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G. Parisi: TOWARD A MEAN FIELD THEORY FOR SPIN GLASSES.

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ABSTRACT

We find an approximate solution of the Sherrington-Kirkpatrick model for spin glasses; the internal energy and the specific heat are in very good agreement with the computer simulations, the zero temperature entropy is unfortunately negative, although it is very small.

To find the exact solution of the Sherrington-Kirkpatrick model[1] for spin glasses is a long standing problem; the simple minded application of the replica theory[2] and of the saddle point method solves correctly the model in the high temperature phase, however in the low temperature phase it desagrees with the computer simulations[3] and the computed entropy is negative at low temperature (In this model the true entropy is always non negative).

It has been realized[4] that this discrepancy is due to a wrong choice of the saddle point: the model is invariant under permutations of the replicas, the saddle point used in ref. (1-2) is invariant under this symmetry, however the saddle point which gives the leading contribution in the thermodynamic limit is not invariant and the replica symmetry is spontaneously broken.

Various patterns of symmetry breaking have been proposed[5, 6] the aim of this note is to present a new one: the novelty is that the pattern is treated as a variational parameter. As a first approximation we consider only very simple patterns of symmetry breaking; doing so, we do not find the exact solution of the model, but we obtain a substantial improvement on the solution with unbroken replica symmetry[1].

Fos completeness we recall that the S-K model consists of N Ising spins interacting via a random infinite range Hamiltonian. The distribution of the values of the bonds is a Gaussian with variance $1/N$. Using the saddle point method in the thermodynamic limit $(N \to \infty)$ it has been shown that the free energy density F is given by:

$$\beta F = \frac{\beta^2}{4} + \lim_{n\to 0} \frac{1}{n} \operatorname{Max} \left[\frac{1}{4} \Sigma_{a,b} Y^2_{a,b} - \ln \operatorname{Tr} \exp (\beta \Sigma_{a,b} Y_{a,b} S_a S_b) \right] , \tag{1}$$

where $\beta = 1/KT$, Y_{ab} is an $n \times n$ matrix, zero on the diagonal, Tr stands for the sum over all the possible values of the n spin variables S_a, the maximum is taken over all the possible choices of the matrix $Y_{a,b}$ and the indices a and b run from 1 to n. The values of the expressions in the limit $n = 0$ are defined as analytic continuation in n from integer n.

We do the following Ansatz:

$$\begin{aligned} Y_{a,b} &= \beta(p+t) \qquad \text{if } \exists K \text{ such that } \begin{cases} Km < a \leq K(m+1) \\ Km < b \leq K(m+1) \end{cases} \\ Y_{a,b} &= \beta p \qquad \text{elsewhere.} \end{aligned} \tag{2}$$

m is an integer number and for $m = 2$ we recover the proposal of ref. (5). It is a standard exercise to find that:

$$\beta F(p,t,m) = -\frac{\beta^2}{4}\left[1 + mp^2 + (1-m)(p+t)^2 - 2(p+t)\right] + \ln 2 -$$

$$- (2\pi)^{-1/2}\int dz \left\{\exp(-z^2/2)\, m^{-1} \cdot \right. \tag{3}$$

$$\left. \cdot \ln\left[(2\pi)^{-1/2}\int dy\, \exp(-y^2/2)\, \mathrm{ch}^m(\beta p^{1/2} z + \beta t^{1/2} y)\right]\right\}$$

Eq. (3) can be analytically continued to non integer m. We look for a solution of the system of equations :

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial p} = \frac{\partial F}{\partial m} = 0\,. \tag{4}$$

Eq. (4) has been solved on a computer and the internal energy U(T), specific heat C(T) and entropy S(T) have been found as functions of the temperature. Excellent agreement has been found with the computer simulations(3); for example in the region $0.2 < T < 0.4$ (we measure T in units of the critical temperature T_c) we have approximately :

$$S(T) \simeq \frac{A}{2}T^2\,; \qquad C(T) \simeq AT^2\,; \qquad U(T) \simeq \bar{U} + \frac{A}{3}T^3$$
$$A \simeq 1.4\,; \qquad \bar{U} \simeq -0.764\,. \tag{5}$$

The computer simulations are well represented by similar expressions where $\bar{U} = 0.76 \pm 0.01$ and A is compatible with the value suggested in ref. (7) ($A = 2\ln \quad \simeq 1.39$). In the same region the theory without breaking of the replica symmetry predicts :

$$S(T) \simeq S(0) + BT\,; \quad C(T) \simeq BT\,; \quad U(T) \simeq U(0) + \frac{1}{2}BT^2$$
$$B \simeq 0.53\,; \quad S(0) \simeq -0.159\,; \quad U(0) \simeq 0.798\,. \tag{6}$$

Unfortunately decreasing T the entropy becomes negative (that happens for $T < 0.1$) and one finds :

$$S(0) \simeq -0.01 \; ; \qquad U(0) \simeq -0.765 \, . \tag{7}$$

We have found only an approximate solution of the S-K model, although it is quite good for $T > 0.2$. It is evident that more elaborate patterns of symmetry breaking are needed to produce the exact solution of the model. The solution with two orders parameters (p and t) works much better than the solution with only one order parameter (i. e. $t = 0$). It is quite likely that an infinite number of order parameters is needed in the correct treatment and the neglected order parameters are negligeable near T_c (e. g. they are proportional to $(T_c - T)^4$) and their presence becomes important only for rather small values of T. As an example we write down a possible generalization of eq. (2) with three order parameters :

$$Y_{a,b} = \beta(p+t+r) \qquad \text{if } \exists K, j \quad \text{such that} \quad \begin{cases} Km < a \leq K(m+1) \\ Km < b \leq K(m+1) \\ jl < a \leq j(l+1) \\ jl < b \leq j(l+1) \end{cases}$$

$$Y_{a,b} = \beta(p+t) \qquad \text{if } \exists K \quad \text{such that} \quad \begin{cases} Km < a \leq K(m+1) \\ Km < b \leq K(m+1) \end{cases}$$

$$Y_{a,b} = \beta p \qquad \text{elsewhere} \, . \tag{8}$$

It is far from being evident that the Ansatz eq. (8) is the correct one and more careful investigations are needed on this point ; this letter should be considered as a first step toward the construction of an infinite set of order parameters : the approximation of keeping only two order parameters works rather well for not too low temperatures.

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