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INTRINSIC TEMPERATURE OF CONFINED SYSTEMS.

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ABSTRACT.

We show that for a general class of confining potentials there exists an intrinsic temperature given by $T = \frac{\langle A \rangle \hbar}{2\pi}$ where $\langle A \rangle$ is the average Euclidean acceleration.

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It is the object of this paper to show that, under rather general conditions, confining potentials lead (quantum mechanically) to an intrinsic temperature. The simplest way to obtain this result is to consider a particle in such a potential and show that its Euclidean Green's function is periodic, which implies a finite temperature. The corresponding Minkowski Green's function will show the appropriate thermal distribution.

The existence of a QM temperature has been shown in a variety of different physical contexts, following Hawking's pioneering work on black holes. The aspect which concerns us is the result that an observer under uniform acceleration A feels a heat bath at temp $T = \frac{A\hbar}{2\pi}$. A physical interpretation has been given in terms of an event-horizon and the associated loss of information[1, 2]. From this point of view there is little conceptual difference between this case and the thermal radiation from a black hole. Another interpretation has been given in terms of topological concepts like winding numbers[3, 4] and gravitational instantons[5]. The possible relevance of the constant acceleration case - which corresponds to a static linear potential - to hadronic momentum distribution in quark models in particle physics has been particularly stressed in refs. (6÷ 8).

We would like to argue that for a very general class of confining potentials there exists a temperature. The main steps are as follows:

a) Consider a static potential V(r) which becomes unbounded only for large r. (We discuss later some consequences of relaxing this condition).

b) The world-line of a particle moving under the above V(r) has a <u>minimum</u> r and there is an event horizon. Conversely the Euclidean world line has a <u>maximum</u> in r.

c) At this point, either one simply accepts the conclusion that every event horizon leads to a loss of information and then to thermal radiation, or equivalently one solves for the Euclidean Green's function of our test particle finding that it is a periodic function of the proper time τ and thus using results from refs (3, 9) this implies a temperature $T = \frac{\hbar \langle A \rangle}{2\pi}$, where $\langle A \rangle$ denotes the average acceleration over one period.

For simplicity, we restrict ourselves to 2-dimensions. The magnitude of the acceleration is given by

$$A(x) = \frac{1}{m} \left| \frac{dV(x)}{dx} \right| \equiv \left| \frac{dU(x)}{dx} \right| \tag{1}$$

Now, we can solve for the 2-velocity (u_o, u_1) and the worldline $(t(\tau), x(\tau))$ in the Euclidean region in the standard manner [c. f. ref. (10)] and obtain

$$u_o(x) = U(x) \tag{2}$$

$$\tau - \tau_o = \int_{\bar{x}}^{x} \frac{dx'}{\sqrt{1 - U^2(x')}} \tag{3}$$

$$t - t_o = \pm \int_{x(\tau_o)}^{x(\tau)} \frac{dx' \; U(x')}{\sqrt{1 - U^2(x')}} \tag{4}$$

Since U(x) becomes unbounded for large x, eq. (2) tells us that the allowed $x \leq \bar{x}$ where $u_o(\bar{x}) = +1$, which is the maximum value for u_o in the Euclidean region. Thus, $\bar{x}$ is the turning point and immediately leads to x and t being periodic functions of τ. From this, we can deduce that it is always possible to find a Euclidean world-line which is indeed a closed periodic curve provided the potential U(x) has no singularities (in the finite region) so as not to produce further turning points.

The case of the linear potential, $U(x) = gx$ has been extensively treated in the literature (2÷8). One simply finds

$$x(\tau) = \frac{1}{g} \cos g\tau$$
$$t(\tau) = \frac{1}{g} \sin g\tau \qquad (5)$$

We consider another interesting example of the harmonic oscillator potential $U(x) = \frac{1}{2} \omega^2 x^2$. The world-line in this case is given analytically in terms of Elliptic functions:

$$x(\tau) = \frac{\sqrt{2}}{\omega} \, cn\,(\omega\tau)$$
$$t(\tau) \pm \left[\tau - \frac{2}{\omega} E(\cos^{-1} cn(\omega\tau), \frac{1}{\sqrt{2}}) \right] \qquad (6)$$

This world-line is shown graphically in the figure 1.

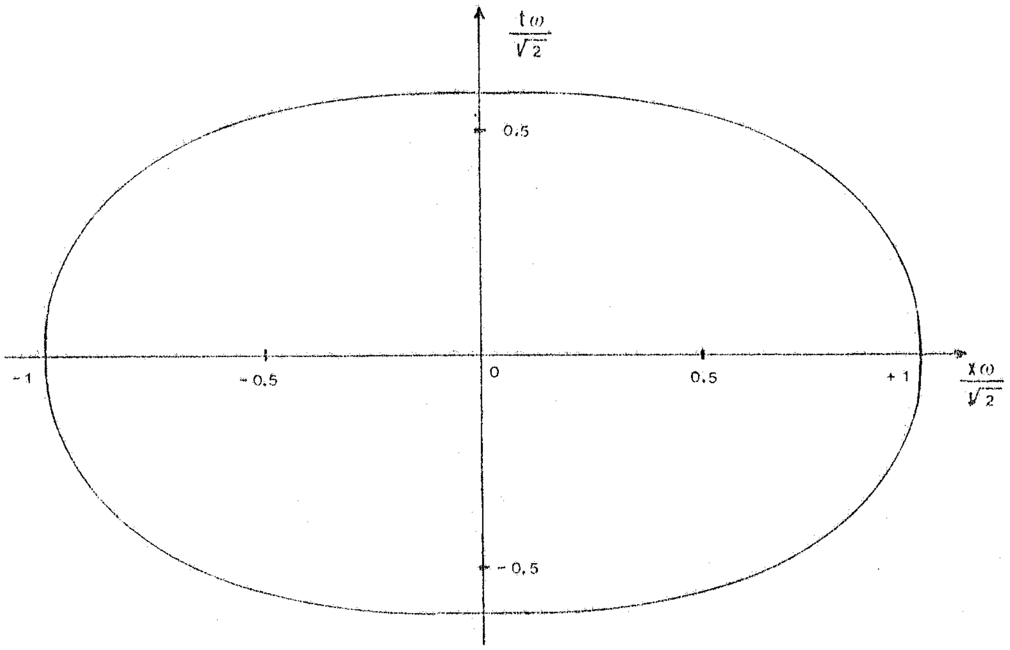

FIG. 1- The Euclidean world-line of a test particle in the harmonic potential $U(x) = (1/2)\,\omega^2 x^2$. The coordinates $(x\ ,t)$ are in units of $\omega/\sqrt{2}$.

This (euclidean) world line is topologically equivalent to the circle eq. (5), obtained in the earlier case of a constant acceleration. The period in τ for the constant acceleration case is $\tau_p = \frac{2\pi}{g}$ whereas now the period is $\frac{4K}{\omega}$ with

$$K = \int_0^{\pi/2} \frac{dz}{\sqrt{1 - \frac{1}{2}\sin^2 z}} \simeq 1.854 \; .$$

It is interesting to note that the average acceleration $\langle A \rangle$, over one period, is computed to be $\langle A \rangle = \frac{\pi\omega}{2K}$, and is equal to $\frac{2\pi}{\tau_p}$. This is a general result. For any potential U(x) which leads to a closed periodic orbit (in the Euclidean domain) we have

$$\langle A \rangle = \frac{2\pi}{\tau_p} \qquad (7)$$

Now, we turn to arguments presented in refs. (3, 5) through which the obove periodicity in τ is reflected in the periodicity of the Euclidean propagator of a test particle at a temperature T. To be precise, let us consider a free field $\Phi(x)$ and define the Green's function

$$G_\beta (x, x') = \frac{\mathrm{Tr} \left\{ e^{-\beta H} \Phi(x) \Phi(x') \right\}}{\mathrm{Tr} \left\{ e^{-\beta H} \right\}} \qquad (8)$$

at a temperature T given by $\beta = \frac{1}{kT}$. This Euclidean Green's function shows a periodicity in τ with period [9]

$$G_\beta (\tau, \tau') = G_\beta (\tau, \tau' + \beta) \qquad (9)$$

Since the free propagator for our test particle shows periodicity in τ (with period τ_p) we conclude that the above system is at a finite temperature given generally by

$$kT = \frac{\hbar}{\tau_p} = \frac{\langle A \rangle \hbar}{2\pi} \quad . \qquad (10)$$

This is our basic result . The particular case of constant acceleration follows immediately.

The free Euclidean propagator at temp.T shows periodicity in τ while the corresponding Minkowski propagator manifests a thermal distribution [9]. In momentum space, with k a real Minkowski 4-vector this propagator shows the Bose-Einstein distribution:

$$D_\beta(k) = \frac{i}{k^2 - \mu^2} + \frac{2\pi}{e^{\beta E} - 1} \delta(k^2 - \mu^2) \tag{11}$$

where $E = \sqrt{k^2 + \mu^2}$. For fermions an analogous result holds giving of course the Fermi-Dirac distribution.

For the two cases worked out explicitly, i. e. linear and harmonic potentials, it is interesting to note that if the parameters are adjusted so as to give the same maximum (Euclidean) acceleration, the corresponding temperatures are numerically almost the same,

$$\frac{T_{Harmonic}}{T_{Linear}} = \frac{\pi}{\sqrt{2}\ K} \simeq 1,2 \tag{12}$$

It is reassuring that the value of the temperature is rather stable against precise details of the potential but depends largely on the boundary condition. Thus, the value $T \simeq 130$ MeV obtained in ref. (6) using a bag model with a linear potential should remain qualitatively the same for a large class of bag models.

We comment briefly about confining potentials which do not satisfy the finiteness criterion states earlier. For such cases, in the Euclidean region these are more turning points and so the orbit while periodic may not be topologically. equivalent to a circle. This is likely to destrory the existence of a finite temperature. An important particular example of such a potential is provided by the standard one used in QCD, i. e. a linear plus a Coulomb potential. This problem is presently under investigation.

We close with some conjectures concerning any field theory which leads to confinement. If confinement necessarily signals the onset of a non-zero temperature then it seems natural that the theory should be stable only for finite temperature. Then, perturbation expansions at zero temperature are not expected to lead to physically relevant results. It would be interesting to check this idea against some known 2-dimensional field theory which shows confinement.

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