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SPIN-DEPENDENT POTENTIALS IN THE FERMION–ANTIFERMION SYSTEM

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The annihilation width of the fermion–antifermion system into one photon is divergent to order m^{-2}. It is shown that this divergence can be removed by properly constraining the strengths of the spin-dependent potentials.

There are many fermion–antifermion (f–a) systems, like e^+e^-, $N\bar{N}$ [1], $c\bar{c}$ [2] and the new heavy quark–antiquark [3], which are interesting to consider in a non-relativistic approximation. When the non-relativistic hamiltonian is derived from a relativistic theory, the potential is purely central and behaves like r^{-1} at the origin, apart from possible logarithms [4]. In this fully non-relativistic approximation the theory of the f–a system is just the same as the theory of the fermion–fermion (f–f) system.

Spin-dependent potentials appear in both theories as relativistic corrections of order m^{-2}, m being the mass of the fermion and antifermion [4]. These spin-dependent potentials are not positive-definite and singular at the origin like $m^{-2}r^{-3}$ or $m^{-2}\delta(r)$, so that the hamiltonian is not bounded from below.

The standard attitude in atomic physics is to consider these spin-dependent potentials as small perturbations, to be treated only in first order perturbation theory. This attitude is not tenable when spin-dependent potentials have large strengths, as is the case for the N–N and $c\bar{c}$ interactions. In nuclear physics the problem is handled by an ad hoc modification of the potential at short distance, done in such a way as to fit the experimental data, with the assumption that nothing should critically depend on the detailed behaviour of the potential at the origin. In this respect the situation is very different for the f–a system, owing to the fact that its annihilation widths are determined by the behaviour at the origin of the wave function which critically depends on the detailed behaviour of the potential. The situation for the f–a system is made more delicate by the recent observation [5] that relativistic corrections of order m^{-2} to the annihilation width are divergent, even with potentials behaving like r^{-1} due to the anomalous behaviour of the coupled partial waves in the presence of a tensor potential. If one wants to include spin effects in the non-relativistic theory, one must include relativistic terms in the hamiltonian, and ask that the hamiltonian be bounded from below and the annihilation widths be non-divergent and non-vanishing.

In the following we will show that a hamiltonian satisfying the above requirements can be constructed by properly constraining the strengths of the different parts of the potential. We will then argue how such constraints can arise in the reduction from a relativistic theory.

Let us start with the most general local hamiltonian

$$H = p^2/2m + U_C(r) + U_\sigma(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + U_T(r)S_T + U_{LS}(r)\boldsymbol{L}\cdot\boldsymbol{S}, \tag{1}$$

where S_T is the standard tensor operator, $\boldsymbol{L}$ the orbital angular momentum operator, and $\boldsymbol{S} = \frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$ the total spin. If some of the potentials are more singular than r^{-2}, either the wave functions vanish at the origin faster than any power, giving rise to vanishing annihilation widths, or the hamiltonian is unbounded from below. We are therefore left with potentials singular like r^{-2} or less, and will discuss for simplicity only the case r^{-1}, which is the one physically relevant:

$$U_\alpha(r) = V_\alpha r_0/r + W_\alpha + ..., \quad (2)$$

where α stands for C, σ, LS, T. Other non-pathological behaviour can be analyzed in a similar way.

One can check that to order m^{-2} only the width of the state $L = 0, 2, S = J = 1$ is divergent. The annihilation current for such a state is

$$j_k = -\sqrt{2}\left(1 + \frac{1}{3}\frac{p^2}{m^2}\right)\chi_k + \frac{1}{\sqrt{2}}\frac{1}{m^2}\sum_{i=1}^{3}(p_i p_k - \tfrac{1}{3}\delta_{ik})\chi_i, \quad (3)$$

where χ_k are the annihilation operators for a spin triplet. The annihilation matrix elements are [6]

$$\langle 0|j_k|\psi_i\rangle = \delta_{ik}\left\{-\frac{1}{\sqrt{2\pi}}\left[R_S(0) - \frac{1}{m^2}R_S''(0)\right] - \frac{5}{4\sqrt{\pi}m^2}R_D''(0) + \Delta\right\}, \quad (4)$$

where R_S and R_D are the S, D radial functions and

$$\Delta = \frac{1}{6\sqrt{\pi}m^2}\lim_{r\to 0}\{2\sqrt{2}r^{-1}R_S'(r) - 2\sqrt{2}R_S''(r) - 3r^{-2}R_D(r) + \tfrac{3}{2}R_D''(r) - 5r^{-1}R_D'(r) + 5R_D''(r)\}. \quad (5)$$

We are now going to evaluate the quantities appearing in eqs. (4) and (5). R_S and R_D satisfy the equations:

$$\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - mU_0(r) + mE\right]R_S - \sqrt{8}\,mU_T(r)R_D = 0,$$

$$\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{6}{r^2} - mU_2(r) + 2mU_T(r) + mE\right]R_D - \sqrt{8}\,mU_T(r)R_S = 0, \quad (6)$$

where E is the energy and

$$U_0(r) = U_C(r) + U_\sigma(r), \quad U_2(r) = U_0(r) - 3U_{LS}(r). \quad (7)$$

Eqs. (6) have two linearly independent solutions:

$$\rho_S^{(1)}(r,E) = 1 + a_S^{(1)}r + b_S^{(1)}r^2 + ...,$$

$$\rho_D^{(1)}(r,E) = a_D^{(1)}r + b_D^{(1)}r^2 + c_D^{(1)}r^2\ln r + ..., \quad (8)$$

$$\rho_S^{(2)}(r,E) = d_S^{(2)}r^3 + ..., \quad \rho_D^{(2)}(r,E) = r^2 + ...$$

Let us explicitly note that there is not a term $r^2\ln r$ in $\rho_S^{(1)}$, and that $\rho_D^{(1)}$ and $\rho_S^{(2)}$ have an anomalous behaviour with respect to pure D and S waves.

The physical solutions to eqs. (6) are those superpositions of the above solutions which vanish at infinity:

$$R_S^{(i)}(r) = \sum_{k=1}^{2}\alpha_k^i\rho_S^{(k)}(r,E_i), \qquad R_D^{(i)}(r) = \sum_{k=1}^{2}\beta_k^i\rho_D^{(k)}(r,E_i), \qquad i = 1,2. \quad (9)$$

We see that the contributions to Δ coming from $\rho_S^{(2)}$ and $\rho_D^{(2)}$ are finite, but the contributions coming from $\rho_S^{(1)}$ and $\rho_D^{(1)}$ are divergent unless

$$a_S^{(1)} = 2\sqrt{2}\,a_D^{(1)}, \quad c_D^{(1)} = 0. \quad (10)$$

If the above conditions are satisfied, $\Delta = 0$.

Following ref. [7] we can now express $a_S^{(1)}$, $a_D^{(1)}$ and $c_D^{(1)}$ in terms of the parameters of the potentials. Putting

$$T = 1/mr_0^2, \quad (11)$$

we have

$$a_S^{(1)} = \tfrac{1}{2}V_0/T, \quad a_D^{(1)} = -\tfrac{1}{4}\sqrt{8}\,V_T/T,$$
$$c_D^{(1)} = (\sqrt{8}/5T^2)[\tfrac{1}{2}V_0V_T - \tfrac{1}{4}V_T(V_2 - 2V_T) + TW_T]. \quad (12)$$

Eqs. (10) and (12) give

$$V_T = -\tfrac{1}{4}V_0, \quad (13)$$

$$V_{LS} = -\tfrac{1}{6}V_0 + \tfrac{16}{3}TW_T/V_0. \quad (14)$$

It should be noted that if $V_0 \neq 0$, $U_0(r) \sim U_T(r) \sim r^{-1}$, while if $V_0 = 0$, $U_0(r) \sim$ const. but $U_T(r) \sim r$. Moreover, if $W_T = 0$, as is usually assumed, and $V_0 \neq 0$, both V_T and V_{LS} are determined in terms of V_0.

It is perhaps worthwhile emphasizing that eqs. (13) and (14) hold true for any f–a system, provided the potentials behave like r^{-1} at the origin. Logarithmic modifications of the potentials, however, can be important and can drastically change eq. (14), so making a distinction between the $N\bar{N}$ and e^+e^- on one side, and the $c\bar{c}$ on the other.

It could be interesting to generalize the above procedure to the (f–a)(f–a) system, and to the case of a system which is a superposition of (f–a) + (f–a)(f–a).

Eqs. (13) and (14) show the non-perturbative character of spin-dependent potentials. Indeed according to the standard expansion in inverse powers of the mass, $V_C \sim m^0$ and $V_T \sim m^{-2}$, so that the divergence due to the D wave is of order m^{-4} and should be omitted in the present calculation. In such a case, the divergence of the S wave, which is of order m^{-2}, could not be removed.

The above result implies that the standard expansion in inverse mass is not permissible. A possible reason is that calculating the annihilation width in the non-relativistic approximation we need both limits $m \to \infty$ and $r \to 0$, and ambiguities can arise in the product mr. According to the uncertainty principle [‡1] $mr \sim 1$ in the annihilation process so that terms like $m^{-2}\delta(r)$ should disappear while terms like $m^{-2}r^{-3}$ should actually behave as r^{-1} renormalizing in a non-perturbative way the strength of the potentials.

We would like to mention that the interaction derived by Schnitzer [4], for $mr = 1$, satisfies eq. (13) for all the values of its parameters, but does not satisfy eq. (14) for the values of the parameters used in the calculations, where $W_T = 0$.

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‡1 This point was suggested by V.I. Zakharov.

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