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Several arrangements of magnets with alternate transverse horizontal and vertical fields have been proposed(1-3) as Siberian Snake configurations, and many have already been analyzed(2, 4, 5). Recently, Steffen(3) proposed a "snake" configuration with small orbit displacement, which has the following properties:

a) the set of vertical fields exhibits symmetry, and
b) the set of horizontal fields exhibits antisymmetry about its central point.

In Ref. (4) it was shown that such an arrangement does not make available a sufficiently wide energy range (in which depolarization can be prevented) under fixed geometry conditions. More importantly, in Ref. (5) it was shown that this configuration, when rotated by $\pi/2$ about the y (velocity) axis, can be used as a very efficient fixed-geometry snake. Since enquiries have been received about these conclusions, it was decided to calculate the effective precession wave number, ν, of the polarization vector as a function of the beam energy, E, for the Steffen's magnetic arrangement rotated by any angle, φ, around the y-axis. Thus, the purpose of this report is to present this generalized expression of $|\cos(\pi\nu)|^{(\varphi)}_{extr}$. The resulting final formula indicates that the $\pi/2$-rotated(5) configuration constitutes the best choice.

Using 2-component spinor algebra(2), we find for the element m_{11} of the transfer matrix through the "snake" insertion

$$\mathrm{Re}(m_{11}) = c(4)\, c(2) \left[c(4) + s^2(4)\right] \left[1 + 2s^2(4)\right] , \tag{1a}$$

and

$$\mathrm{Im}(m_{11}) = (1/2) s^2(8)\, s(1) \left[1 + 2s^2(4)\right] \cos\varphi , \tag{1b}$$

where

$$c(k) \equiv \cos(\phi/k)\,, \qquad s(k) \equiv \sin(\phi/k)\,, \qquad (k = 1;2;4;8) \tag{2a}$$

and

$$\phi = \pi E/E_o\,. \tag{2b}$$

Here, E_o is the reference energy. $\varphi = 0$ corresponds to the snake configuration as originally proposed by Steffen(3); $\varphi = \pi/2$ refers to the same arrangement, except that the V(H) magnets are replaced by H(V) magnets in it(5).

(To facilitate comparison with our previous work(4, 5), we note that the r.h.s. of Eq. (1a), the r.h.s. of A as given in Ref. (4) and the r.h.s. of Eq. (2) of Ref. (5) are three identical expressions. Besides, the r.h.s. of Eq. (1b), with $\varphi = 0$, is identical to the expression of B as given in Ref. (4). For $\varphi = \pi/2$, we have $m_{11} (= m_{22}) = (1/2)\mathrm{Tr}$, so that Eq. (3b) of Ref. (5) holds).

Since, at any φ, we have $m_{22} \equiv m_{11}^*$, $\left|\cos(\pi\nu)\right|_{extr}^{(\varphi)}$ is given by the equation

$$\left|\cos(\pi\nu)\right|_{extr}^{(\varphi)} = \sqrt{\mathrm{Re}^2(m_{11}) + \mathrm{Im}^2(m_{11})}\,. \tag{3}$$

Inspection of Eqs. (1) - (3) above readily reveals that at $\varphi = \pi/2$ the "snake" spoils the interaction which takes place in the main bending arcs.

Thus, in order to utilize fully the capability of any "snake", the following condition must be satisfied:

"symmetry of the horizontal fields pattern, and antisymmetry of the vertical fields pattern about the snake's midpoint".

References.

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(2) - B. W. Montague, LEP-70/76 (1978).
(3) - K. Steffen, DESY PET-78/11 (1978).
(4) - A. Turrin, LNF-78/54 (1978).
(5) - A. Turrin, LNF-78/59 (1978).