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LEPTOPRODUCTION AND DRELL-YAN PROCESSES BEYOND THE LEADING APPROXIMATION IN CHROMODYNAMICS *

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The problem of going beyond the leading logarithmic approximation in QCD for leptoproduction and Drell-Yan processes is considered. All the coefficient functions for leptoproduction are evaluated to order $\alpha_s(Q^2)$ (apart from two-loop corrections to logarithmic exponents). Existing calculations are thus completed and in part corrected. Particular attention is given to the constraint imposed by the validity at all Q^2 of the Adler sum rule. The question of a convenient definition of effective parton densities appropriate at this level of accuracy is discussed. Phenomenological consequences for leptoproduction are considered with special emphasis on the problem of extraction from the data of the small sea densities which are particularly sensitive to the corrections. The modifications of the Drell-Yan formula relevant for proton-nucleus processes are also explicitly calculated to order $\alpha_s(Q^2)$.

1. Introduction

The gauge theory of colored quarks and gluons (QCD) ** is at present the best candidate for a fundamental theory of the strong interactions. The asymptotic freedom of QCD offers a natural explanation for the approximate validity of Bjorken scaling in the deep inelastic region of leptoproduction. However even in this limit

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** See, for example, the reviews of ref. [1].

QCD predicts [2] a definite pattern of logarithmic scaling violations which can be computed in a perturbative expansion in $\alpha_s(Q^2)$, the logarithmically decreasing effective coupling constant. These firm predictions provide us with the most promising possibility of a quantitative test of QCD. In leading order in $\alpha_s(Q^2)$ the predictions of QCD for scaling violations have been completely derived [2] and their comparison with experiments appear to be reasonably satisfactory. However, there are several reasons why the inclusion of the next order corrections in $\alpha_s(Q^2)$ is now opportune. In the present range of Q^2, $\alpha_s(Q^2)$ is not sufficiently small to make the lowest order approximation as precise as desired. On the other hand, if the theory is correct, the scaling violations are decreasing with $\alpha_s(Q^2)$, so that analysis of the non-scaling effects for a given precision of the data forces one to work with moderate values of Q^2.

To lowest order in $\alpha_s(Q^2)$ the structure functions are given by the results of the naive parton model re-expressed in terms of effective Q^2 dependent parton densities that evolve in Q^2 with derivatives linear in $\alpha_s(Q^2)$ [3,4]. In the next order in $\alpha_s(Q^2)$ various effects arise. First the form of $\alpha_s(Q^2)$ is modified in a known way. Secondly, the naive parton model expressions for the structure functions change by terms of order $\alpha_s(Q^2)$. Finally the derivatives of the effective parton densities are corrected by terms of order $\alpha_s^2(Q^2)$. All effects are equally important. The calculation of the two-loop corrections to the logarithmic exponents, relevant to the last of the above effects, was started in ref. [5] and is presently being completed. In this paper we consider the corrections of order $\alpha_s(Q^2)$ to the naive parton model expressions for the structure functions. In fact the existing computations [6–8] are not complete and we found that they contain errors *. In particular our results are in agreement with the validity of the Adler sum rule at all Q^2, whereas the results of refs. [6,7] are not **.

The explicit form of the corrections depends on the exact definition of effective parton densities beyond the leading approximation. We discuss this problem in detail and propose a definition that appears to us as the most convenient for phenomenology. We then discuss the applications of the improved forms for the structure functions. When fitting parton densities from the data at moderate values of Q^2 the corrections are found to be important in some cases. In particular the non-leading terms are crucial for a determination beyond a purely qualitative level of the small sea densities (and of the gluons). At present Q^2 values the corrections are of the same magnitude as the sea densities themselves so that in this case the complications of the second-order formulas cannot be avoided.

In the last part of the present paper we deal with the related problem of the next order corrections to the Drell-Yan formula [9] for lepton pair production in hadro-

* We shall see that the corrections depend on the definitions adopted. The definitions which we use (massless quarks, off shell partons) are those of ref. [6], apart from their use of the Landau gauge, whereas in refs. [7,8] massive quarks and on shell partons are used.

** In ref. [8] the relevant coefficients are not considered.

nic collisions. These terms are *a priori* of particular importance for proton nucleus scattering where the leading quark-antiquark term is small, being proportional to sea densities. We therefore compute the corrections arising from quark-gluon and quark-antiquark contributions and discuss their quantitative effects.

2. Definition of the coefficient functions

In this section we specify our notation and the definition of the coefficient functions which will be calculated in sect. 3.

We consider the well-known structure functions * of deep inelastic leptoproduction $F_i(x, Q^2)$, $(i = 1, 2, 3)$. Q^2 is the absolute value of q^2, where q is the momentum carried by the current, and

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2M\nu}, \tag{1}$$

is the Bjorken variable (P and M are the target momentum and mass). We also define the related quantities $\mathcal{F}_i(x, Q^2)$ by

$$(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3) = (2F_1, F_2/x, F_3), \tag{2}$$

and their moments

$$M_i^{(n)}(Q^2) = \int_0^1 \mathrm{d}x x^{n-1} \mathcal{F}_i(x, Q^2). \tag{3}$$

The asymptotic forms of $M_i^{(n)}$ and $\mathcal{F}_i$ for large Q^2 are determined by renormalization group equations applied to the coefficient functions [2] of the local operators appearing in the light-cone expansion for the current product. If powers of m^2/Q^2 are neglected (with m any target or quark mass) only contributions from the minimum twist operators survive and the asymptotic results are determined by the zero-mass theory. In QCD there are three families of leading twist operators which govern the behavior of $M_i^{(n)}(Q^2)$ at large Q^2; the quark operator which is a non-singlet under flavor group transformations, the singlet quark operator and the gluon operator [2]. For simplicity we temporarily restrict our attention to non-singlet components of $M_i^{(n)}$. The general solution of the renormalization group equations has in this case the structure [2]:

$$M_i^{(n)}(Q^2) = c_i^{(n)}(\alpha_s(t))\, q_0^{(n)} \exp \int_{\alpha_s}^{\alpha_s(t)} \frac{\gamma^{(n)}(\alpha)}{\beta(\alpha)} \mathrm{d}\alpha. \tag{4}$$

* See, for example, ref. [10].

Here we set

$$t = \ln (Q^2/\mu^2)\,, \qquad \alpha_s(0) \equiv \alpha_s\,, \tag{5}$$

with μ the subtraction point mass. $\beta(\alpha)$ is the Gell-Mann–Low function related to the running coupling constant $\alpha_s(t)$ by [1]:

$$\frac{\mathrm{d}\alpha_s(t)}{\mathrm{d}t} = \beta(\alpha_s(t))\,, \tag{6}$$

($\alpha_s \equiv g_s^2/4\pi$, where g_s is the quark-gluon coupling constant [2]). $\gamma^{(n)}(\alpha)$ are the anomalous dimension functions [1] for the local (non-singlet) operators whose matrix elements between target states are indicated by the constants $q_0^{(n)}$. Finally $c_i^{(n)}(\alpha_s(t))$ are the coefficient functions $c_i^{(n)}(\alpha_s, t)$ for the local operators in the light-cone expansion evaluated at $t = 0$ with α_s replaced by $\alpha_s(t)$ [1]:

$$c_i^{(n)}(\alpha_s(t)) \equiv c_i^{(n)}(\alpha_s(t), 0)\,. \tag{7}$$

The scale of $q_0^{(n)}$ is fixed in such a way that $c_{1,2}^{(n)}(0) = 1$ for electroproduction on a unit charge parton. Expanding in α_s we define

$$\begin{aligned} \beta(\alpha) &= -b\alpha^2(1 + b'\alpha + ...)\,, \\ \gamma^{(n)}(\alpha) &= g^{(n)}\alpha(1 + g'^{(n)}\alpha + ...)\,, \\ c_i^{(n)}(\alpha) &= (1 + c_i'^{(n)}\alpha + ...)\,, \end{aligned} \tag{8}$$

where b and b' are independent of the renormalization procedure and known [2,11]. The corrected form of $\alpha_s(t)$ is then given by

$$\frac{1}{\alpha_s(t)} \simeq \frac{1}{\alpha_s} + bt + b' \ln(1 + b\alpha_s t) = b \ln\frac{Q^2}{\Lambda^2} + b' \ln\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right], \tag{9}$$

with Λ defined by

$$b\alpha_s \ln(\mu^2/\Lambda^2) = 1\,. \tag{10}$$

We also have

$$\exp \int_{\alpha_s}^{\alpha_s(t)} \frac{\gamma^{(n)}(\alpha)}{\beta(\alpha)}\,\mathrm{d}\alpha \equiv \frac{H^{(n)}(\alpha_s(t))}{H^{(n)}(\alpha_s)} \simeq \left(\frac{\alpha_s}{\alpha_s(t)}\right)^{g^{(n)}/b} \left[1 + \frac{g^{(n)}}{b}(g'^{(n)} - b') \times (\alpha_s - \alpha_s(t)) + ...\right]. \tag{11}$$

We recall that although $c_i^{(n)}(\alpha_s(t))$ and $H^{(n)}(\alpha_s(t))$ are separately dependent upon the precise definition of the operator (the renormalization prescription), their product is not [5]. The t-dependent factor in $M_i^{(n)}(t)$ has the following expansion in terms of $\alpha_s(t)$ (given by eq. (9)):

$$c_i^{(n)}(\alpha_s(t))\, H^{(n)}(\alpha_s'(t)) \simeq (\alpha_s(t))^{-g^{(n)}/b}\left\{1 + \left[c_i'^{(n)} - \frac{g^{(n)}}{b}\,(g'^{(n)} - b')\right]\alpha_s(t) + \ldots\right\}. \tag{12}$$

The ambiguities in $c_i'^{(n)}$ (the same for any i) and $g'^{(n)}$ cancel in the coefficient of $\alpha_s(t)$. Since $\alpha_s(t)$ is of first order in α_s while $\alpha_s - \alpha_s(t)$ is of second order, $c_i'^{(n)}$ and $g'^{(n)}$ arise in different orders in a perturbation expansion in α_s but contribute to the same order in $\alpha_s(t)$.

The set of constants $c_i'^{(n)}$ are evaluated from the diagrams in fig. 1 where the point-like quark-current cross section is corrected by the emission of real and virtual gluons to order α_s. In the massless theory infrared singularities appear for a parton that emits a collinear parton. We specify our definition of $c_i'^{(n)}$ by working in the massless theory with off-mass-shell partons [6]. In sect. 3, where the calculations are presented, the above statement will be made completely explicit. This choice of regularisation is made both for physical reasons, (the partons in the target are indeed off mass shell), and for practical reasons of simplicity, and because the calculations of $g'^{(n)}$ have been made [5] (or are in progress) with the present definition. A non-singlet moment is then calculated from the diagrams of order α_s in the form:

$$M_i^{(n)}(t) = q_0^{(n)}\left[1 + \alpha_s\left(t + \ln\frac{\mu^2}{-p^2}\right)g^{(n)} + \alpha_s c_i'^{(n)}\right]. \tag{13}$$

The set of constants $g^{(n)}$ (which are independent of the regularization procedure and of i) and $c_i'^{(n)}$ are then extracted. The $\ln(\mu^2/-p^2)$ term, where p^2 is the virtual mass

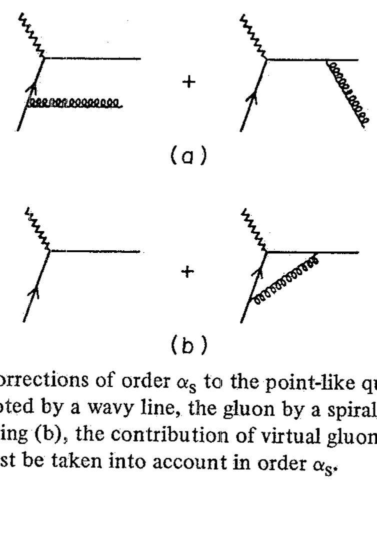

Fig. 1. Diagrams giving the corrections of order α_s to the point-like quark-current cross sections. The incoming current is denoted by a wavy line, the gluon by a spiralling line and the quarks by a continuous line. In calculating (b), the contribution of virtual gluon exchange, the quark wavefunction renormalization must be taken into account in order α_s.

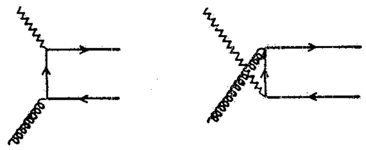

Fig. 2. Diagrams of order g_s contributing to gluon-current scattering.

of the off-shell incoming parton, is to be reabsorbed in the operator matrix element, corresponding to a renormalized operator that coincides with the bare one at $-p^2 = \mu^2$ ★. When this has been done the Mellin transform of eq. (13) is of the form (one flavor, unit charge)

$$\mathcal{F}_i(x, t) = \int_x^1 \frac{dy}{y} q_0(y)\left[\delta\left(1 - \frac{x}{y}\right) + \alpha_s t P\left(\frac{x}{y}\right) + \alpha_s f_i\left(\frac{x}{y}\right) + ...\right], \tag{14}$$

where $q_0(z)$, $P(z)$ and $f_i(z)$ are the inverse Mellin transforms of $q_0^{(n)}$, $g^{(n)}$ and $c_i'^{(n)}$ respectively, i.e. for example:

$$c_i'^{(n)} \equiv \int_0^1 dz z^{n-1} f_i(z) . \tag{15}$$

Dropping the limitation to non-singlet quantities the diagrams with an initial gluon (fig. 2) must also be taken into account in order α_s and we obtain through the same steps

$$\mathcal{F}_i(x, t) = \int_x^1 \frac{dy}{y} \left\{\sum_l a_l^i q_{0l}(y)\left[\delta\left(1 - \frac{x}{y}\right) + \alpha_s t P_{qq}\left(\frac{x}{y}\right) + \alpha_s f_{q,i}\left(\frac{x}{y}\right) + ...\right]\right.$$

$$\left. + \left(\sum_l a_l^i\right) G_0(y)\left[\alpha_s t P_{qG}\left(\frac{x}{y}\right) + \alpha_s f_{G,i}\left(\frac{x}{y}\right) + ...\right]\right\}, \tag{16}$$

or equivalently,

$$M_i^{(n)}(t) = \sum_l a_l^i q_{0l}^{(n)}[1 + \alpha_s t g_{qq}^{(n)} + \alpha_s c_{q,i}'^{(n)} + ...]$$

$$+ \left(\sum_l a_l^i\right) G_0^{(n)}[\alpha_s t g_{qG}^{(n)} + \alpha_s c_{G,i}'^{(n)} + ...] . \tag{17}$$

In the previous formulae we have restored all flavors (the index l runs over quarks and antiquarks of any flavor). The labels q and G distinguish quark and gluon operators, and the numbers a_l^i are the appropriate coupling factors (in particular $\Sigma_l\, a_l^3 = 0$).

★ This may not be true in some renormalization schemes, e.g. ref. [5].

3. Calculation of coefficients to order α_s

We start by giving a detailed account of the calculation of $f_{q,i}(z)$ or equivalently the set of coefficients $c'^{(n)}_{q,i}$ defined in eqs. (8, 13, 16, 17). Their evaluation is more involved than that of $f_{G,i}(z)$ and $c'^{(n)}_{G,i}$ because of the presence of both real and virtual gluons in the diagrams of fig. 1. We recall that we stipulated that the singularities of the massless theory would be regularized by taking the initial quark off mass shell by an amount $p^2 < 0$ (the "virtual mass"). The functions $f_{q,i}(z)$ turn out to be distributions with a singular behavior near $z = 1$. In order to make these distributions well-defined and therefore make the subsequent evaluation of moments completely unambiguous, it is also necessary, in the intermediate stages of the computation, to set the final quark off mass shell with a "virtual mass" $p'^2 > 0$. This also makes the virtual diagrams infrared convergent (with massless gluons).

The variable z has the invariant meaning given by:

$$z = \frac{Q^2}{2p \cdot q}\,, \tag{18}$$

where p is the incoming parton four momentum.

With the stated prescriptions the total contribution of the real diagrams in fig. 1a to $\mathcal{F}_2(z, t)$ (the structure function, see eq. (2), for electroproduction off a parton quark of unit charge) is found to be:

$$\mathcal{F}_2(z, t)|_{\text{real}} = \frac{4}{3}\frac{\alpha_s}{2\pi}\Bigg[-\left[\frac{1+z^2}{(1-z)_+} - 2\ln\omega'\delta(1-z)\right]\ln\omega$$
$$-\tfrac{3}{2}\left[\frac{1}{(1-z)_+} - \delta(1-z)\ln\omega'\right]$$
$$+ 1 + 3z - \frac{2(1+z^2)}{1-z}\ln z + \delta(1-z)\Bigg]\,, \tag{19}$$

where

$$\omega = -p^2/Q^2 > 0\,, \qquad \omega' = p'^2/Q^2 > 0\,. \tag{20}$$

and the distribution $(1-z)_+^{-1}$ is defined by:

$$\int_0^1 \mathrm{d}z\,\frac{h(z)}{(1-z)_+} \equiv \int_0^1 \mathrm{d}z\,\frac{h(z)-h(1)}{(1-z)}\,. \tag{21}$$

The following remarks concerning the more delicate steps should allow the reader to reproduce eq. (19) easily. The distribution $(1-z)_+^{-1}$ arises as follows. With $\omega, \omega' \neq 0$ the singular denominator is actually $(1 - z - \omega)$. On the other hand phase space demands:

$$z \leqslant (1 - \omega - \omega')\,. \tag{22}$$

Thus for moments we would find, in the limit $\omega, \omega' \to 0$:

$$\int_0^{1-\omega-\omega'} dz \frac{z^{n-1}}{(1-z-\omega)} \simeq \int_0^1 dz \frac{z^{n-1}-1}{(1-z)} + \int_0^{1-\omega-\omega'} dz \frac{1}{(1-z-\omega)}$$

$$= -\sum_{k=1}^{n-1} \frac{1}{k} - \ln \omega' , \tag{23}$$

showing the equivalence

$$(1-z-\omega)^{-1} = (1-z)_+^{-1} - \delta(1-z) \ln \omega' . \tag{24}$$

In the same way the last term in eq. (19) arises from the replacement

$$\frac{\omega'}{(1-z-\omega)^2} = \delta(1-z) , \tag{25}$$

because of the relation

$$\lim_{\omega' \to 0} \omega' \int_0^{1-\omega-\omega'} \frac{dz z^{n-1}}{(1-z-\omega)^2} = 1 . \tag{26}$$

Finally we note that working in a reference frame where the energy component of q vanishes, and θ is the angle between the incoming quark and the final gluon, the singularities $(1-\cos\theta)^{-1}$ of the massless theory become in our treatment

$$\left[1 - \cos\theta + \frac{2\omega z^3}{1-z}\right]^{-1}$$

$(-1 \leqslant \cos\theta \leqslant 1)$. However we also find a double pole term

$$\frac{-2\omega z^3}{1-z}\left[1 - \cos\theta + \frac{2\omega z^3}{1-z}\right]^{-2} ,$$

leading after integration over $\cos\theta$ (in the limit $\omega \to 0$) to an additional -1 which was included in eq. (19). To summarise terms in the numerator proportional to ω and ω' may not be neglected in the presence of double poles.

The total contribution to $\mathcal{F}_2(z, t)$ from the virtual diagrams in fig. 1b (the vertex diagram interference with the lowest order diagram corrected by the appropriate wave-function renormalisation factors) is found to be *

$$\mathcal{F}_2(z, t)|_{\text{virtual}} = \frac{4}{3}\frac{\alpha_s}{2\pi}\left\{-1 - \frac{2\pi^2}{3} - \frac{3}{2}\ln\omega - \frac{3}{2}\ln\omega' - 2\ln\omega\ln\omega'\right\}\delta(1-z) . \tag{27}$$

* The quark-wave function renormalizations are defined by subtraction at the "virtual mass" of the quark.

We note here that all the virtual diagrams have been calculated in the Feynman gauge. In the Feynman gauge dimensional or Pauli-Villars regularization must be used in intermediate stages of the calculation but the ultraviolet singularities cancel in the sum.

By adding together the real and virtual contributions in eqs. (19, 27) all terms in $\ln \omega'$ cancel out and we are left with the result;

$$\mathcal{F}_2(z,t) = \frac{4}{3}\frac{\alpha_s}{2\pi}\left\{-\left[\frac{1+z^2}{(1-z)_+} + \tfrac{3}{2}\delta(1-z)\right]\ln\omega + 1 + 3z - \frac{3}{2}\frac{1}{(1-z)_+} - 2\left(\frac{1+z^2}{1-z}\right)\ln z - \tfrac{2}{3}\pi^2\delta(1-z)\right\}. \tag{28}$$

The coefficient of $\ln Q^2$ in the above equation coincides with the well-known result in the generating function $P_{qq}(z)$ of the lowest order logarithmic exponents $g_{qq}^{(n)}$ in the quark operator [4]. According to eq. (14) we finally obtain for the coefficient function $f_{q,2}(z)$ the result:

$$\alpha_s f_{q,2}(z) = \frac{4}{3}\frac{\alpha_s}{2\pi}\left[1 + 3z - \frac{3}{2}\frac{1}{(1-z)_+} - 2\frac{(1+z^2)}{(1-z)}\ln z - \tfrac{2}{3}\pi^2\delta(1-z)\right], \tag{29}$$

or equivalently by eq. (15):

$$\alpha_s c_{q,2}^{\prime(n)} = \frac{4}{3}\frac{\alpha_s}{2\pi}\left[\frac{-1}{2n} + \frac{3}{n+1} + \frac{3}{2}\sum_{k=1}^{n}\frac{1}{k} + \frac{2}{n^2} - \frac{2}{(n+1)^2} - 4\sum_{k=1}^{n}\frac{1}{k^2}\right]. \tag{30}$$

The computation of quark coefficients is then readily completed because the differences $f_{q,2} - f_{q,1}$ and $f_{q,2} - f_{q,3}$ are well-defined even in the massless theory. In particular the result for $f_{q,2} - f_{q,1}$ is well-known [12]; We have

$$\alpha_s[f_{q,2}(z) - f_{q,1}(z)] = \frac{4}{3}\frac{\alpha_s}{2\pi}2z, \tag{31}$$

and [13]

$$\alpha_s[f_{q,2}(z) - f_{q,3}(z)] = \frac{4}{3}\frac{\alpha_s}{2\pi}(1+z). \tag{32}$$

Note that for a current $\gamma_\mu(a - b\gamma_5)$ the previous results for $\mathcal{F}_{1,2}$ are to be multiplied by $a^2 + b^2$ and $\mathcal{F}_3$ by $2ab$ as in the naive quark model.

The results for eqs. (29, 30) for $f_{q,2}$ and $c_{q,2}^{\prime(n)}$ are in disagreement with those of ref. [6] where the same definitions are apparently adopted. It is therefore important to observe that our results are in agreement with the constraint imposed by the Adler sum rule [14] which is true at all Q^2. When powers of m^2/Q^2 are neglected the Adler sum rule is given by

$$\int_0^1 \frac{dx}{x} [F_2^{\nu p}(x,t) - F_2^{\nu n}(x,t)] = A_0 , \qquad (33)$$

with A_0 a given number depending on the flavor content of the theory. We remark that our calculation applies to any combination of vector and axial vector currents. Therefore the validity of the Adler sum rule at all Q^2 implies that for any possible flavor content of the theory the first moment (charge) of the non-singlet part of $\mathcal{F}_2$ is independent of Q^2 and fixed. From eqs. (4, 8) this amounts to the requirement,

$$[1 + \alpha_s(t)\, c'^{(1)}_{q,2} + ...] \exp \int_{\alpha_s}^{\alpha_s(t)} \frac{\gamma^{(1)}(\alpha)}{\beta(\alpha)} d\alpha = 1 . \qquad (34)$$

Because of the necessary vanishing of $\gamma^{(1)}(\alpha)$ for the non-singlet quark operator (conservation of charge), the integral is zero. We conclude that:

$$c'^{(1)}_{q,2} = 0 . \qquad (35)$$

The same conclusion can be obtained from the first-order expansion in α_s in eq. (13) taking into account that $g^{(1)} = 0$. The constraint in eq. (35) is satisfied by our eq. (30) but not by the results quoted in ref. [6] and in ref. [7] *.

The other current algebra sum rules have a different status because they do not hold at all Q^2. For example there are corrections of order $\alpha_s(t)$ to the Gross-Llewellyn Smith sum rule that are determined by $c'^{(1)}_{q,3}$. We find:

$$\int_0^1 dx [F_3^{\bar{\nu} p}(x,t) + F_3^{\nu p}(x,t)] \simeq L_0(1 + \alpha_s(t)\, c'^{(1)}_{q,3}) = L_0\left(1 - \frac{\alpha_s(t)}{\pi}\right) , \qquad (36)$$

where L_0 is the predicted asymptotic value. Since eq. (32) is independent of the regularization procedure, so is eq. (36), when eq. (35) is taken into account.

We now turn to the calculation of the gluon coefficients determined by the real diagrams in fig. 2. In this case giving a virtual mass $p^2 < 0$ to the initial gluon is sufficient for a complete definition of moments. In fact the results are regular near $z = 1$ and no distributions need to be defined. Here also attention must be given to a double-pole contribution from the $\cos\theta$ integration as in the quark case. The contribution of gluon current scattering to $\mathcal{F}_2(x, t)$ is found to be [13]

$$\tfrac{1}{2}\mathcal{F}_2(z,t) = \frac{\alpha_s}{2\pi}\left\{\tfrac{1}{2}[z^2 + (1-z)^2]\left[\ln\frac{Q^2}{-p^2} - 2\ln z - 1\right] - \tfrac{1}{2}(1-2z)^2\right\}, \qquad (37)$$

* In ref. [7] on-shell quarks with finite rest mass are considered. The Adler sum rule should nevertheless be valid. Our eqs. (32) and (36) also disagree with ref. [7].

corresponding to (according to eq. (16)):

$$\alpha_s f_{G,2}(z) = \frac{-\alpha_s}{2\pi} \frac{1}{2} \{z^2 + (1-z)^2](2 \ln z + 1) + (1-2z)^2\} , \tag{38}$$

or equivalently

$$\alpha_s c'^{(n)}_{G,2} = \frac{\alpha_s}{2\pi} \left\{ \frac{1}{n^2} + \frac{2}{(n+2)^2} - \frac{2}{(n+1)^2} - \frac{1}{n} + \frac{3}{(n+1)} - \frac{3}{(n+2)} \right\} . \tag{39}$$

It must be remarked that the above expression for $\mathcal{F}_2$ is appropriate for electroproduction with one quark flavor of unit charge. Also according to eq. (16) $f_{G,2}$ is defined through $\frac{1}{2}\mathcal{F}_2$ because this corresponds to the gluon correction for *each* given quark *or* antiquark. In other words, $\mathcal{F}_2$ is always given by a combination of quark plus antiquark terms (of different flavors for charged currents), one pair for each term in the current (hence with equal couplings). To each quark plus antiquark pair with unit coupling there corresponds a gluon correction equal to $2\alpha_s f_{G,2}$. With this convention the coefficient of $\ln Q^2$ in eq. (37) is seen to be in agreement with the well-known result for $P_{qG}(z)$ (as defined in eq. (16)) [4].

The set of gluon coefficients is then completed by the well-known result [8,12] for the gluon contribution in the longitudinal structure function. In our notation (independent of the regularization procedure)

$$\alpha_s (f_{G,2}(z) - f_{G,1}(z)) = \frac{\alpha_s}{2\pi} 2z(1-z) , \tag{40}$$

or

$$\alpha_s (c'^{(n)}_{G,1} - c'^{(n)}_{G,1}) = \frac{\alpha_s}{2\pi} \left(\frac{2}{n+1} - \frac{2}{(n+2)} \right) . \tag{41}$$

We finally remark that the gluon contribution to $\mathcal{F}_3$ is zero:

$$f_{G,3}(z) = c'^{(n)}_{G,3} = 0 . \tag{42}$$

4. Partons in QCD beyond the leading logarithmic approximation

For practical applications the familiar parton model language is very useful. This is particularly true for a comparison of different processes like leptoproduction by an e, ν or $\bar{\nu}$ beam or the generalization to Drell-Yan processes.

It is well-known that in the leading logarithmic approximation, i.e. when the functions defined in eq. (4), $\beta(\alpha_s)$, $\gamma^{(n)}(\alpha_s)$ and $c_i^{(n)}(\alpha_s)$ (for all relevant operators) are computed to lowest order in α_s, the results of QCD can be interpreted by saying that the $\mathcal{F}_i$ are given by the naive parton model formulae expressed in terms of t dependent effective parton densities. Moreover the parton densities satisfy the evolution equations [4]:

$$\frac{dq_l(x,t)}{dt} = \alpha_s(t) \int_x^1 \frac{dy}{y} \left[q_l(y,t) P_{qq}\left(\frac{x}{y}\right) + G(y,t) P_{qG}\left(\frac{x}{y}\right) \right] , \tag{43}$$

$$\frac{dG(x,t)}{dt} = \alpha_s(t) \int_x^1 \frac{dy}{y} \left[\sum_l q_l(y,t) P_{Gq}\left(\frac{x}{y}\right) + G(y,t) P_{GG}\left(\frac{x}{y}\right) \right] . \tag{44}$$

Where terms of next order in α_s are also considered, besides the change in the form of $\alpha_s(t)$ (*cf.* eq. (9)) due to b' in eq. (8), two other changes occur. Firstly, the expressions for $\mathcal{F}_i$ in terms of effective parton densities deviate from the naive parton model formulae by terms of order $\alpha_s(t)$ induced by $f_{q,i}$ and $f_{G,i}$. Secondly, the derivatives of parton densities with respect to t are modified by terms of order $\alpha_s^2(t)$, arising, for example, from the next to the leading terms in the expansion of $\gamma^{(n)}(\alpha)$ for each operator. The explicit form of the corrections to structure functions and to derivatives of parton densities depends to some extent on the definition of effective parton densities beyond the leading logarithmic approximation. To discuss this problem we go back to eq. (4) for non-singlet moments:

$$M_i^{(n)}(t) = c_i^{(n)}(\alpha_s(t))\, q_0^{(n)} \exp \int_{\alpha_s}^{\alpha_s(t)} \frac{\gamma^{(n)}(\alpha)}{\beta(\alpha)}\, d\alpha . \tag{45}$$

To lowest order $c_i^{(n)}(0) = 1$ and the (non-singlet) moments of the effective quark densities are directly related to $M_i^{(n)}(t)$ with $\gamma^{(n)}(\alpha)$ and $\beta(\alpha)$ evaluated in lowest order. To go beyond the lowest-order approximation we could for example define the effective parton densities by [15]:

$$\tilde{q}^{(n)}(t) = q_0^{(n)} \exp \int_{\alpha_s}^{\alpha_s(t)} \frac{\gamma^{(n)}(\alpha)}{\beta(\alpha)}\, d\alpha , \tag{46}$$

with the obvious generalization to non-singlet moments. The effective parton densities defined in this way reduce in lowest order to the previous ones and satisfy two natural criteria which we require any good definition of effective parton densities to satisfy. First, effective parton densities should not depend on the index i, i.e. for a given target they must be the same for all structure functions and all possible processes. Second, they should obey the conservation of charge and momentum:

$$\int_0^1 dx\, [q_l(x,t) - \bar{q}_l(x,t)] = v_l , \tag{47}$$

$$\int_0^1 dx\, x \left[\sum_l q_l(x,t) + G(x,t) \right] = 1 , \tag{48}$$

where v_l is the valence value for the given flavor. Effective parton densities defined as in eq. (46) obey both criteria. In fact they are obviously independent of i, and validity of eqs. (47, 48) is guaranteed by the conservation of the charge operators and of the energy momentum tensor operator, i.e. by the vanishing of the corresponding logarithmic exponents. For this definition of effective parton densities the corrected expansions for the structure functions would be obtained from the results of sect. 3 as follows:

$$\mathcal{F}_i(x,t) = \int_x^1 \frac{dy}{y} \left\{ \sum_l a_l^i \tilde{q}(y,t) \left[\delta\left(1 - \frac{x}{y}\right) + \alpha_s(t) f_{q,i}\left(\frac{x}{y}\right) \right] + (\sum_l a_l^i)\, \tilde{G}(y,t)\, \alpha_s(t) f_{G,i}\left(\frac{x}{y}\right) \right\}, \tag{49}$$

where as usual l sums over both quarks and antiquarks, and a_l^i are the relevant couplings. This corresponds in perturbation theory to eq. (16) with the terms proportional to t reabsorbed in the effective quark densities. The order $\alpha_s^2(t)$ corrections are then completely specified by $\gamma^{(n)}(\alpha)$ and $\beta(\alpha)$ as discussed in sect. 2.

However this perfectly legitimate definition of effective parton densities suffers from the disadvantage that it depends on the precise definition of the relevant operators, because as we have mentioned $f_q(z)$ and $f_G(z)$ depend on the renormalization prescription adopted.

We have therefore chosen a more physical definition of the parton densities. In addition to the above mentioned criteria we further specify the parton densities by demanding that $\mathcal{F}_2(x,t)$ expressed in terms of them should have the same form as in the naive quark model (up to corrections of order $\alpha_s^2(Q^2)$). Thus, for example the non-singlet moments of the quark densities are defined by,

$$q^{(n)}(t) = c_2^{(n)}(\alpha_s(t))\, q_0^{(n)} \exp \int_{\alpha_s}^{\alpha_s(t)} \frac{\gamma^{(n)}(\alpha)}{\beta(\alpha)}\, d\alpha\,. \tag{50}$$

The structure function $\mathcal{F}_2$ is given a special status because it satisfies the Adler sum rule. This property of $c_2^{(1)}(\alpha_s(t))$, explicitly demonstrated to order $\alpha_s(t)$ in sect. 3, guarantees that eq. (47) are verified by the present definition of effective parton densities. The prescription that $\mathcal{F}_2$ is not modified fixes quark densities in the singlet case as well. For example, to first order the relation between $q_l(x,t)$ and $\tilde{q}_l(x,t)$, $\tilde{g}(x,t)$ (the latter defined as in eq. (46)) is given by:

$$q_l^{(n)}(t) = \tilde{q}_l^{(n)}(t)[1 + \alpha_s(t)\, c_{q,2}^{\prime(n)}] + \alpha_s(t)\, \tilde{G}^{(n)}(t)\, c_{G,2}^{\prime(n)}\,, \tag{51}$$

which implies

$$q_l(x,t) = \int_x^1 \frac{dy}{y} \left\{ \tilde{q}_l(y,t) \left[\delta\left(1 - \frac{x}{y}\right) + \alpha_s(t) f_{q,2}\left(\frac{x}{y}\right) \right] + \alpha_s(t)\, \tilde{G}(y,t)\, f_{G,2}\left(\frac{x}{y}\right) \right\}. \tag{52}$$

Returning to eq. (16) with $i = 2$ we see that in our definition of $q(x, t)$ we have absorbed not only the effects P_{qq} and P_{qG} but also those due to $f_{q,2}$ and $f_{G,2}$.

Some freedom still remains in the definition of $G(x, t)$. We use this freedom to ensure that the momentum sum rule eq. (48) is satisfied. If we were to take $G = \widetilde{G}$ eq. (48) for $q^{(2)}$ and $G^{(2)}$ would not be satisfied because it immediately follows from eqs. (30, 39) that $c'^{(2)}_{q,2}$ and $c'^{(2)}_{G,2}$ are both non-zero. Therefore since $\widetilde{q}^{(2)}$ and $\widetilde{G}^{(2)}$ do satisfy eq. (48), in order for $q^{(2)}$ and $G^{(2)}$ to also satisfy it, we must suitably specify $G(x, t)$ in terms of $\Sigma_l \widetilde{q}_l(x, t)$ and $\widetilde{G}(x, t)$. For example we could set

$$\sum_l q_l(x, t) + G(x, t) = \sum_l \widetilde{q}_l(x, t) + \widetilde{G}(x, t)\,, \tag{53}$$

implying

$$G^{(n)}(t) = \widetilde{G}^{(n)}(t)[1 - 2f\alpha_s(t)\, c'^{(n)}_{G,2}] - \alpha_s(t) \sum_l \widetilde{q}_l^{(n)}(t)\, c'^{(n)}_{q,2}\,, \tag{54}$$

where f = number of flavors. Note however that to first order in $\alpha_s(t)$ the expressions for $\mathcal{F}_i$ are not affected by the difference between G and $\widetilde{G}$ because the gluons first appear in $\mathcal{F}_i$ only at order $\alpha_s(t)$.

As for the t derivatives of the effective parton densities we first observe that at order $\alpha_s(t)$ the evolution equations (43, 44) are not modified with the present definition. This is because

$$\frac{d}{dt} = \beta(\alpha_s(t)) \frac{d}{d\alpha_s(t)} \simeq -b\alpha_s^2(t) \frac{d}{d\alpha_s(t)}\,. \tag{55}$$

Thus the inclusion of coefficient functions in the definition of parton densities only affects the derivatives in the next order in $\alpha_s(t)$. Therefore the terms of order $\alpha_s^2(t)$ in the evolution equations will now not only depend on $\gamma^{(n)}(\alpha)$ (for quark and gluon operators) and $\beta(\alpha)$ but also on $c^{(n)}_{q,2}$ and $c^{(n)}_{G,2}$.

In the following we shall adopt this latter definition of parton densities, with $\mathcal{F}_2$ given to order $\alpha_s(t)$ by the same form as in the naive quark model. Then $\mathcal{F}_i$ are given in terms of q_l and G by eq. (49) with $f_{q/G,i}$ replaced by $f_{q/G,i} - f_{q/G,2}$. More explicitly for one flavor, V – A currents and unit couplings (see eq. (2)):

$$\mathcal{F}_2(x, t) = 2[q(x, t) + \bar{q}(x, t)]\,, \tag{56}$$

$$\mathcal{F}_1(x, t) = 2\int_x^1 \frac{dy}{y} \left\{ [q(y, t) + \bar{q}(y, t)] \left[\delta\left(1 - \frac{x}{y}\right) + \alpha_s(t)\left(f_{q,1}\left(\frac{x}{y}\right) - f_{q,2}\left(\frac{x}{y}\right)\right)\right] \right.$$
$$\left. + 2\alpha_s(t)\, G(y, t)\left(f_{G,1}\left(\frac{x}{y}\right) - f_{G,2}\left(\frac{x}{y}\right)\right)\right\}\,, \tag{57}$$

$$\mathcal{F}_3(x, t) = 2\int_x^1 \frac{dy}{y} [-q(y, t) + \bar{q}(y, t)] \left[\delta\left(1 - \frac{x}{y}\right) + \alpha_s(t)\left(f_{q,3}\left(\frac{x}{y}\right) - f_{q,2}\left(\frac{x}{y}\right)\right)\right]\,, \tag{58}$$

Similarly for the moments

$$M_2^{(n)}(t) = 2[q^{(n)}(t) + \bar{q}^{(n)}(t)] , \tag{59}$$

$$M_1^{(n)}(t) = 2\{[q^{(n)}(t) + \bar{q}^{(n)}][1 + \alpha_s(t)(c'^{(n)}_{q,1} - c'^{(n)}_{q,2})] + 2\alpha_s(t)\, G^{(n)}(t)(c'^{(n)}_{G,1} - c'^{(n)}_{G,2})\} , \tag{60}$$

$$M_3^{(n)}(t) = 2[-q^{(n)}(t) + \bar{q}^{(n)}(t)][1 + \alpha_s(t)(c'^{(n)}_{q,3} - c'^{(n)}_{q,2})] . \tag{61}$$

In this way all dependence on the renormalisation prescription drops out in the differences. Similarly in the corrections of order $\alpha_s^2(t)$ to the t derivatives all dependence on the renormalisation prescription cancels between $c'^{n}_{q/G,2}$ and $g'^{(n)}$ as described in sect. 2.

5. Implications for leptoproduction phenomenology

The quality of the available data on leptoproduction is at present not sufficient for a determination of the parton densities to the level of accuracy implied by the retention of the next order terms in $\alpha_s(t)$. The corrections are in fact of the order of the longitudinal structure function which is badly known at present. Note that for a general leptoproduction process the longitudinal strcuture function can always be cast in the form

$$F_L(x, t) = \frac{\alpha_s(t)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} \left[\tfrac{8}{3}F_2(y, t) + 2ayG(y, t)\left(1 - \frac{x}{y}\right)\right] \tag{62}$$

(see eqs. (31, 40)), where a is the sum of all coefficients of quarks *and* antiquarks in the parton-model expression for $\mathcal{F}_2 = F_2/x$. For example, in the four-quark model, $a = \frac{20}{9}$ in electroproduction and $a = 8$ for ν and $\bar{\nu}$ scattering on matter (see eqs. (68, 69)). Consequently for the total longitudinal cross section we have,

$$\sigma_L(t) = \int_0^1 dx F_L(x, t) = \frac{\alpha_s(t)}{2\pi}\{\tfrac{8}{9}\int_0^1 F_2(x, t)\,dx + \tfrac{1}{6}a\int_0^1 xG(x, t)\,dx\} . \tag{63}$$

Whilst F_2 allows the precise determination of valence quark densities, we see that good data on F_L could be translated into a reasonable knowledge of the gluon density. Once the gluon corrections have been reliably estimated the small sea densities can finally be determined. As the present experimental situation on F_L is confused a detailed fitting procedure based on the above strategy and the results of the previous sections, would certainly be premature. We therefore limit ourselves in the following to a number of phenomenological remarks mainly concerning the effect of gluon corrections on the sea densities. The change in the sea densities will be large, since they themselves are small ★.

★ The importance of the $\alpha_s(Q^2)$ corrections for the determination of the sea and gluon densities was pointed out in ref. [21], in the context of a different definition of the parton densities.

Consider first the total fractional momentum carried by partons, a quantity which is related to total cross sections. We review a number of different ways that are used to estimate the momentum carried by sea quarks and study the effects of gluon corrections in each case. We recall that from eqs. (31,32,40) it follows that:

$$\alpha_s \Delta c_{q,1}^{(2)} = \alpha_s [c'^{(2)}_{q,1} - c'^{(2)}_{q,2}] = -\frac{8}{9}\frac{\alpha_s}{2\pi} = 0.141\alpha_s \ , \tag{64}$$

$$\alpha_s \Delta c_{q,3}^{(2)} = \alpha_s [c'^{(2)}_{q,3} - c'^{(2)}_{q,2}] = -\frac{10}{9}\frac{\alpha_s}{2\pi} = -0.177\alpha_s \ , \tag{65}$$

$$\alpha_s \Delta c_{G}^{(2)} = \alpha_s [c'^{(2)}_{G,1} - c'^{(2)}_{G,2}] = -\frac{1}{6}\frac{\alpha_s}{2\pi} = -0.027\alpha_s \ . \tag{66}$$

Consider the right-handed cross section:

$$\sigma_{\mathrm{RH}}(t) \sim \tfrac{1}{2} \int_0^1 \mathrm{d}x (2xF_1 + xF_3) \ . \tag{67}$$

For V – A currents this quantity is determined in the naive parton model by the momentum carried by antiquarks. In the four-quark model, above charm threshold (the case to which we shall always refer in this section) we have (on matter)

$$\int_0^1 F_2^{\nu}(x, t)\, \mathrm{d}x = U + D + 2S + \bar{U} + \bar{D} + 2\bar{C} \ , \tag{68}$$

$$\int_0^1 F_2^{\bar{\nu}}(x, t)\, \mathrm{d}x = \bar{U} + \bar{D} + 2\bar{S} + U + D + 2C \ , \tag{69}$$

where we used the notation:

$$U \equiv U(t) = \int_0^1 \mathrm{d}x x u(x, t) \tag{70}$$

etc, for the momentum carried by parton quarks. Therefore, recalling eqs. (56–58) and (64–66):

$$\begin{aligned} \sigma^{\nu}_{\mathrm{RH}}(t) &= (\bar{U} + \bar{D} + 2\bar{C})(1 + \tfrac{1}{2}\alpha_s(t)\, \Delta c^{(2)}_{q,1} + \tfrac{1}{2}\alpha_s(t)\, \Delta c^{(2)}_{q,3}) \\ &\quad + (U + D + 2S)(\tfrac{1}{2}\alpha_s(t)\, \Delta c^{(2)}_{q,1} - \tfrac{1}{2}\alpha_s(t)\, \Delta c^{(2)}_{q,3}) + 8G\tfrac{1}{2}\alpha_s(t)\, \Delta c^{(2)}_{G} \\ &= (\bar{U} + \bar{D} + 2\bar{C})(1 - 0.159\alpha_s(t)) + 0.018\alpha_s(t)(U + D + 2S) - 0.106\alpha_s(t)\, G \ . \end{aligned} \tag{71}$$

Similarly for $\bar{\nu}$ one only needs to exchange $\bar{C} \to \bar{S}$ and $S \to C$. We see that in order

$\alpha_s(t)$ quarks and gluons also contribute to this quantity which in lowest order is entirely determined by antiquarks. For Q^2 in the range between 3–10 GeV^2, $\alpha_s(t) \sim \frac{1}{2}$ since Λ in eq. (10) is believed to be ~0.5 GeV. For $G \sim \frac{1}{2}$ (the gluons carry about one half of the proton momentum) we see that the gluon correction is about 0.03, while the quark correction is roughly ten times smaller. Thus the gluon correction is of the same order as the momentum carried by antiquarks and therefore certainly cannot be neglected for a precise quantitative determination of the sea densities.

A related estimate of sea densities is obtained from the study of y distributions in ν and $\overline{\nu}$ scattering. From

$$\sigma^{\nu,\overline{\nu}}(y) \sim (1-y)\int \mathrm{d}x F_2^{\nu,\overline{\nu}} + \tfrac{1}{2}y^2 \int \mathrm{d}x 2xF_1^{\nu,\overline{\nu}} \mp y(1-\tfrac{1}{2}y)\int \mathrm{d}x xF_3^{\nu,\overline{\nu}} \,, \quad (72)$$

we have

$$\sigma^{\nu}(0) - \sigma^{\nu}(1) \sim \int \mathrm{d}x (F_2^{\nu} - xF_1^{\nu} + \tfrac{1}{2}xF_3^{\nu}) \,, \quad (73)$$

$$\sigma^{\overline{\nu}}(1) \sim \int \mathrm{d}x (xF_1^{\overline{\nu}} + \tfrac{1}{2}xF_3^{\overline{\nu}}) \,. \quad (74)$$

The above quantities are both proportional to sea quarks in lowest order (the normalization can be set by observing that $\sigma^{\nu,\overline{\nu}}(0) = \int \mathrm{d}x F_2^{\nu,\overline{\nu}}$ are the same for ν and $\overline{\nu}$ as is clear from eqs. (68, 69)). We have already analysed the expression for $\sigma^{\overline{\nu}}(1)$ in eq. (74). For the other one we have:

$$\begin{aligned}\int \mathrm{d}x (F_2^{\nu} - xF_1^{\nu} + \tfrac{1}{2}xF_3^{\nu}) &= (\overline{U} + \overline{D} + 2\overline{C})(1 - \tfrac{1}{2}\alpha_s(t)\,\Delta c_{q,1}^{(2)} \\ &\quad + \tfrac{1}{2}\alpha_s(t)\,\Delta c_{q,3}^{(2)}) + (U + D + 2S)(-\tfrac{1}{2}\alpha_s(t)\,\Delta c_{q,1}^{(2)} - \tfrac{1}{2}\alpha_s(t)\,\Delta c_{q,3}^{(2)}) \\ &\quad - 8G\tfrac{1}{2}\alpha_s(t)\,\Delta c_G^{(2)} \\ &= (\overline{U} + \overline{D} + 2\overline{C})(1 - 0.018\alpha_s(t)) + 0.159\alpha_s(t)(U + D + 2S) + 0.106\alpha_s(t)\,G \,. \end{aligned} \quad (75)$$

Here the quark correction is important, (roughly 0.04), and adds to the gluon correction which is opposite in sign to the correction in eq. (71). It is clear that quarks and gluons play a non-negligible role in determining the deviations from flatness in $\sigma^{\nu}(y)$.

Another quantity that is used to extract sea densities from the data is the charm changing total cross section in $\overline{\nu}$ scattering which in lowest order is essentially determined by strange antiquarks. We have (neglecting the Cabibbo angle)

$$\begin{aligned}\sigma^{\overline{\nu}}_{\Delta c=1} &= 2\overline{S}\,(1 + \tfrac{1}{6}\alpha_s(t)\,\Delta c_{q,1}^{(2)} + \tfrac{1}{3}\alpha_s(t)\,\Delta c_{q,3}^{(2)}) \\ &\quad + 2C(\tfrac{1}{3} + \tfrac{1}{6}\alpha_s(t)\,\Delta c_{q,1}^{(2)} - \tfrac{1}{3}\alpha_s(t)\,\Delta c_{q,3}^{(2)}) + 4G(\tfrac{1}{6}\alpha_s(t)\,\Delta c_G^{(2)}) \\ &= 2\overline{S}\,(1 - 0.083\alpha_s(t)) + 2C(\tfrac{1}{3} + 0.036\alpha_s(t)) - 0.018\alpha_s(t)\,G \,. \end{aligned} \quad (76)$$

This is a peculiar case in that all correction terms are particularly small: the gluon correction in roughly 10% of the measured effect.

In going from total cross sections to x distributions the form of the correction terms is clearly more drastically affected by our ignorance of the gluon density. From the previous discussion it is seen that a quantity at given x which in lowest order is determined by a sea density is in general corrected by terms of order $\alpha_s(t)$ involving a convolution with quark and gluon densities at $y \gtrsim x$, as in eqs. (56–58) The convolution integrals are of the same form as the evolution equations that govern the t dependence of the densities (in lowest order eqs. (43–44)). Thus, taking the natural view that the sea is generated from the gluons, and the gluons from the valence through the t evolution equations, we do not expect the x shape of the corrective terms to the sea densities arising in order $\alpha_s(t)$ from the convolutions with quarks and gluons to differ substantially from the x shape of the lowest order term. If the very naive ansatz is made that at a given t the sea and gluon densities can each be parametrised in terms of a single power of $(1-x)$, then the two exponents should differ by about one unit. Consider the specific example of the right handed structure function for $\bar{\nu}$ scattering on matter in a V – A theory, defined by (see eq. (67)):

$$F_{\mathrm{RH}}^{\bar{\nu}}(x,t) = \tfrac{1}{2}x\,[2F_1^{\bar{\nu}}(x,t) + F_3^{\bar{\nu}}(x,t)]\ . \tag{77}$$

Introducing for shorthand the notation

$$\bar{Q}(x,t) = \bar{u}(x,t) + \bar{d}(x,t) + 2\bar{s}(x,t)\ , \tag{78}$$

$$Q(x,t) = u(x,t) + d(x,t) + 2c(x,t)\ , \tag{79}$$

we have

$$F_{\mathrm{RH}}^{\bar{\nu}}(x,t) = x\int_x^1 \frac{\mathrm{d}y}{y}\left\{\bar{Q}(y,t)\left[\delta\left(1-\frac{x}{y}\right) + \tfrac{1}{2}\alpha_s(t)\left(f_{\mathrm{q},1}\left(\frac{x}{y}\right) + f_{\mathrm{q},3}\left(\frac{x}{y}\right) - 2f_{\mathrm{q},2}\left(\frac{x}{y}\right)\right)\right]\right.$$
$$\left. + Q(y,t)\,\alpha_s(t)\,\tfrac{1}{2}\left[f_{\mathrm{q},1}\left(\frac{x}{y}\right) - f_{\mathrm{q},3}\left(\frac{x}{y}\right)\right] + 4G(y,t)\left[f_{\mathrm{G},1}\left(\frac{x}{y}\right) - f_{\mathrm{G},2}\left(\frac{x}{y}\right)\right]\right\}\ . \tag{80}$$

Note that both $f_{\mathrm{q},1}(z) - f_{\mathrm{q},3}(z)$ and $f_{\mathrm{G},1}(z) - f_{\mathrm{G},2}(z)$ are proportional to $(1-z)$ (see eqs. (31, 32, 40)) so that if $G(x,t)$ behaves like $(1-x)^a$ near $x = 1$, then the corresponding convolution behaves like $(1-x)^{a+2}$. In fact this is always so for the gluon corrections because $f_{\mathrm{g},1} - f_{\mathrm{g},2}$ is the only non-vanishing difference. A similar result, true only in this particular case, also holds for $Q(x,t)$. Thus if the gluon density does not dominate the antiquark density by more than two powers of $(1-x)$ near $x \sim 1$, the gluon correction, which is largest because of the factor of 4 in eq. (80), never overwhelms the leading term, even near $x \sim 1$. A simple numerical example is shown in fig. 3 where $F_{\mathrm{RH}}^{\bar{\nu}}$ is plotted for $\alpha_s(t) \simeq \frac{1}{2}$ and parton densities

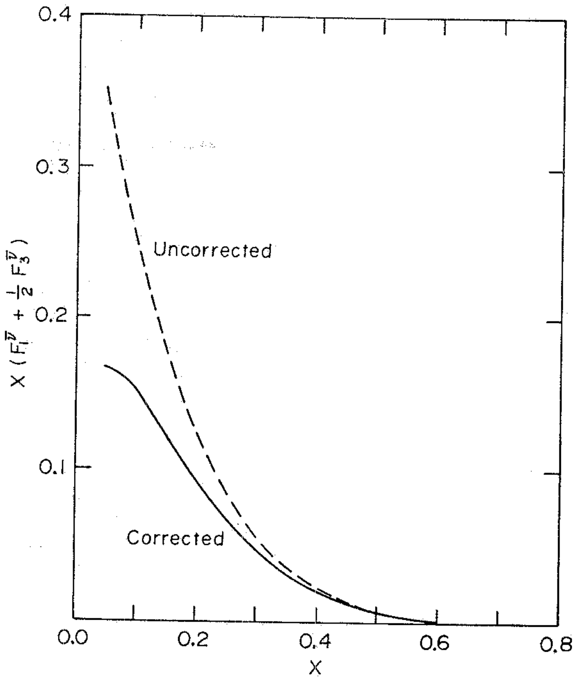

Fig. 3. The right-handed structure function $F_{RH}^{\bar{\nu}} = x(F_1^{\bar{\nu}} + \frac{1}{2}F_3^{\bar{\nu}})$ for antineutrino scattering on matter in the V – A four-quark model. The dashed curve only includes the leading term from sea densities. The solid curve is the total including the quark and gluon corrections as in eq. (80) with $\alpha_s(t) \simeq \frac{1}{2}$ and parton densites specified as in eq. (81).

given by (SU(3)) symmetric sea, no charm) [17]:

$$u - \bar{u} \simeq 1.79(1-x)^3(1+2.3x)/\sqrt{x} ,$$
$$d - \bar{d} \simeq 1.07(1-x)^{3.1}/\sqrt{x} ,$$
$$xs(x) \simeq 0.12(1-x)^6 ,$$
$$xG(x) = A_G(1-x)^5 , \tag{81}$$

with A_G fixed by momentum conservation. The two curves in fig. 3 refer to the leading term alone and to the total result including the correction. The correction is concentrated at small x because the quarks, due to their small coefficient (see also eq. (71)) only take over at large x where the whole cross section is very small.

In conclusion the correction terms, although important for quantities dominated by the small sea densities in lowest order, are always of reasonable magnitude for integrated quantities; for the unintegrated x densities this is only true if the x distributions of the relevant densities are correlated in a reasonable fashion.

6. Corrections to the Drell-Yan formula

In the naive parton model the production of a lepton pair of mass Q^2 in hadron-hadron collisions is given by the well-known Drell-Yan formula [9]:

$$\frac{\mathrm{d}\sigma^{\mathrm{DY}}}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{9SQ^2}\int \frac{\mathrm{d}x_1\,\mathrm{d}x_2}{x_1x_2}\,[\sum_f e_f^2 q_{0f}^{(1)}(x_1)\,\overline{q}_{0f}^{(2)}(x_2) + (1 \leftrightarrow 2)]\;\delta\left(1-\frac{\tau}{x_1x_2}\right), \tag{82}$$

where $\sqrt{S}$ is the invariant mass of the incoming hadronic system, $\tau = Q^2/S$, the flavor index f sums over quarks (not antiquarks) and the labels 1 and 2 refer to the two incoming hadrons. Assuming that this is true to zeroth order in α_s, one can compute the corrections from the order α_s processes

$$\mathrm{G} + \mathrm{q}(\overline{\mathrm{q}}) \to \gamma^* + \mathrm{q}(\overline{\mathrm{q}})\,, \tag{83}$$

$$\mathrm{q} + \overline{\mathrm{q}} \to \gamma^* + \mathrm{G}\,, \tag{84}$$

together with the virtual corrections to the simplest process $\mathrm{q} + \overline{\mathrm{q}} \to \gamma^*$. It is not difficult to see that the structure of the result will be (all obvious factors and sums are omitted):

$$\begin{aligned}\frac{\mathrm{d}\sigma^{\mathrm{DY}}}{\mathrm{d}Q^2} = \int_0^1 \frac{\mathrm{d}x_1}{x_1}\int_0^1 \frac{\mathrm{d}x_2}{x_{2c}}\Bigg\{&[q_0^{(1)}(x_1)\,\overline{q}_0^{(2)}(x_2) + (1 \leftrightarrow 2)]\Bigg[\delta(1-z_{12})\\ &+ \alpha_s\theta(1-z_{12})\left(P_{\mathrm{qq}}^{\mathrm{DY}}(z_{12})\ln\frac{(Q^2)^2}{p_1^2p_2^2} + f_{\mathrm{q}}^{\mathrm{DY}}(z_{12})\right)\Bigg]\\ &+ \Bigg[(q_0^{(1)}(x_1) + \overline{q}_0^{(1)}(x_1))\,G_0^{(2)}(x_2)\,\alpha_s\theta(1-z_{12})\Bigg[P_{\mathrm{qG}}^{\mathrm{DY}}(z_{12})\ln\frac{Q^2}{-p_2^2}\\ &+ f_{\mathrm{G}}^{\mathrm{DY}}(z_{12})\Bigg] + (1 \leftrightarrow 2)\Bigg]\Bigg\}\,,\end{aligned} \tag{85}$$

where $p_1^2, p_2^2 < 0$ are the "virtual masses" of the partons in the first and second hadron and

$$z_{12} = \tau/x_1x_2\,. \tag{86}$$

This formula is the exact counterpart of eq. (16) for $\mathcal{F}_2(x, t)$ in electroproduction which was also obtained by adding the order α_s corrections to the naive parton-model expression. In the same style as above we rewrite eq. (16) as:

$$\begin{aligned}\mathcal{F}_2 \simeq \int_0^1 \frac{\mathrm{d}y}{y}\Bigg\{&q_0(y)\left[\delta(1-z) + \alpha_s\theta(1-z)\left(P_{\mathrm{qq}}(z)\ln\frac{Q^2}{-p^2} + f_{\mathrm{q},2}(z)\right)\right]\\ &+ G_0(y)\,\alpha_s\theta(1-z)\left(P_{\mathrm{qG}}(z)\ln\frac{Q^2}{-p^2} + f_{\mathrm{G},2}(z)\right)\Bigg\}\end{aligned}$$

$$\equiv q(x, t)\,, \tag{87}$$

where $z = x/y$ and the last equality follows because we decided to define quark densities beyond the leading approximation in such a way as to keep $\mathcal{F}_2$ fixed in its naive parton model form.

It has been demonstrated [16] that the log Q^2 coefficients in eqs. (85) and (87) are in fact equal:

$$P^{\mathrm{DY}}_{\mathrm{qq}}(z) = P_{\mathrm{qq}}(z)\,, \tag{88}$$

$$P^{\mathrm{DY}}_{\mathrm{qG}}(z) = P_{\mathrm{qG}}(z)\,. \tag{89}$$

It is then a simple matter of integral manipulation to show that (to order α_s)

$$\frac{\mathrm{d}\sigma^{\mathrm{DY}}}{\mathrm{d}Q^2} = \int \frac{\mathrm{d}x_1\,\mathrm{d}x_2}{x_1 x_2} \{ [q^{(1)}(x_1, t)\,\overline{q}^{(2)}(x_2, t) + (1 \leftrightarrow 2)]\,[\delta(1 - z_{12})$$
$$+ \alpha_s(t)\,\theta(1 - z_{12})(f^{\mathrm{DY}}_{\mathrm{q}}(z_{12}) - 2f_{\mathrm{q},2}(z_{12}))]$$
$$+ [(q^{(1)}(x_1, t) + \overline{q}^{(1)}(x_1, t))\,G^{(2)}(x_2, t) + (1 \leftrightarrow 2)]$$
$$\times [\alpha_s(t)\,\theta(1 - z_{12})(f^{\mathrm{DY}}_{\mathrm{G}}(z_{12}) - f_{\mathrm{G},2}(z_{12}))]\}\,, \tag{90}$$

where α_s has been renormalization group improved to $\alpha_s(t)$.

Restoring all factors we therefore write the corrected Drell-Yan formula (to order $\alpha_s(t)$):

$$\frac{\mathrm{d}\sigma^{\mathrm{DY}}}{\mathrm{d}Q^2} \simeq \frac{4\pi\alpha^2}{9SQ^2} \int_0^1 \frac{\mathrm{d}x_1}{x_1} \int_0^1 \frac{\mathrm{d}x_2}{x_2} \{ [\sum_f e_f^2 q_f^{(1)}(x_1, t)\,\overline{q}_f^{(2)}(x_2, t) + (1 \leftrightarrow 2)]$$
$$[\delta(1 - z_{12}) + \alpha_s(t)\,\theta(1 - z_{12})(f^{\mathrm{DY}}_{\mathrm{q}}(z_{12}) - 2f_{\mathrm{q},2}(z_{12}))]$$
$$+ [\mathcal{F}_2^{(1)}(x_1, t)\,G^{(2)}(x_2, t) + (1 \leftrightarrow 2)]\,[\alpha_s(t)\,\theta(1 - z_{12})$$
$$(f^{\mathrm{DY}}_{\mathrm{G}}(z_{12}) - f_{\mathrm{G},2}(z_{12}))]\}\,. \tag{91}$$

The explicit evaluation of $f^{\mathrm{DY}}_{\mathrm{G}}(z)$ and $f^{\mathrm{DY}}_{\mathrm{q}}(z)$ can now be carried out from the diagrams in figs. 4 and 5 in the massless theory with the initial partons off mass shell with virtual masses $p_1^2, p_2^2 < 0$. The results are ⋆:

$$\alpha_s\,[f^{\mathrm{DY}}_{\mathrm{G}}(z) - f_{\mathrm{G},2}(z)] = \frac{\alpha_s}{2\pi}\,[\tfrac{1}{2}(z^2 + (1 - z)^2)\ln(1 - z) + \tfrac{9}{4}z^2 - \tfrac{5}{2}z + \tfrac{3}{4}]\,, \tag{92}$$

⋆ Note that in $f_{\mathrm{G},2}(z)$ the double-pole term in the $\cos\theta$ integral (see sect. 3) is cancelled by a similar term in the cross section for process (84). Note that,

$$\int_0^1 \mathrm{d}z z^{n-1} \left(\frac{\ln(1 - z)}{1 - z}\right)_+ \equiv \int_0^1 \mathrm{d}z\, \frac{(z^{n-1} - 1)}{(1 - z)} \ln(1 - z)\,.$$

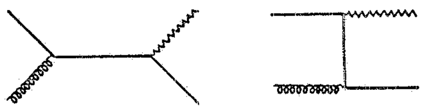

Fig. 4. Diagrams of order g_s contributing to the process $q(\bar{q}) + G \to q(\bar{q}) + \gamma^*$. The wavy line indicates the massive photon.

$$\alpha_s[f_q^{DY}(z) - 2f_{q,2}(z)] = \frac{4}{3}\frac{\alpha_s}{2\pi}\left[\frac{3}{(1-z)_+} - 6 - 4z + 2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + \left(\frac{\pi^2}{3} - 1\right)\delta(1-z)\right]. \quad (93)$$

We stress once again that these are the corrections appropriate for effective quark densities related to $\mathcal{F}_2$ *via* the naive quark model expression. Note that the difference in eq. (92) does not vanish at $z = 1$, unlike the quantity for $f_{G,1}(z) - f_{q,2}(z)$ which governs the gluon corrections in leptoproduction.

In practice, corrections of order $\alpha_s(t)$ arising from the gluon term are of importance only in the case of proton-nucleon collisions, while they are expected to be negligible in proton-antiproton collisions. However, the $q\bar{q}$ terms of order α_s which are surprisingly large can never be neglected ★.

In order to estimate the quantitative importance of the gluon correction we plotted in fig. 6 the ratio of gluon corrections to the Drell-Yan formula and the unmodified expression (i.e., without the correction terms in eq. (91)) for $\sqrt{S} \simeq$ 27 GeV as in the data of ref. [18]. We also set $\Lambda \simeq 0.5$ GeV in eq. (9) (with the b' term neglected). As a further simplification. We considered densities with no t dependence given by eqs. (81), except that the SU(3) symmetric sea (with no charm) was taken as:

$$xS(x) = (N_s + 1)\, 0.034(1-x)^{N_s}\,, \quad (94)$$

where N_s is a number to which we assign various integer values for the purposes of illustration.

We see that the gluon correction is in fact negligible in $\bar{P}$-nucleus collisions, where the main correction arises from the $q\bar{q}$ terms of order $\alpha_s(t)$ which are rather large.

On the other hand in the P-nucleus case the size of the correction is in general much larger. It is clearly very dependent on the detailed x behavior of sea *versus* gluon densities. In fact for fixed τ the densities enter in eq. (91) at $x \geqslant \tau$, so that if the gluons drop much less rapidly in x than the sea, the correction becomes catastrophic, even for moderate τ values as indicated by the curve with $N_s = 5$ or 7 the correction, although important, has a more reasonable magnitude. We take this as

★ Numerical details will be given in a forthcoming publication. In the following we consider only the gluon corrections.

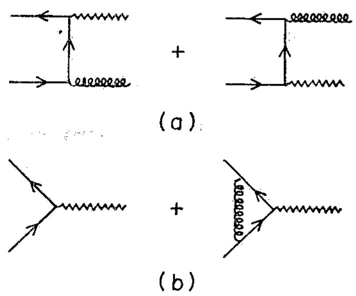

Fig. 5. (a) Diagrams of order α_s contributing to the process $q + \bar{q} \to G + \gamma^*$. (b) The corrections due to the exchange of virtual gluons. Quark wave-function renormalization must be taken into account.

a strong argument against too naive parametrisations of the parton densities in terms of a single power of $(1 - x)$. In any case it is unreasonable, as stressed in sect. 5, to allow the gluons to dominate the sea densities too drastically at large x. In fact it is physically implausible that the correction term in eq. (91) becomes too large, because the sea density itself is to be ascribed to the gluon corrections through the leading logarithmic terms which were absorbed in the definition of the densities. Thus we expect the subleading series to result in a moderate correction as is the case in fig. 5 for $N_s \simeq 5$–6. We recall that the high value of N_s ($N_s \sim 9$) found in fitting the data with the naive Drell-Yan formula and t independent densities is system-

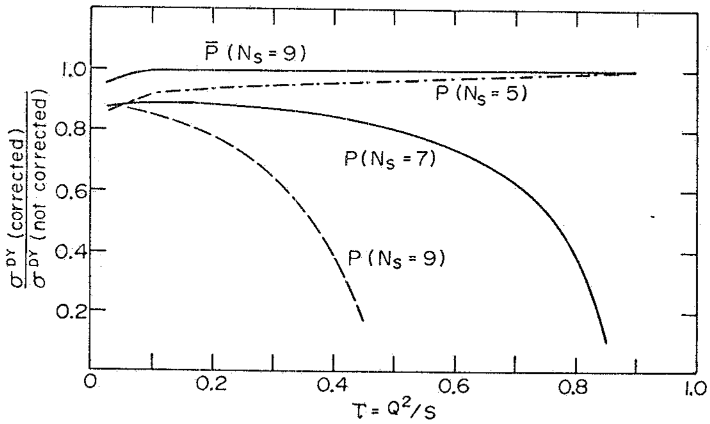

Fig. 6. The ratio of the Drell-Yan formula including the gluon corrections in eq. (91) to the naive version eq. (82) for $\bar{P}$- or P-nucleus scattering with $\sqrt{s} \simeq 27$ GeV. Parton densities are fixed according to eq. (81) for valence and gluons and to eq. (94) for the sea. In particular the gluons are taken to behave as $(1 - x)^5$ and the sea as $(1 - x)^{N_s}$. $\alpha_s(t)$ is taken to be $\alpha_s(t) \simeq (b \ln Q^2/\Lambda^2)^{-1}$ with $\Lambda \simeq 0.5$ GeV. The $q\bar{q}$ corrections are also important and should be taken into account.

atically and substantially reduced by taking the t dependence into account as shown in ref. [19]. We are therefore of the opinion that by taking the t dependence into proper account and assuming a more realistic parametrisation of the parton densities the data can be fitted by the corrected Drell-Yan formula with a reasonably small contribution from the gluon terms (not exceeding 25% for $\tau \leqslant 0.3$). In other words we do not agree with the pessimistic view expressed in ref. [20] and we also think that order $\alpha_s^2(t)$ corrections arising from qG or qq terms are indeed negligible over almost the whole range of τ. We plan to demonstrate these statements by a detailed comparison with the data in a forthcoming publication.

7. Summary and conclusions

In the previous sections we gave a precise definition of the coefficient functions for leptoproduction and described their evaluation in detail. We then proposed a physically motivated definition of the parton densities. Our definition has the merit that both the corrections to the structure functions in subleading order in $\alpha_s(t)$ and the corrections to the t derivatives of the parton densities are independent of the renormalisation prescription. However, since the calculations of $c'^{(n)}_{q/G,2}$ were given in full the reader may modify the definition of the parton densities at will.

The correction terms are of practical importance especially for quantities determined in lowest order by the small sea densities. As a case in point we considered the gluon corrections to the Drell-Yan formula for proton-nucleus collisions, in terms of the densities measured in electroproduction. The main lesson which we learn is that the valence, sea and gluon densities must respect the strong correlations in their x distributions implied by the evolution equations and the form of the correction terms. If these correlations are neglected, paradoxes arise in the form of unreasonably large corrective terms in some regions of phase space. These paradoxes are simply manifestations of the incompatability of the proposed parametrization with the underlying theoretical structure. We re-emphasize the utmost importance of precise measurements of longitudinal structure function in leptoproduction; this quantity provides one of the most direct tests of the theory and supplies information on the distribution of gluons in the nucleon. Better knowledge of the gluon distribution is the key to a considerable improvement of the predictive power of QCD for other processes.

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