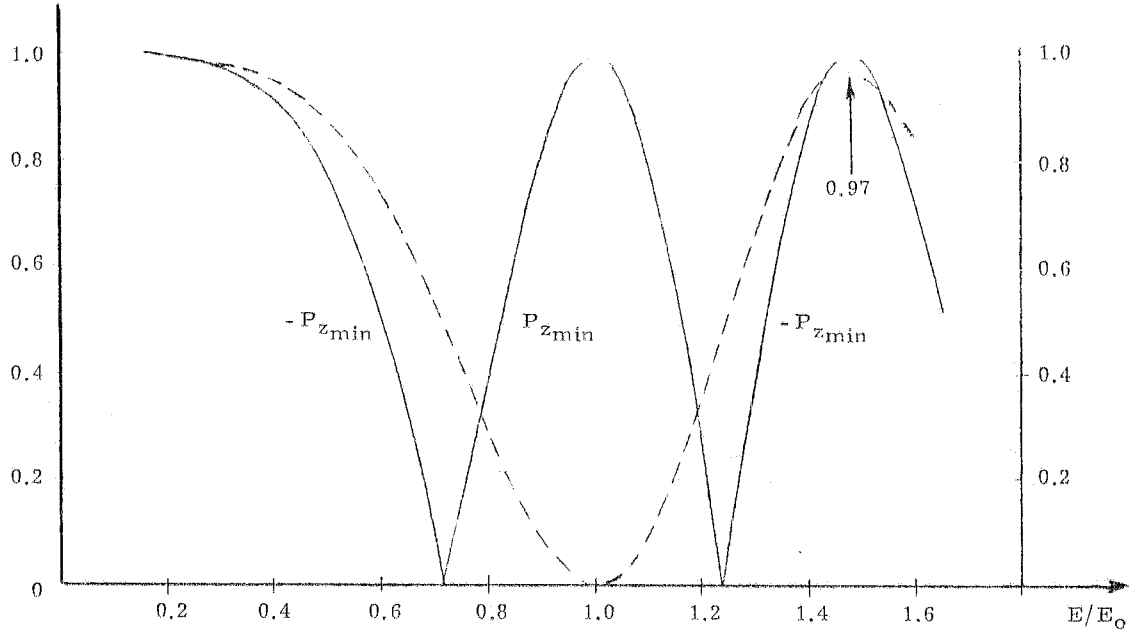

FIG. 2 - $\left| \cos(\pi\nu) \right|_{extr}$ (with $\varphi = 0$) for the Double Siberian Snake (dashed line) and $P_{z_{min}}$ (solid line).

fixed-geometry conditions is excluded with this Double Snake.

The polarization eigenvector can be expressed in terms of the normalized eigenvectors of the transfer matrix, $\| a \|$, around one revolution.

One obtains for P_z at the mid-point of one of the two main bending arcs

$$P_z = \left| a_{12} \right|^2 \left[(2/N_1)^2 - (2/N_2)^2 \right] , \qquad 7)$$

where

$$\left| a_{12} \right|^2 = (A^2 + B^2)\gamma^2 + (\alpha^2 + \beta^2)C^2 - 2\gamma C \left[p\cos(\varkappa/2) + r\sin(\varkappa/2) \right] , \qquad 8)$$

$$(N_{1,2}/2)^2 = \left| a_{12} \right|^2 + \left[Im(a_{11}) \stackrel{+}{-} \sqrt{1 - \cos^2(\pi\nu)} \right]^2 , \qquad 9)$$

$$Im(a_{11}) = \gamma C + p\cos(\varkappa/2) + r\sin(\varkappa/2), \qquad 10)$$

and
$$\cos(\pi\nu) = r \cos(\varkappa/2) - p \sin(\varkappa/2).$$
$$(\varphi = 0)$$

11)

Here, $p = A\beta + B\alpha$, $r = A\alpha - B\beta$,

12a,b)

and $\varkappa = 2\pi G \, E/(mc^2)$ is the precession phase per revolution (G is the gyromagnetic anomaly).

The plot of $P_{z_{min}}$ is shown in Fig. 2 (solid line).

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(4) K. Steffen, DESY PET-78/11 (1978).
(5) A. Turrin, LNF-79/8(1979).