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RESONANT SPACE AND TIME-LIKE PION FORM FACTOR.

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ABSTRACT.

Excellent fit to the high quality data on pion form factor, which shows much structure up to 1.6 GeV, is obtained for which all 4 resonances $\varrho(776)$, $\varrho_A(1100)$, $\varrho'(1250)$ and $\varrho''(1540)$ are found essential. Both radiative corrections and interference between resonances are important for obtaining the visible structure. Our parametrization is also found to be in very good agreement with the space like data.

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With the advent of e^+e^- colliding beams, pion form factor in the time-like region were measured up to ~ 1 GeV by Orsay[1] and Novosibirsk[2] groups. These data were successfully "explained" in terms of the Gounaris-Sakurai formula using the ϱ-pole only[3]. More recent Novosibirsk high quality data[2] between (0.8 - 1.34) GeV and higher energy data (1.3 - 3) GeV from BCF[4], Frascati[5], DCI[6], DESY[7] and SPEAR[8] have shown that the simple ϱ-pole is quite inadequate in this region (See Fig. 1). On the other hand, various experiments have revealed a number of high mass vector mesons. In particular, $\varrho'(1250)$ and $\varrho''(1600)$ were first observed by the $\mu\pi$ group at ADONE[9]. Recent DCI experiments[10] have given evidence for $\varrho''(1550)$ in the 4π channel, thus confirming the earlier Frascati result. Also, Frascati-DESY collaboration[11] has confirmed the existence of $\varrho'(1250)$. Morover, the latter group[11] has found a new resonance (through its e^+e^- decay) - which we call ϱ_A - at a mass of 1100 MeV and with a width ~ 30 MeV. Its hadronic decays are as yet unknown. In the present work we analyse the time-like pion form factor under the hypothesis that all of the above resonances are coupled to it. Our object is not so much to find the best values for the masses and partial widths etc. of these resonaces - in fact, we use published values of these parameters whenever available - but rather to show that these resonances are <u>necessary</u> to explain the above data. For this analysis, radiative corrections are very important (especially due to the radiative tails) and rather delicate when there are interfering resonances. For this purpose, we have used the formalism developed in ref. (12). Our form factor is then continued into the space-like region and compared successfully with the existing data.

The resonance contribution to the form factor $F_\pi(s)$ has been parametrized as follows

$$F_\pi(s) = \sum_i \frac{\varepsilon_i m_i^2 e^{i\phi_i}}{m_i^2 - s - i\left(\frac{s}{m_i^2}\right) m_i \Gamma_i} , \qquad (1)$$

where m_i and Γ_i are the mass and width respectively of the i^{th} resonance and ε_i is proportional to the product of the resonance coupling to the photon and to two-pions. The arbitrary phases ϕ_i are chosen, for simplicity, to be zero. As we shall see later the quality of our fit (with $\phi_i = 0$) is very good. Thus, the introduction of a host of extra parameters ϕ_i was deemed superfluous for our analysis. With the above assumption, each new resonance is introduced into F_π with only 1 unknown parameter ε_i.

In a scattering process such as this one, which proceeds via the formation of resonances, the cross section is very sensitive to an energy loss in the initial state, thus requiring special treatment of the radiative corrections. These are discussed in detail in ref. (12). We simply quote below the multiplicative factors which account for these corrections.

Let us call R_i and I_i the real and imaginary parts of the i^{th} resonance contribution to $F_\pi(s)$. For 3 resonances only, i.e. ϱ, ϱ_A, ϱ' we have explicitly

$$\begin{aligned}
\left|F_\pi\right|^2 = & \ (R_\varrho^2 + I_\varrho^2) \\
& + (R_{\varrho_A}^2 + I_{\varrho_A}^2)(\sin\delta_{\varrho_A})^{-\beta_e}\left[1 + C_{Res}(\varrho_A)\right] \\
& + (R_{\varrho'}^2 + I_{\varrho'}^2)(\sin\delta_{\varrho'})^{-\beta_e}\left[1 + C_{Res}(\varrho')\right] \\
& + 2R_\varrho R_{\varrho_A}(\sin\delta_{\varrho_A})^{-\beta_e}\left[1 + C_{INT}(\varrho_A)\right] \\
& + 2I_\varrho I_{\varrho_A}(\sin\delta_{\varrho_A})^{-\beta_e}\left[1 + C_{Res}(\varrho_A)\right] \\
& + 2R_\varrho R_{\varrho'}(\sin\delta_{\varrho'})^{-\beta_e}\left[1 + C_{INT}(\varrho')\right] \\
& + 2I_\varrho I_{\varrho'}(\sin\delta_{\varrho'})^{-\beta_e}\left[1 + C_{Res}(\varrho')\right] \\
& + 2R_{\varrho_A}R_{\varrho'}(\sin\delta_{\varrho_A})^{-\beta_e}(\sin\delta_{\varrho'})^{-\beta_e}\left[1 + C_{INT}(\varrho_A) + C_{INT}(\varrho')\right] \\
& + 2I_{\varrho_A}I_{\varrho'}(\sin\delta_{\varrho_A})^{-\beta_e}(\sin\delta_{\varrho'})^{-\beta_e}\left[1 + C_{Res}(\varrho_A) + C_{Res}(\varrho')\right],
\end{aligned} \tag{2}$$

where

$$C_{Res}(i) = \beta_e \frac{(W - m_i)}{(\Gamma_i/2)} \delta(i) \ , \quad (3a)$$

$$C_{INT}(i) = - \beta_e \frac{(\Gamma_i/2)}{(W - m_i)} \delta(i) \ , \quad (3b)$$

$$\tan \delta(i) = \frac{(\Gamma_i/2)}{(W - m_i)} \ , \quad (4)$$

$$\beta_e = \frac{4\alpha}{\pi} (\ln \frac{W}{m_e} - \frac{1}{2}) \ . \quad (5)$$

Here $W = \sqrt{s}$ is the total CM energy and m_e the mass of the electron.

In the above formulae, we have made no radiative correction for the ϱ-term, because it is rather broad and its parameter ε_ϱ has already been determined using the low-energy data ($W \lesssim 0.8$ GeV)(1). On the other hand, as stated earlier, the radiative tails of the other resonances (ϱ_A, ϱ', etc.) have been included and their interferences have been radiatively corrected. The generalization to more resonances is straightforward.

We have used C_{INT} factor for the correction to the interference between the real parts of two resonances. This is exactly what happens when one considers the interference between the QED term (which is real) and a resonance(12). For the interference between the imaginary parts of two resonances, however, we have used C_{Res} factors, since this case is similar to the resonance term itself. Of course, the formulae presented in eqs. (2) and (3) are strictly correct only for the energy loss $\Delta\omega \gg \Gamma$, the width of the resonance under consideration, which is not the case here.

In Fig. 1 the data on $|F_\pi|^2$ are presented from (0.8 - 2) GeV(2,4,5,6). The phenomenological fit to the data includes 4 resonances, ϱ, ϱ_A, ϱ' and ϱ'' with their quoted mass and widths(10, 11) :

$$m_\varrho = 0.776\,\text{GeV};\quad m_{\varrho_A} = 1.1\,\text{GeV};\quad m_{\varrho'} = 1.266\,\text{GeV};\quad m_{\varrho''} = 1.54\,\text{GeV}$$
$$\Gamma_\varrho = 0.155\,\text{GeV};\quad \Gamma_{\varrho_A} = 0.03\,\text{GeV};\quad \Gamma_{\varrho'} = 0.11\,\text{GeV};\quad \Gamma_{\varrho''} = 0.22\,\text{GeV}\,. \tag{6}$$

The parameter ε_ϱ is taken to be 1.08[1]. The remaining 3 parameters have been varied to fit the data. Our best values are

$$\varepsilon_{\varrho_A} = -0.02\,;\qquad \varepsilon_{\varrho'} = -0.07\,;\qquad \varepsilon_{\varrho''} = -0.06\,. \tag{7}$$

This fit is shown as the solid line in Fig. 1, while the ϱ-tail (dashed curve) is also shown for comparison. As is clear from the figure, the fit is very good (χ^2/d.f. $\simeq$ 1) for energies up to $\simeq$ 1.6 GeV. Beyond this energy, we need some other mechanism to keep the form factor above the ϱ-level. In the spirit of the Regge recurrances of the vector mesons, it is quite natural to assume that there exist other resonances like ϱ', ϱ'', etc. In fact, the trend of the data seem to favor this possibility. To illustrate this idea, let us postulate the existence of a ϱ''' at a mass of $\simeq$ 1.82 GeV and a width ($\simeq$ 0.22 GeV) comparable to ϱ''. The contribution of such a term with $\varepsilon_{\varrho'''} \simeq -0.04$ is also shown in Fig. 1 (dotted curve). This type of argument based on duality suggests that for (atleast moderately) large s, $|F_\pi(s)|$ should roughly be twice the ϱ-value, in rough accord with the high energy data. Just as through the ψ-decay it was possible to determine $|F_\pi|$ at 3.1 GeV[7,8], it would be very interesting to obtain $|F_\pi|$ at $\sim$10 GeV using the Υ-decay and thus test our "asyptotic" estimate.

While $\varrho'(1250)$ and $\varrho''(1540)$ are quite natural to be included in our analysis, one might question the necessity of $\varrho_A(1100)$ - especially since it does not seem to belong to any spectroscopic model which we are aware of. To investigate this point, we show on top of Fig. 1, on an expanded scale, the fit to $|F_\pi|^2$ with and without the ϱ_A term. Without this term, the curve provides a good average about the data point (missing however 6 out of 11 points between (1.0 - 1.2) GeV).

On the other hand the data seem to indicate an interference behavior in remarkable accord with our curve with the ϱ_A term included. This type of interference pattern is typical (e.g. ϕ in $e^+e^- \rightarrow \mu^+\mu^-$) of a resonance weakly coupled to both initial and final states.

A better way to appreciate the quality of the fit is to consider an energy integral of $|F_\pi|^2$ which eliminates the local fluctuations due to the radiative effects as well as the resonances. Such a comparison is shown in Fig. 2. The circles are the experimental points, the solid line is our curve and the dashed line shows the ϱ-term alone. Since the DCI and BCF group points are in agreement with those of Novosibirsk, in performing the integral we have used the Novosibirsk values.

It is clearly seen from Fig. 2 that the local fluctuations in $|F_\pi|^2$ due to resonances and their interferences have completely disappeared in this integral and also the relative error is considerably reduced ($\sim$5 %) due to the summation. Again, ϱ-term alone is ruled out. The excellent agreement between our expression for this integral (the solid curve) and the integrated experimental values (circles) leads us to conclude that the chosen couplings of these resonances are indeed the right ones. Of course, as more higher mass resonances will be added, their exact values will change, but they will remain of the same order of magnitude.

Having fixed all the parameters in eq. (1), it is quite natural to do a continuation into the space-like region $Q^2 = -s \geq 0$ and study $F(Q^2)$. Since the energy dependence of Γ_i are not determined, we shall follow the standard procedure of neglecting the widths (for $Q^2 > 0$) and normalizing the form factor to 1 at $Q^2 = 0$. We have explicitly then

$$F_\pi(Q^2) = \frac{\sum_i \varepsilon_i \left(\frac{m_i^2}{m_i^2 + Q^2}\right)}{\sum_i \varepsilon_i} \qquad (8)$$

In Fig. 3, we have plotted $Q^2F_\pi(Q^2)$ vs. Q^2 and compared with the available data[7, 13, 14, 15] and the asymptotic ϱ-value. The agreement is surprisingly good especially since there are no free parameters and thus it is a true prediction. We would like to take this as providing support for our resonance - dominated form factor.

The pion charge radius $\langle r_\pi^2 \rangle^{1/2}$ can be computed using eqs. (6), (7) and (8). We find $\langle r_\pi^2 \rangle^{1/2} \simeq 0.69$ fermi, to be compared to the data $\langle r_\pi^2 \rangle^{1/2} \simeq (0.7 - 0.8)$ fermi[14, 16, 17, 18].

In view of this level of agreement between our model and experiment for $Q^2F_\pi(Q^2)$, we believe that atleast some of the quoted experimental values of $\langle r_\pi^2 \rangle^{1/2}$ are too large by $\sim (10 - 15)\%$[14, 16, 17, 18]. We also refer the reader to another curious point. It appears that the "asymptotic" value of $Q^2F_\pi(Q^2)$ is approximately 2/3 of the ϱ-term and that this value is already reached for $Q^2 \simeq 4$ GeV^2. Simple analyticity arguments require that also in the time-like region, $s\,|F_\pi(s)|$ equal this value for large s. Such does not seem to be the case for the experimental data till $s \simeq 9$ GeV^2. We interpret it to mean that there are other (weakly coupled) resonances whose local presence and interference with the ϱ-term keeps the data above the ϱ-value and only much later does it fall back to its (space-like) asymptotic value. In this respect the measurement of F_π at the Υ-resonace will be most crucial.

In conclusion, we have presented a model for the pion form factor containing 4-resonances ϱ, ϱ_A, ϱ' and ϱ'' for an analysis of the data till $W \simeq 1.6$ GeV. The fit is very good and practically reproduces all the local structures visible in the data. Radiative corrections appropriate for resonances is quite essential for the quality of this fit. Our expression then allows us to make "asymptotic" estimates for the time-like region using duality as well as absolute predictions for the space-like region. We find excellent agreement with the space-like data. We hope that soon high quality data of the type presented by DCI and Novosibirsk will also be available at energies just beyond 1.5 GeV through high luminosity machines like ALA at Frascati.

REFERENCES.

(1) - D. Benaksas et al., Phys. Letters 39B, 289 (1972).
(2) - A. D. Bukin et al., Phys. Letters 73B, 226 (1978).
(3) - G. Gounaris and J. J. Sakurai, Phys. Rev. Letters 21, 244 (1968).
(4) - D. Bollini et al., Lett. Nuovo Cimento 14, 418 (1975).
(5) - B. Esposito et al., Phys. Letters 67B, 239 (1977).
(6) - G. Cosme et al., Phys. Letters 63B, 349 (1976).
(7) - B. H. Wiik and G. Wolf, DESY 77/01 (1977).
(8) - F. Vannucci et al., Phys. Rev. D15, 1814 (1977).
(9) - M. Conversi et al., Phys. Letters 52B, 493 (1974); G. Barbarino et al., Lett. Nuovo Cimento 3, 689 (1972).
(10) - G. Penso, Invited talk at "Topical Meeting on e^+e^- Annihilation at Medium Energies", May 18-19, 1978, Frascati (Italy).
(11) - S. Bartalucci et al., Frascati Report LNF-78/24 (1978), submitted to Nuovo Cimento.
(12) - M. Greco, G. Pancheri-Srivastava and Y. Srivastava, Nuclear Phys. B101, 234 (1975).
(13) - C. J. Bebek et al., Phys. Rev. D13, 25 (1975).
(14) - G. Adylov et al., Phys. Letters 51B, 402 (1974).
(15) - G. Wolf, Proceedings of the 1975 Intern. Symposium on Lepton and Photon Interactions at High Energies, Stanford, Ca. (USA).
(16) - G. Bardin et al., Nuclear Phys. B120, 45 (1977).
(17) - A. Del Guerra et al., Phys. Letters 50B, 487 (1974).
(18) - S. F. Berezhnev et al., Yad. Fiz. 18, 102 (1973).

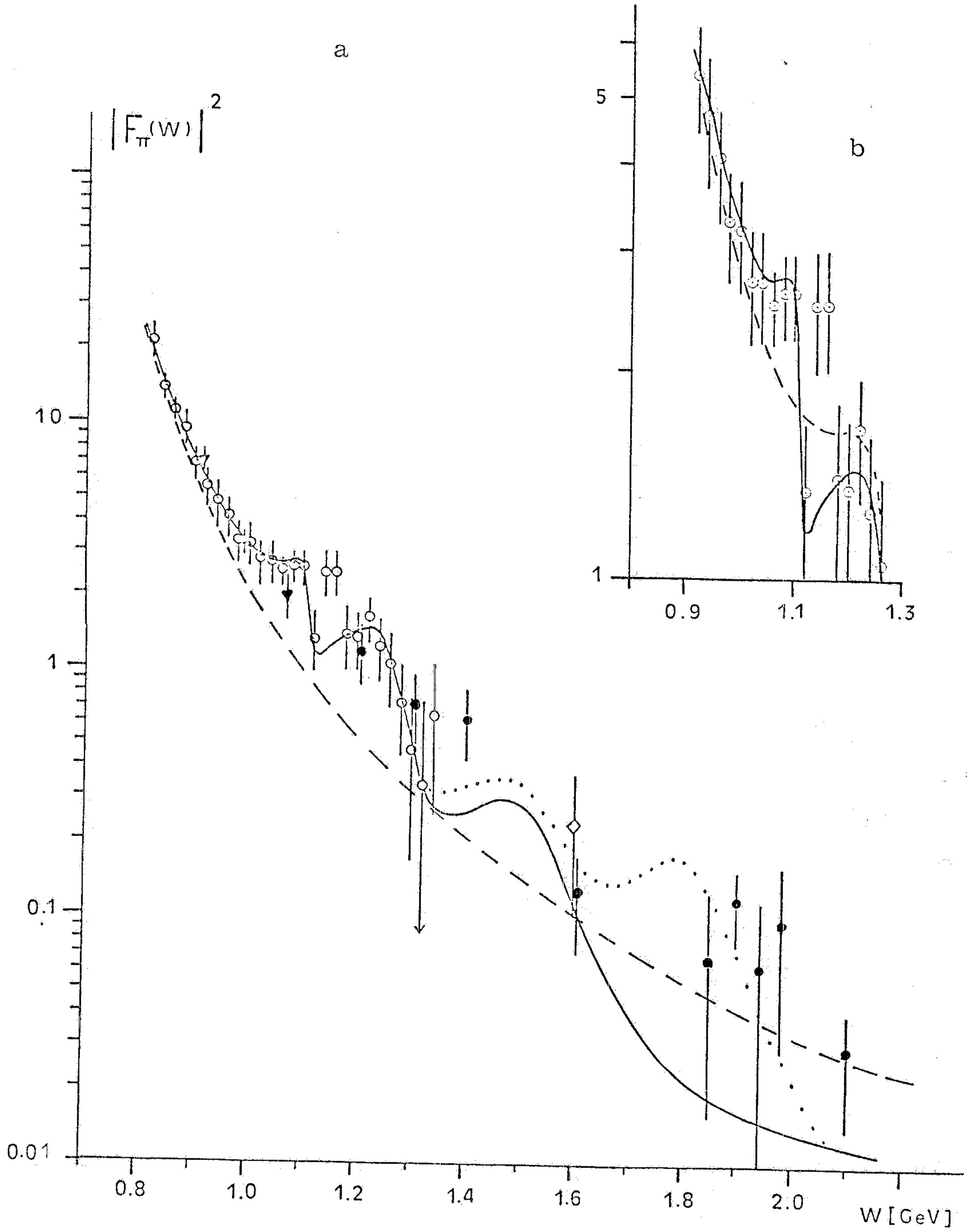

FIG. 1 - (a) Solid line is the theoretical curve and dashed curve is the ϱ-term alone. Dotted curve (....) is with ϱ''' added (see text). Open circles (ϕ) are Novosibirsk data(2); triangles ($\blacktriangledown$) are DCI data(6); Solid circles ($\bullet$) are Frascati BCF(4) and diamonds ($\diamond$) are from Frascati MEA(5).

(b) Fit to $|F_\pi|^2$ with (solid line) and without (dashed line) the ϱ_A term.

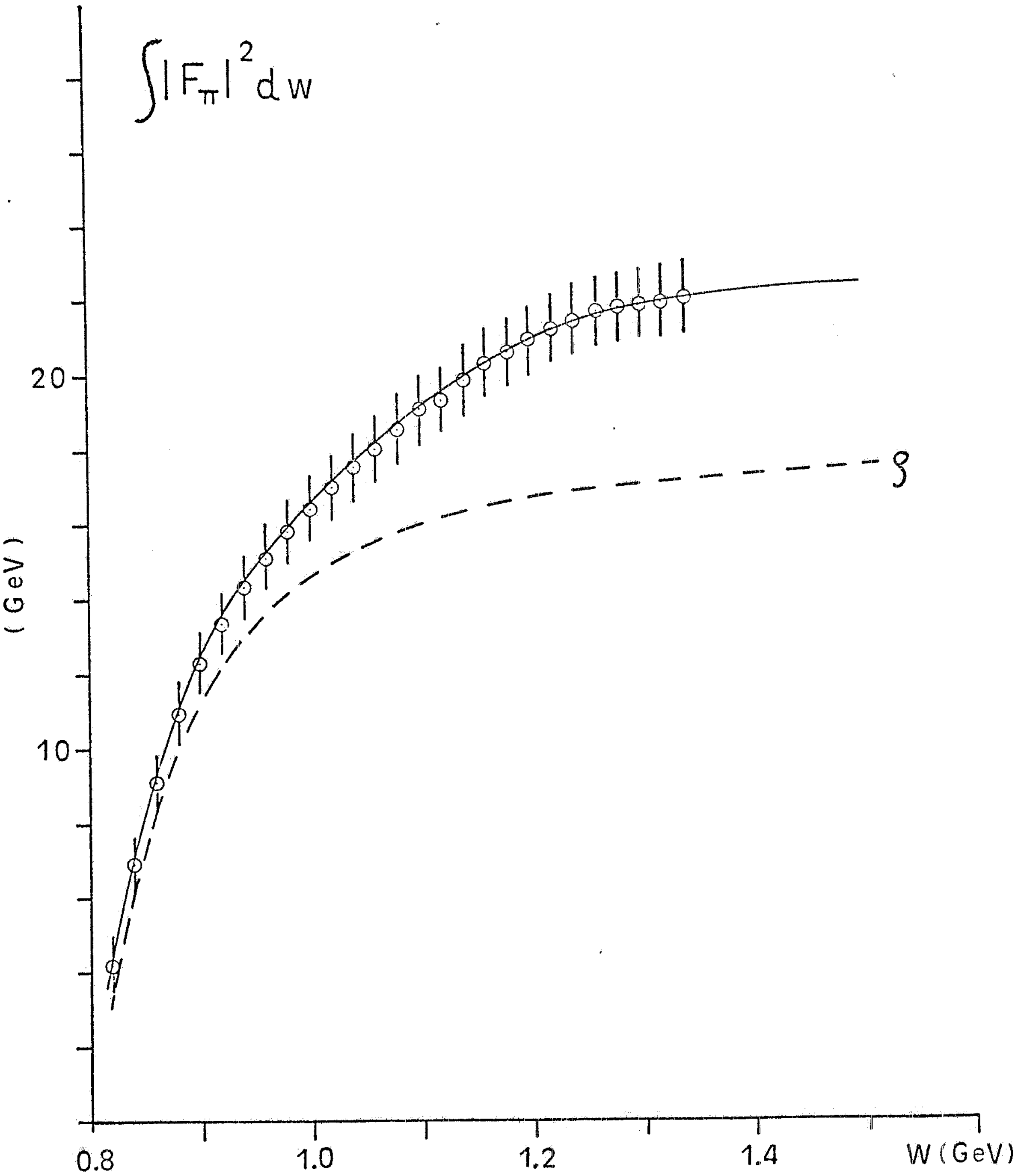

FIG. 2 - $\int |F_\pi|^2$ dW vs. W. Solid line is the theoretical curve and the circles are integrals over the experimental values from ref. (2). Dashed curve is the integral of the ϱ-term.

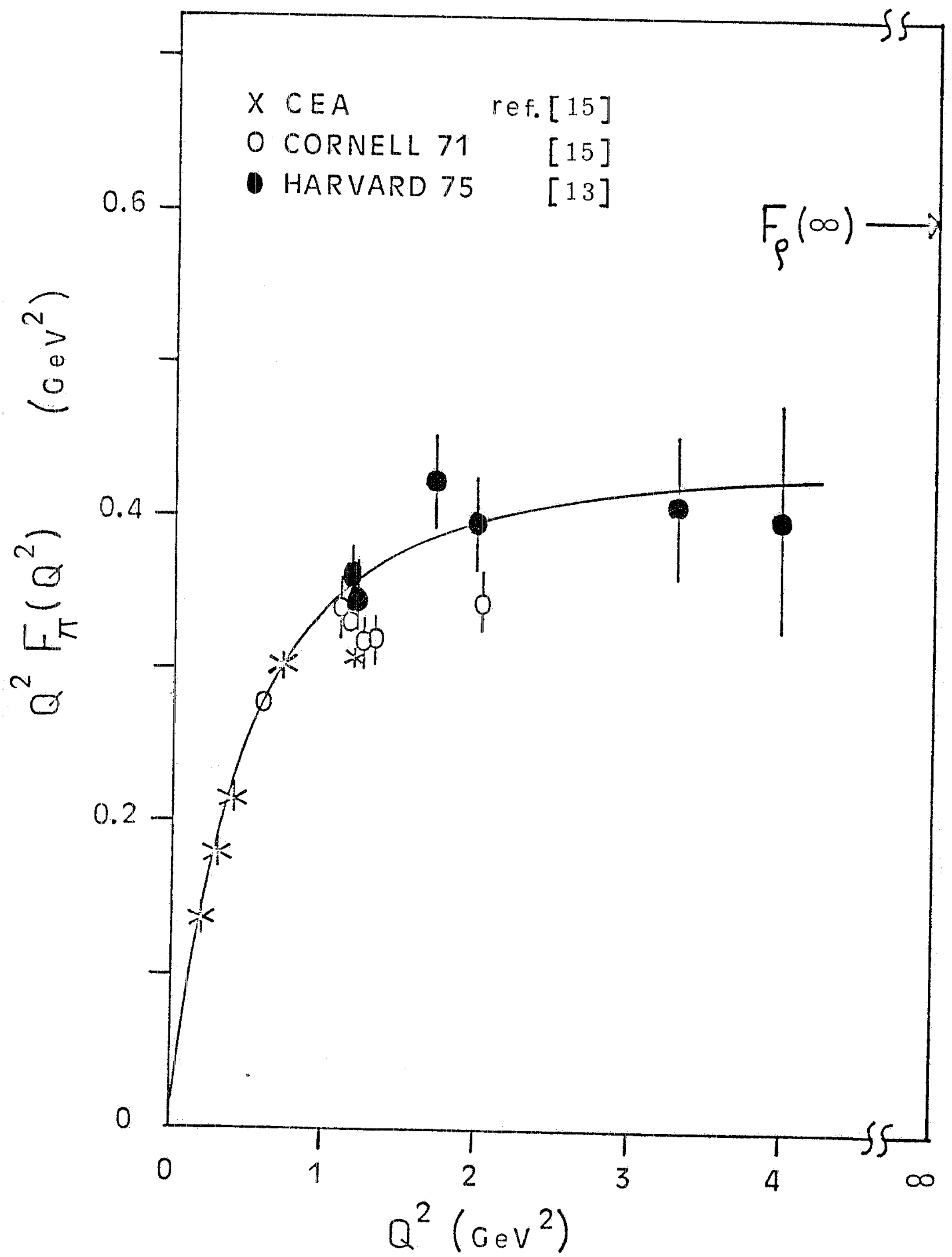

FIG. 3 - $Q^2F_\pi(Q^2)$ vs. Q^2. Solid line is our prediction eq. (8). The symbol $F_\pi(\infty)$ refers to the asymptotic value ($\simeq$ 0.6 GeV^2) given by the ρ-term alone.

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