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Laboratori Nazionali di Frascati

LNF-79/20(R)
29 Marzo 1979

M. Greco, G. Pancheri-Srivastava and Y. Srivastava:

RADIATIVE CORRECTIONS TO $e^+e^- \to \mu^+\mu^-$ NEAR THE Z^o-RESONANCE.

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We present radiatively corrected expressions for the direct production of Z^o in the process

$$e^+e^- \to \mu^+\mu^-$$

before and in the vicinity of the Z^o mass.

Our formulae incorporate the soft photon corrections in exponentiated form (i.e. a summation to all orders of terms with $\alpha \ln \Delta\omega$ or $\alpha \ln \Gamma$) as well as finite corrections of order α obtained through perturbation theory. These formulae are thus valid for $\Delta\omega \ll E$, otherwise, hard bremsstrahlung corrections also need to be included. Weak effects are considered only to first order. The diagrams explicitly taken into account are shown in Figs. 1 and 2.

Notation.

$$e^-(p_1) + e^+(p_2) \to \mu^-(p_3) + \mu^+(p_4)$$

$$s = W^2 = 4E^2 = (p_1 + p_2)^2 ,$$

$$z = \cos\theta = \hat{p}_1 \cdot \hat{p}_3 = \hat{p}_2 \cdot \hat{p}_4 ,$$

$$a = \sin\theta/2 , \quad b = \cos\theta/2 ,$$

(x) Permanent Address: Northeastern University, Boston, Mass., USA.
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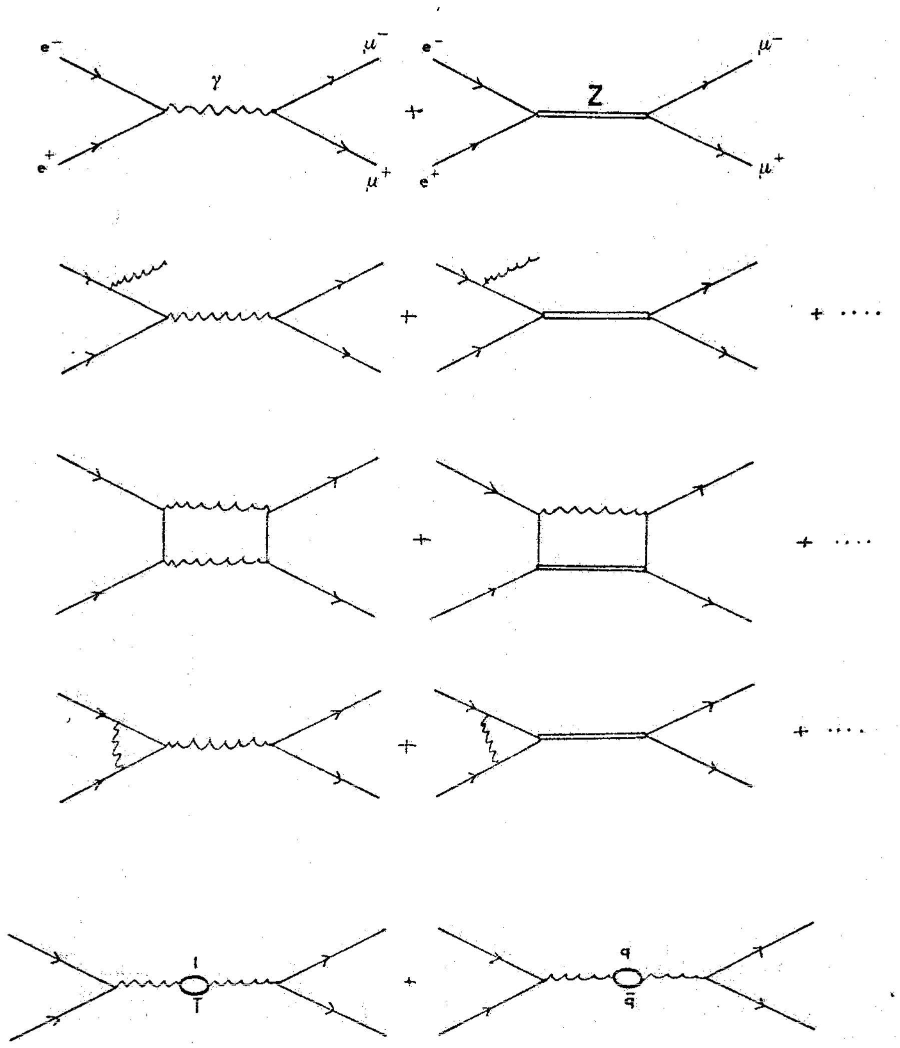
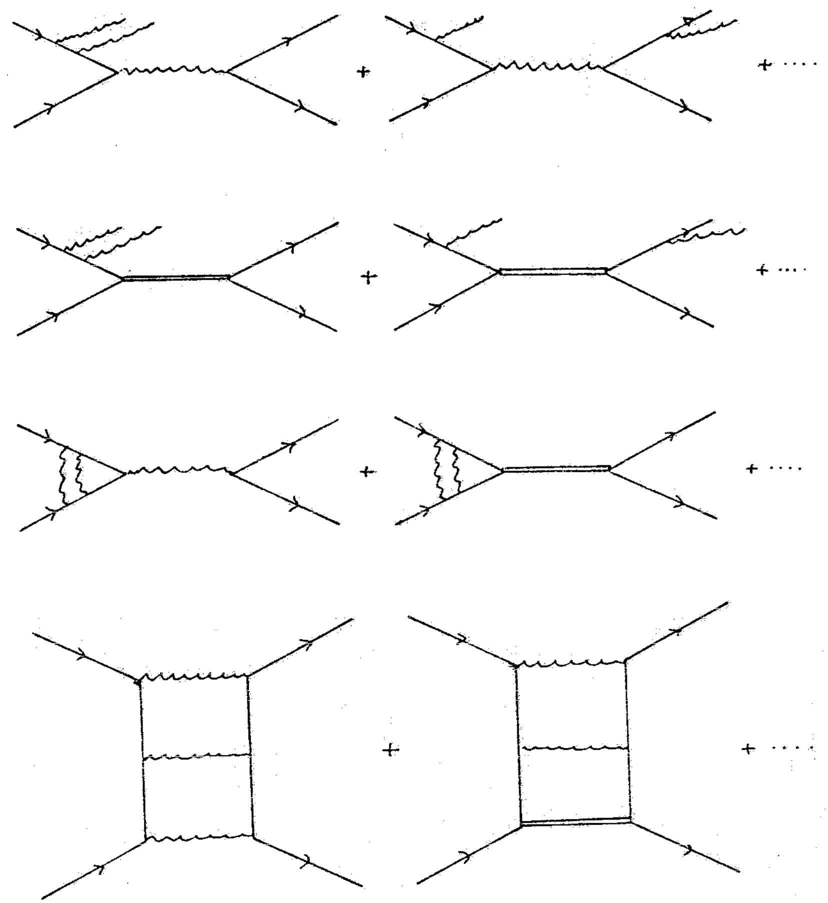


FIG. 1 - Graphs contributing to the Born term and to the first order corrections.



+ higher orders.

FIG. 2 - Some representative graphs included in the soft photon approximation to obtain the exponentiated factors C_{infra}.

$$\beta_e = \frac{4\alpha}{\pi}\left[\ln\frac{W}{m_e} - \frac{1}{2}\right] ,$$

$$\beta_\mu = \frac{4\alpha}{\pi}\left[\ln\frac{W}{m_\mu} - \frac{1}{2}\right] ,$$

$$\beta_{int} = \frac{4\alpha}{\pi}\ln(\tan\frac{\theta}{2}) ,$$

$$\Delta \equiv \frac{\Delta\omega}{E} = \text{fractional energy resolution} .$$

The weak boson Z^o is taken to be a resonance of mass M and total width Γ, such that the phase shift δ_R is

$$\tan\delta_R(s) = \frac{M\Gamma}{M^2 - s} .$$

Also, we define

$$\chi(s) = (\frac{3\Gamma_e}{\alpha M})(\frac{s}{s - M^2 + iM\Gamma})$$

where Γ_e is the leptonic width. In terms of the Weinberg angle θ_W, we have

$$r_V \equiv \frac{\Gamma_e^V}{\Gamma_e} = \frac{(1 - 4\sin^2\theta_W)^2}{1 + (1 - 4\sin^2\theta_W)^2} ,$$

$$r_A \equiv \frac{\Gamma_e^A}{\Gamma_e} = \frac{1}{1 + (1 - 4\sin^2\theta_W)^2} .$$

Formulae.

The differential cross section can be written as follows :

$$(\frac{d\sigma}{d\Omega})^{corr} = C^{res}_{infra}(\frac{d\sigma_{res}}{d\Omega})(1 + C^{res}_F) + C^{int,V}_{infra}(\frac{d\sigma_{int,V}}{d\Omega})(1 + C^{int,V}_F) + C^{int,A}_{infra}(\frac{d\sigma_{int,A}}{d\Omega})(1 + C^{int,A}_F) + C^{QED}_{infra}(\frac{d\sigma_{QED}}{d\Omega})(1 + C^{QED}_F) \quad (1)$$

where the Born cross-sections are given by

$$\left(\frac{d\sigma_{QED}}{d\Omega}\right) = \left(\frac{\alpha^2}{4s}\right)(1 + z^2) ,$$

$$\left(\frac{d\sigma_{int,V}}{d\Omega}\right) = \left(\frac{\alpha^2}{4s}\right)(1 + z^2)(2\,\mathrm{Re}\chi)\, r_V ,$$

$$\left(\frac{d\sigma_{int,A}}{d\Omega}\right) = \left(\frac{\alpha^2}{4s}\right)(2z)(2\,\mathrm{Re}\chi)\, r_A ,$$

$$\left(\frac{d\sigma_{res}}{d\Omega}\right) = \left(\frac{\alpha^2}{4s}\right)\left\{(1 + z^2) + 8 z\, r_V\, r_A\right\}|\chi|^2 .$$

Infrared Factors.

The factors C_{infra} in eq. (1) take into account the contribution from soft photons to all orders[1, 2, 3]. We calculate them using the techniques developed in ref. (1) for the ψ-resonance, where $\Gamma \ll \Delta\omega$ and generalized in ref. (2), via the coherent states method, to include the case $\Gamma \gtrsim \Delta\omega$.

$$C^{QED}_{infra} = \left(\frac{\Delta\omega}{E}\right)^{\beta_e + \beta_\mu + 2\beta_{int}} ,$$

$$C^{int,V}_{infra} = C^{int,A}_{infra} = \left(\frac{\Delta\omega}{E}\right)^{\beta_\mu + \beta_{int}} \frac{1}{\cos\delta_R} \cdot$$

$$\cdot\ \mathrm{Re}\left\{ e^{i\delta_R} \left(\frac{\Delta}{1 + \Delta\left(\frac{s}{M\Gamma}\right) e^{i\delta_R} \sin\delta_R}\right)^{\beta_e} \left(\frac{\Delta}{\Delta + \left(\frac{M\Gamma}{s}\right)\frac{e^{-i\delta_R}}{\sin\delta_R}}\right)^{\beta_{int}} \right\}$$

$$C^{res}_{infra} = \left(\frac{\Delta\omega}{E}\right)^{\beta_\mu} \left|\frac{\Delta}{1 + \Delta\left(\frac{s}{M\Gamma}\right) e^{i\delta_R} \sin\delta_R}\right|^{\beta_e} \cdot$$

$$\cdot \left|\frac{\Delta}{\Delta + \left(\frac{M\Gamma}{s}\right)\frac{e^{-i\delta_R}}{\sin\delta_R}}\right|^{2\beta_{int}} \left[1 - \beta_e\, \delta_R \cot\delta_R\right] .$$

Some typical graphs which build up these infrared factors are shown in Fig. 2.

Finite contributions to order α.

To obtain the finite contributions C_F one calculates the first order corrections to the Born term (Fig. 1). A comparison with the first order expansion of eq. (1) will then uniquely define $C_F^{(i)}$. For the case of a vector resonance the lowest order radiative corrections may be found in refs. (2,4,5). We have extended these calculations to include the axial contributions as well. We find

$$C_F^{QED} = \frac{13}{12}(\beta_e + \beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{17}{18}\right) + X_V \;,$$

$$C_F^{int,V} = \frac{11}{12}(\beta_e + \beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{13}{18}\right) + \frac{1}{2} X_V \;,$$

$$C_F^{int,A} = \frac{11}{12}(\beta_e + \beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{13}{18}\right) + \frac{1}{2} X_A \;,$$

$$C_F^{res} = \frac{3}{4}(\beta_e + \beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \frac{r_V Y_V + r_A Y_A}{1 + \frac{8z}{1+z^2} r_V r_A} +$$

$$+ \frac{2\bar{R}\alpha}{9}\left(\frac{\alpha \Gamma M^2}{\Gamma_e s}\right) \frac{r_V + \frac{2z}{1+z^2} r_A}{1 + \frac{8z}{1+z^2} r_V r_A}$$

where

$$X_V = -\frac{4\alpha}{\pi}\left\{\frac{1}{1+z^2}\left[z\left((\ln a)^2 + (\ln b)^2\right) + a^2 \ln b - b^2 \ln a\right] + \right.$$

$$\left. + (\ln b)^2 - (\ln a)^2 + \frac{1}{2} Li_2(a^2) - \frac{1}{2} Li_2(b^2)\right\} \;,$$

$$X_A = -\frac{2\alpha}{\pi}\left\{Li_2(a^2) - Li_2(b^2) - (\ln a)^2 + (\ln b)^2 - \frac{b^2}{z}\ln a - \frac{a^2}{z}\ln b\right\} \;,$$

$$Y_V = -\frac{2\alpha}{3}\left(\frac{\alpha \Gamma M^2}{\Gamma_e s}\right)\frac{1}{1+z^2}\left[z - 2z(\ln a + \ln b) + 2(1+z^2)\ln\frac{a}{b}\right] \;,$$

$$Y_A = -\frac{2\alpha}{3}\left(\frac{\alpha \Gamma M^2}{\Gamma_e s}\right)\frac{2}{1+z^2}\left[z \ln\frac{a}{b} + \frac{1}{2}\right] ,$$

and

$$\bar{R} = \sum_{\substack{i = \text{leptons} \\ + \text{quarks}}} Q_i^2 .$$

Numerical results and discussions.

We present numerical results for $(d\sigma/d\Omega)$, σ and the integrated asymmetry for the following choice of the parameters in the (standard) Weinberg-Salam model. We have considered 6 quarks and 6 leptons and assumed $\sin^2\theta_W = 1/4$. This gives

$$M_{Z^o} = 86.4 \text{ GeV} ,$$

$$\Gamma(Z \rightarrow e^+e^-) = 70 \text{ MeV} = \frac{1}{32}\,\Gamma(Z \rightarrow \text{all}) ,$$

$$\Gamma(Z \rightarrow \text{all}) = 2.24 \text{ GeV} .$$

The results are shown in Figs. 3, 4, 5 for values of $\Delta = \Delta\omega/E = 5 \times 10^{-3}$, 10^{-2}, 5×10^{-2} and 10^{-1}. The comparison is always made with the Born terms called "naive" in the figures.

These figures show that radiative corrections change substantially the naive results. A more complete discussion and details of the derivation shall be presented elsewhere.

References.

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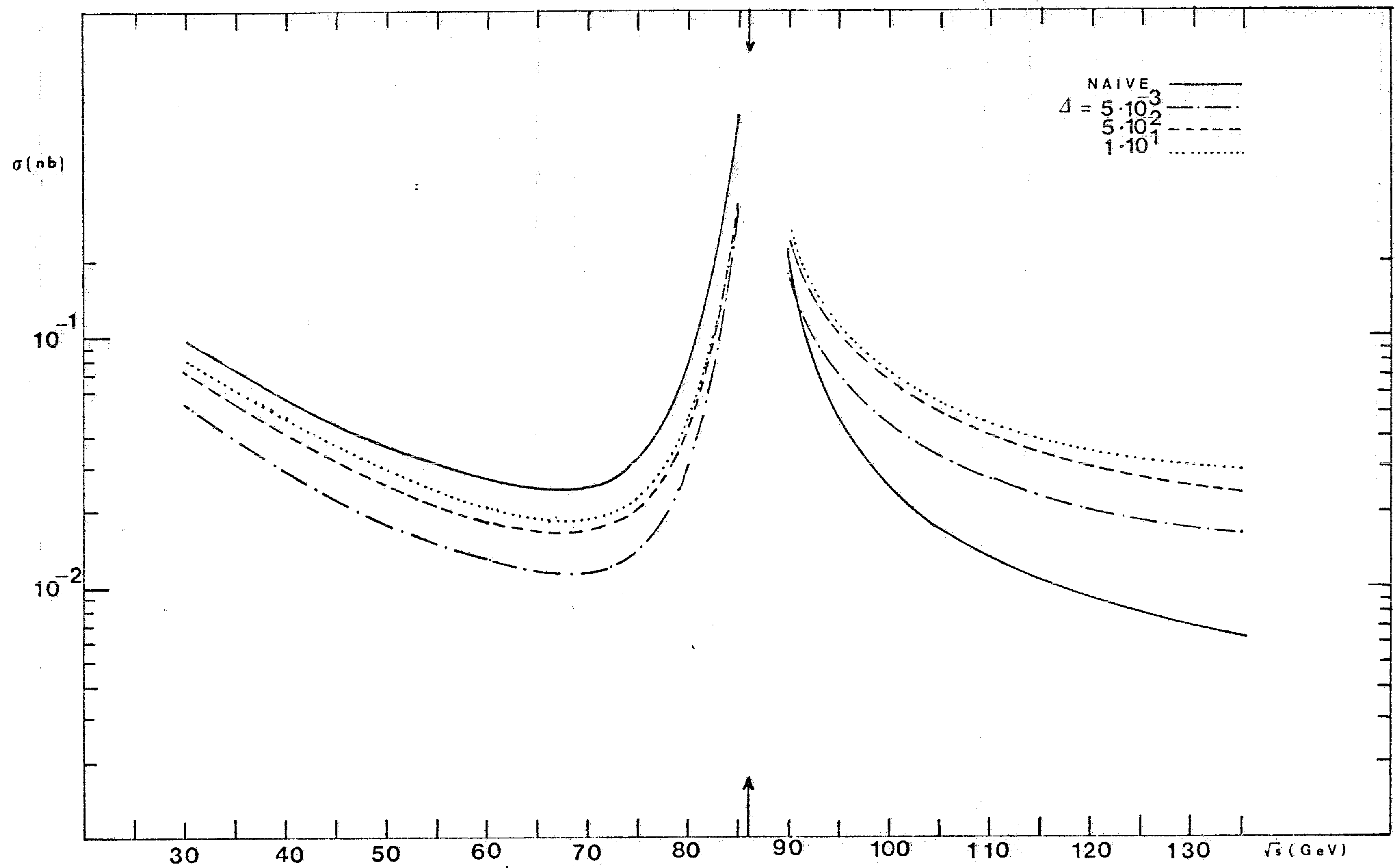

FIG. 3 - Total cross-section σ vs. $\sqrt{s}$ with and without radiative corrections (naive). Various values of the experimental resolution $\Delta\omega/E = \Delta$ are considered.

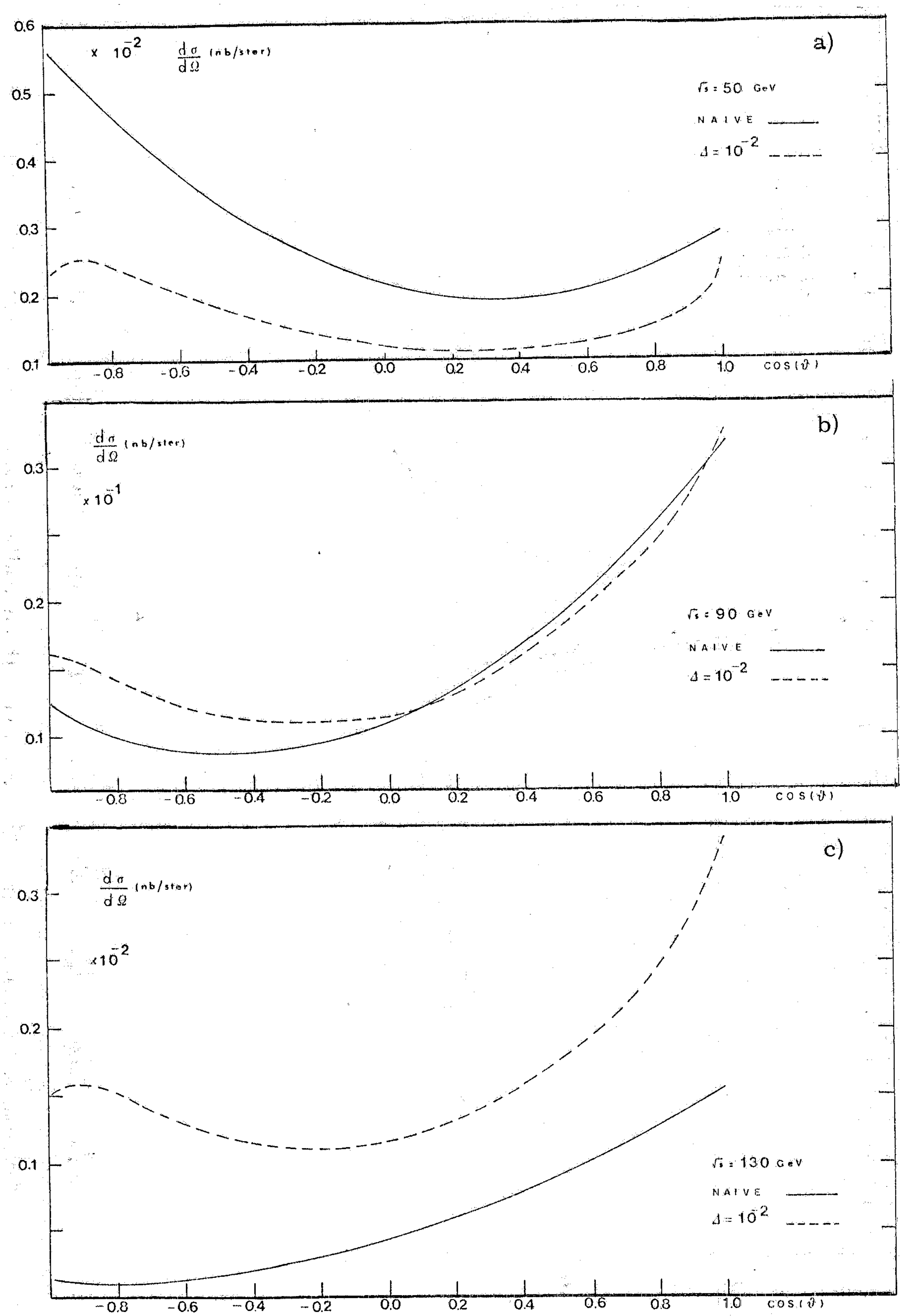

FIG. 4 - Differential cross-sections $d\sigma/d\Omega$ vs. $\cos\theta$, for various energies, with and without radiative corrections.

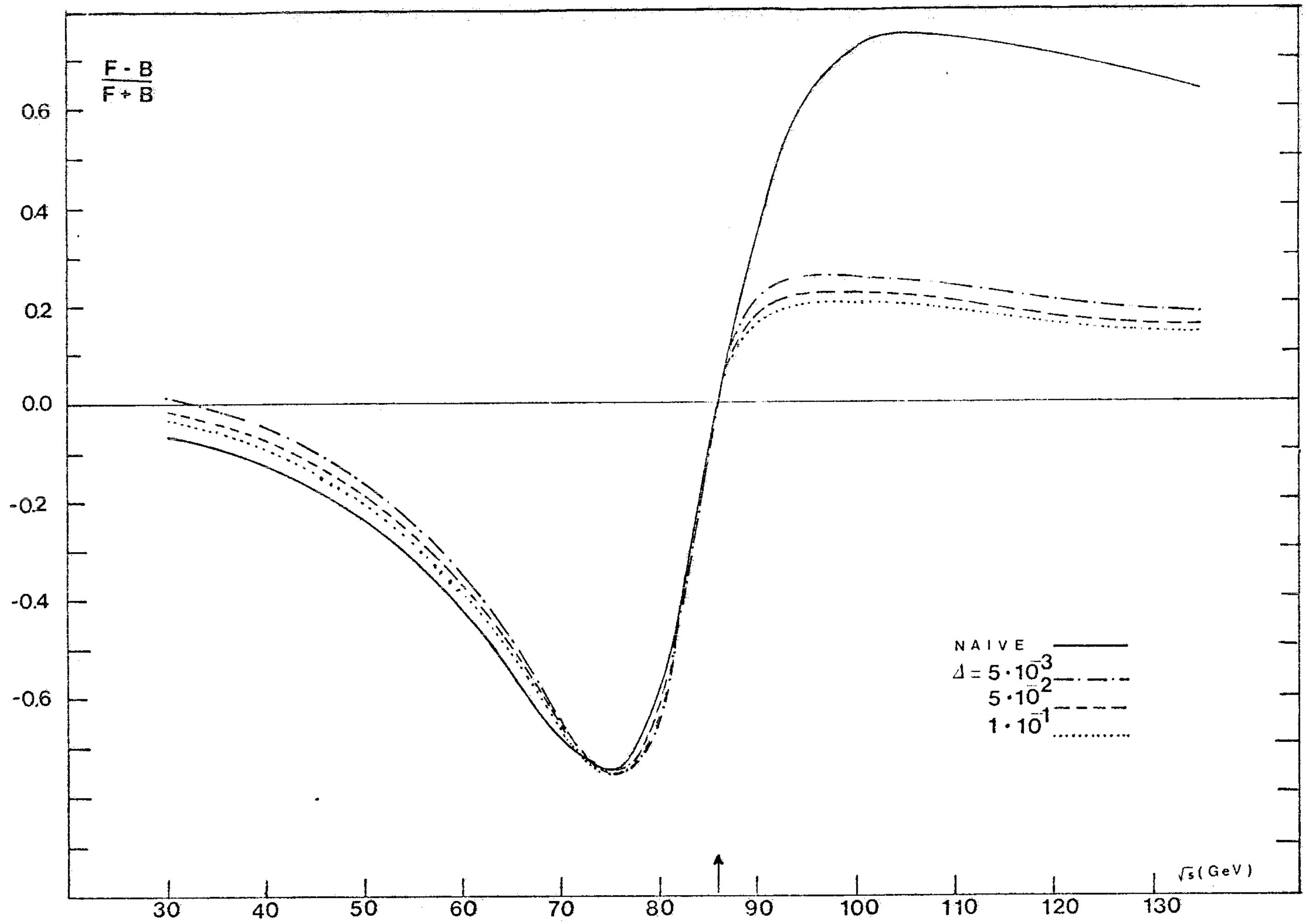

FIG. 5 - Integrated asymmetry vs. s, for various values of the experimental resolution (dotted lines) and without radiative corrections (full line).