Invited talk at the Sanibel Symposium on Fundamental Problems of Quantum Field Theory, Flagler Beach, Florida (USA) 26-28 February 1979

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S. Ferrara: NEW INSIGHTS IN SUPERSYMMETRY AND SUPERGRAVITY.

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ABSTRACT.

Two different topics in supersymmetry are discussed. First we present an analysis of invariants quadratic in the Riemann tensor for SO(N)-extended supergravity. On pure group theoretical grounds we discuss possible implications for quantum corrections, structure of auxiliary fields and superconformal theories.

Secondly we derive a (quadratic) mass formula relating boson and fermion masses in a wide class of spontaneously broken supersymmetric Lagrangian field theories. This formula states that the graded trace of the square mass operator of the theory vanishes.

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In the present talk I would like to discuss some recent work on two rather different areas of supersymmetric field theories.

The first subject is an investigation on higher-order invariants in extended supergravity, the second topic is a general mass formula in spontaneously broken supersymmetry.

In quantum gravity there are two possible one-loop counterterms[1] which are quadratic in the Rieman tensor. They are $C^2_{\mu\nu\varrho\sigma}$, the square of the Weyl tensor $C_{\mu\nu\varrho\sigma}$ and R^2, the square of the scalar curvature.

The first term is not only Einstein invariant but also Weyl invariant, it is nothing but the Lagrangian of conformal gravity and coincides, up to a form-divergence with $R^2_{\mu\nu} - 1/3\ R^2$. In simple supergravity these two invariants admit a supersymmetric extension and vanish, as in pure gravity, on the supergravity mass-shell. The linearized part of these two invariants containing the spin 2 and 3/2 fields reads[2,3]

$$\mathcal{L}_1 = R^2_{\mu\nu} - \frac{1}{3} R^2 - \frac{2}{3} K^2 \bar{\psi}_\mu (\Box \not{\partial} \psi_\mu - \partial_\mu \not{\partial} \partial \cdot \psi + \frac{1}{2} \Box \gamma_5 \gamma_\nu \varepsilon^{\mu\nu\varrho\sigma} \partial_\varrho \psi_\sigma)$$

$$\mathcal{L}_2 = R^2 + 4 K^2 \bar{\psi}_\mu (\Box \delta_{\mu\nu} - \partial_\mu \partial_\nu) \gamma_\nu (\not{\partial} \gamma \cdot \psi - \partial \cdot \psi) \qquad (1)$$

The completion of $\mathcal{L}_1$ is the conformal supergravity action[3].

On the other hand it is known that the supersymmetric extension of the Einstein theory, whose (linearized) spin 2, 3/2 sector is

$$\mathcal{L}_E = - \frac{1}{2K^2} R - \frac{1}{2} \varepsilon^{\mu\nu\varrho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu \partial_\varrho \psi_\sigma \qquad (2)$$

exists up to N = 8 spin 3/2 fields (SO(N)-extended supergravity). The standard lore is that SO(N)-extended supergravity exists up to N = 8 because the higher helicity state cannot be lower than $\frac{N}{4}$ (N even) or $\frac{N+1}{4}$ (N add)[4]. For instance in N = 9 (SO(9) supergravity) one would obtain 1 spin-5/2 state, 10 spin-2 states, 120 spin-1 states, 210 spin-1/2 states and 250 spin-0 states.

It is amazing to observe that recently a consistent Lagrangian for free massless spin 5/2 fields has been constructed[5]. This Lagrangian has the only peculiar property that it is not the massless limit of a massive spin-5/2 Lagrangian.

The extension to SO(N) supergravity ($N \geq 2$) of the two invariants $\mathcal{L}_1$ and $\mathcal{L}_2$ is not known and the implication of their possible existence is reported here.

This discussion will cover completely some work done in collaboration with B. de Wit[6].

Our observation is the following: when terms quadratic in the curvature are added to the original Einstein action, then the theory describes three types of graviton states, a massless and a massive (ghost) spin 2-state as well as a massive spinless state[7]. Of course this analysis is independent of the presence of supersymmetry.

However in the case of supergravity, these states must be extended to fit in massless and massive supermultiplets. This has been analyzed explicitely for N = 1 supergravity[8] and in this case one has a massless spin (2, 3/2) multiplet, a massive spin (2, 3/2, 3/2, 1) multiplet (ghost) and two massive spin ($1/2$, $0^{\pm}$) multiplets. The spin 2 and 3/2 modes and two spin 1/2 modes come from the graviton and gravitino fields but the spin 1 mode and the remaining spinless modes come from the six auxiliary fields of supergravity[9].

We learn from this analysis that higher order derivative equations give us information on the auxiliary field structure of supermultiplets.

This fact was pointed out already in ref. 2 in the case of conformal supergravity and of a model of global supersymmetry.

It is clear that in SO(N) supergravity the massive graviton states must be assigned to massive multiplets which must contain particles of at least spin N/2. This observation immediately implies that for $N > 4$ the invariants in eq. (1) must describe propagating states with spin higher than 2. From this fact one is tempted to conclude, in analogy with the argument given in SO(N) supergravity for $N > 8$, that the invariants in eq. (1) do not have a supersymmetric completion for $N > 4$. We will discuss this observation later on.

We consider the coupling of the invariants $\mathcal{L}_1$, $\mathcal{L}_2$ (eq. (1)) to the supergravity Lagrangian $\mathcal{L}_E$ (eq. (2)) so that we obtain massive propagating states.

It is important to mention that in order to carry out this analysis we need a tensor calculus and therefore we assume the existence of a complete set of auxiliary fields for SO(N) - supergravity up to N = 8 (closure of the off-shell gauge algebra). This is because we make new invariants by taking linear combinations of separate invariants.

For convenience of the reader we report the list of the relevant massive representations of N-extended supersymmetry in Table I.

When we couple the invariant $\mathcal{L}_1$ to $\mathcal{L}_E$ we must extend the massive (2, 3/2, 3/2, 1) multiplet of simple to extended supergravity. It is clear from the Table that for $N \leq 4$ the spin 2 massive and massless representations contain precisely the right number of spin 2 and 3/2 states that are described by a singlet graviton and a N-plet of Majorana Rarita-Schwinger fields subject to their higher-derivative field equations. We can also eventually obtain a lower bound of propagating modes of a given spin present in $\mathcal{L}_1 + \mathcal{L}_E$. This will give some insight into the auxiliary field structure of extended supergravity.

TABLE I
Some massive representations of extended supersymmetry

	N = 1	N = 2	N = 3	N = 4	N = 5
5/2					1
2	1	1	1	1	10
3/2	1 2	1 4	1 6	8	44
1	1 2 1	1 4 6	6 15	27	110
1/2	1 2 1	4 6 4	14 20	48	165
0	2 1	5 4 1	14 14	42	132

For $N > 4$ the previous analysis completely breaks down (see Table I for $N = 5$).

Two alternative situations can be envisaged: either higher spin fields are propagating or there is no SO(N)-supersymmetric extension of $\mathcal{L}_1$ (and $\mathcal{L}_2$) for $N > 4$.

It seems that the second alternative is excluded by explicit calculations which show that the contribution of all particles of any spin to $\mathcal{L}_1$ (and $\mathcal{L}_2$) has always the same sign in quantum gravity and so one cannot make them to vanish. However it should be enphasized that these results are gauge-dependent and the chosen gauge is not supersymmetric. As a consequence these results cannot be used as they stand. Certainly there is something funny going on with supersymmetric gauge conditions. This is suggested by the puzzle on the axial anomaly in supersymmetric Yang-Mills theories in which, using together a symmetry argument and known results, contradictory conclusions are obtained[10].

The first alternative does not look natural either. This is because for $N \leq 4$ auxiliary fields with spin $S > 1$ seen not to be needed while for $N > 4$ they should have spin $S \geq 5/2$, giving a jump of at least three half-units of spin.

Moreover $\mathcal{L}_1$ is the superconformal action and it would require conformal fields with higher spin. For dimensional reasons it seems difficult to assign these fields to the same multiplet as the conventional ones. Finally conformal supergravity is the gauge theory of the superconformal group and it seems unlikely that it requires propagating fields with spin $S > 2$. Analogous considerations apply to the second invariant $\mathcal{L}_2$.

If the second alternative (absence of $\mathcal{L}_1$, $\mathcal{L}_2$) turns out to be true this would be extremely interesting because it would mean that there is a softening of quantum divergences by increasing N. This is precisely what happens in global supersymmetry for Yang-Mills theories with $N > 2$ in which the Callan-Symanzik function $\beta(g)$ vanishes at the first two-loop orders[11]. Furthermore, as a corollary, this would also imply that superconformal theories can exist with a U(N) gauge symmetry up to $N = 4$, in contrast with supergravity theories which exist with an SO(N) symmetry up to $N = 8$ [(x)].

Therefore superconformal theories, apart from ghost problems, are worse then Einstein-supergravity theories as far as a grand-unification scheme of fundamental interactions is concerned.

The second topic I would like to discuss is a general (quadratic) mass formula which holds in a wide class of global and local supersymmetric theories.

This result has been obtained in collaboration with L. Girardello and F. Palumbo[12].

The mass formula reads as follows

$$\sum_J (-)^{2J} (2J+1)\, m_J^2 = 0 \qquad (3)$$

where the sum is performed over each (real) particle state of spin J.

This formula has been derived in all global supersymmetric models with and without gauge invariance with an arbitrary self-interaction of the chiral matter multiplet of particle content $(0^{\pm}, 1/2)$.

In local supersymmetry (supergravity) the formula was already obtained in ref. 13 for the coupling of a chiral multiplet to the supergravity Lagrangian.

It is remarkable that this formula is also true with an explicit breaking provided it fullfills some conditions.

It is possible that eq. (3) can be derived on pure group theoretical grounds with a certain additional requirements on the breaking. This is suggested by the fact that (3) is nothing but the vanishing of

(x) This result has been obtained independently by M. Gell-Mann using group-theoretical methods.

the graded trace of the square mass operator of the theory.

As a matter of fact eq. (3) is trivially satisfied in absence of breaking stating the equal number of bosonic and fermionic degrees of freedom in each multiplet.

Let us first consider the self-interaction of a chiral multiplet $\Sigma^a = (z^a, \chi^a_L, H^a)$ with flavor index $a = 1, \ldots, N$. A flavor singlet function is

$$f(\Sigma) = \sum_{n=0}^{\infty} C_{a_1 \ldots a_n} \Sigma^{a_1} \ldots \Sigma^{a_n} \qquad (4)$$

$C_{a_1 \ldots a_n}$ are real symmetric functions.

The Lagrangian is given by the kinetic part added to the F component of $f(\Sigma)$

$$\mathcal{L}_{ss} = -\frac{1}{2}\partial_\mu z^a \partial_\mu z^{*a} - \bar{\chi}^a_L \not\partial \chi^a_R + \frac{1}{2} H^a H^{*a} + f_a(z) H^a +$$

$$+ f_a(z^*) H^{*a} - f_{ab}(z) \chi^a_L \chi^b_L - f_{ab}(z^*) \chi^a_R \chi^b_R \qquad (5)$$

By elimination of the auxiliary fields one gets the bosonic potential

$$V = 2 f_a(z) f_a(z^*) \qquad (6)$$

The extremum condition is

$$V_b = 2 f_{ab}(z) f_a(z^*) = 0 \qquad (7)$$

and the trace of the bosonic square mass is

$$\mathrm{Tr}\,\mathcal{M}^2_{ab} = 4 \frac{\partial}{\partial z^a} \frac{\partial}{\partial z^{*a}} V = 8 f_{ab}(z) f_{ab}(z^*) \qquad (8)$$

On the other hand the fermionic mass matrix is given by

$$-\frac{1}{2} M_{ab} \bar{\chi}^a \chi^b + \frac{1}{2} N_{ab}\, i \bar{\chi}^a \gamma_5 \chi^b \qquad (9)$$

$M_{ab} = f_{ab}(z) + f_{ab}(z^*)$, $N_{ab} = i(f_{ab}(z) - f_{ab}(z^*))$ are real symmetric matrices.

The trace of the square mass matrix is

$$\mathrm{Tr}\mathcal{M}^2_{Fab} = \mathrm{Tr}(M^2 + N^2) = 4f_{ab}(z)\, f_{ab}(z^*) \qquad (10)$$

From (8) and (10) we get

$$\sum_{J=0} m^2_B = 2 \sum_{J=1/2} m^2_F \qquad (11)$$

which is nothing but eq. (3).

Note that for $N = 1$ spontaneous symmetry breaking is impossible because the extremum condition implies $m^2_A + m^2_B = 0$. Then one of the scalar particles is a tachyon and the (non supersymmetric) solution is always unstable (unless $f_{zzz} = 0$ also so that $m_A = m_B = 0$).

We observe at this point that the mass formula (3) is still valid if we add to the Lagrangian (5) an explicit breaking, provided it satisfy

$$\frac{\partial}{\partial z^a} \frac{\partial}{\partial z^{*a}} \mathcal{L}_{BREAK} = 0 . \qquad (12)$$

Any function of the form $\mathrm{Re}\, g(z)$ where $g(z)$ is analytic in z^a would be a solution of (12).

Note that the Wess-Zumino model[14] with a soft breaking[15], for which the mass relation (3) was established, fullfills eq. (12) with $g(z) = z$.

We can extend these results to a gauge interaction with arbitrary self-interaction of chiral multiplet with, eventually, flavor quantum numbers.

Let us consider first the case in which the gauge group is U(1) and a chiral doublet Σ^a transforms as follows under the gauge group

$$\delta \Sigma_a = \Lambda \varepsilon_{ab} \Sigma_b \qquad (13)$$

The vector potential V is a pseudoscalar superfield transforming as

$$\delta V = \frac{i}{g} (\Lambda - \Lambda^*) \qquad (14)$$

and superfield notations have been used.

In this case the trace of the scalar mass matrix is

$$\mathrm{Tr}\,\mathcal{M}^2_{ab} = 4\,\mathrm{Tr}\,\frac{\partial^2}{\partial z^a\,\partial z^{*a}}\,V = g^2 z_a z^*_a + 8 f_{ab}(z) f^*_{ab}(z)\,. \qquad (15)$$

The square mass of the vector boson is $M^2_V = g^2 z_a z^*_a$ so that the trace of the bosonic square mass is

$$\mathrm{Tr}\,\mathcal{M}^2_{ab} + 3 M^2_V = 4 g^2 z_a z^*_a + 8 f_{ab} f^*_{ab}\,. \qquad (16)$$

The fermionic mass term has the form

$$-f_{ab}\chi_{aL}\chi_{bL} - f^*_{ab}\chi_{aR}\chi_{bR} - g\dot{\varepsilon}_{ab}(\lambda_L \chi_{aL} z^*_b + \lambda_R \chi_{aR} z_b) \qquad (17)$$

and λ is the spin - 1/2 particle belonging to the vector multiplet.

The trace of the fermionic square-mass matrix can be easily computed and gives

$$\mathrm{Tr}\,\mathcal{M}^2_F = \mathrm{Tr}(M^2 + N^2) + 2g^2 z_a z^*_a = 4 f_{ab} f^*_{ab} + 2g^2 z_a z^*_a\,. \qquad (18)$$

Comparing with (16) we finally get

$$\sum_{J=0} m^2_B + 3 m^2_V = 2 \sum_{J=1/2} m^2_F\,. \qquad (19)$$

In the non-abelian can the previous analysis goes unchanged provided the following substitutions are made

$$\mathrm{Tr}\,\mathcal{M}^2_{\text{scalar}} = 8 f_{ab} f^*_{ab} + C(R)\, z^a_\alpha z^{*a}_\alpha\,,$$

$$\mathrm{Tr}\,\mathcal{M}^2_{\text{vector}} = C(R)\, z^a_\alpha z^{*a}_\alpha\,, \qquad (20)$$

$$\mathrm{Tr}\,\mathcal{M}^2_F = 4 f_{ab} f^*_{ab} + 2\, C(R)\, z^a_\alpha z^{*a}_\alpha\,.$$

a is the color index, C(R) is the quadratic Casimir operator of the representation R to which the Σ^a belong and an extra index $\alpha = 1, 2$ is needed if R is not a real representation. One can check that models[16], constructed as an attempt to unify fundamental interactions, without axial gauge fields, have mass patterns that indeed fullfill eq. (3). Exception to the mass formula (3) are models with axial gauge invariance(*), the simplest of them being the so called supersymmetric Higgs model[17]. We observe however that these models are faced with the "axial anomaly" problem.

In the case of local supersymmetry (supergravity) eq. (3) has been established for a general class of interactions (with canonical kinetic terms) under the assumption of absence of cosmological term[13].

Remarkably it also holds in a model of SO(8) spontaneusly broken supergravity recently obtained by Scherk and Schwarz[18].

In local supersymmetry (supergravity) the absence of a cosmological term (12) is crucial for the validity of (3). This is clearly related to the structure of the global symmetry algebra.

For the class of models discussed in ref. 13 eq. (3) becomes

$$m_A^2 + m_B^2 = 4\, m_\psi^2 \qquad (21)$$

where A, B are the scalar (pseudoscalar) particles of the chiral multiplet. The spin 3/2 gravitino field ψ_μ has became massive through the super Higgs effect, by absorption of the spin 1/2 Goldstone fermion χ, and has therefore four helicity states.

In the SO(8) spontaneously broken supergravity theory of ref. 18 the eight gravitinos acquire a mass through the super Higgs mechanism. Also the usual Higgs mechanism for most of the twenty-eight vector bosons takes place. There is no induced cosmological constant in the resulting theory. All masses are calculated in terms of three arbitrary (real) parameters m_1, m_2, m_3.

In this model eq. (3) becomes

$$\sum_{J=0,1} (2J+1)\, m_J^2(\text{bosons}) = \sum_{J=1/2,\,3/2} (2J+1)\, m_J^2(\text{fermions}) \qquad (22)$$

and again it is satisfied.

(*) We thank P. Fayet for a discussion on this point.

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