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M. Greco, G. Penso and Y. Srivastava:

QCD AND DUALITY IN e^+e^- ANNIHILATION.

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ABSTRACT.

We present a new method, based on duality, for smoothing the ratio $R \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. Our method allows us to extract from all available data the three light and charm quark contributions seperately which are successfully compared with QCD predictions. We find $m_c = 1.45 \pm 0.05$ GeV and $\Lambda^2 = 0.5 \div 0.6$ GeV^2. The method itself provides a self consistency check of semilocal duality.

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Recent experiments in $e^+ e^-$ annihilation into hadrons have provided an accurate determination of $R(s) \equiv \frac{\sigma(e^+ e^- \to \text{hadrons})}{\sigma(e^+ e^- \to \mu^+ \mu^-)}$ also in the low and medium energy region. Thus, R is reliably known from threshold up to 8 GeV$^{(1)}$. Such high quality data ought naturally to provide for precise tests of proposed models and theories. However, a direct confrontation with theoretical predictions has been hampered by the local fluctuations in R due to resonances as well as the presence of multihadron thresholds. To this end, in this paper we present a method based on duality which helps smooth out R. Our procedure simultaneously allows for a check of the concept of duality itself in $e^+ e^-$ annihilation $^{(2)}$.

Earlier, to overcome the above-mentioned difficulty two different methods have been suggested. In the first one $^{(3,4)}$, the experimental data are extrapolated from the time-like to the space-like region via dispersion relations, and then compared with the theoretical QCD predictions for the function $D(Q^2)$ defined as

$$D(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R(s)\, ds}{(Q^2 + s)^2} = \left(\frac{3\pi}{\alpha}\right) Q^2 \left. \frac{d\Pi(s)}{ds} \right|_{s=-Q^2} \qquad (1)$$

where $\Pi(s)$ is the hadronic contribution to the vacuum polarization tensor. Ignorance of R beyond the energy region accessible experimentally, directly introduces an indeterminacy in $D(Q^2)$, depending upon the assumptions-regarding the number and mass of heavy flavors-made for the extrapolation of the data.

In the second approach $^{(5)}$ a smearing procedure has been suggested which is applied directly in the time-like region. To be precise, the quantity

$$\bar{R}(s, \Delta) = \left(\frac{\Delta}{\pi}\right) \int_{4m_\pi^2}^{\infty} \frac{ds'\, R(s')}{(s' - s)^2 + \Delta^2} \qquad (2)$$

is compared with the theoretical prediction for R(s) given by QCD perturbation theory. The price one has to pay here is the introduction of an arbitrary parameter Δ.

As stated before, our smoothing procedure is based on duality, which has been successfully applied earlier for this process (2). Given the experimental value, $R_{exp}(s)$, we construct the zero-th moment

$$M(\bar{s}) = \int_{4 m_\pi^2}^{\bar{s}} R_{exp}(s)\, ds \qquad (3)$$

Figure (1a) shows the values of $R_{exp}(s)$ used by us. The data have been organized as follows. For W ($\equiv \sqrt{s}$) from 0.1 to 0.9 GeV, the dominat contribution is due to the ϱ-resonance for which the Gounaris-Sakurai form (6) has been assumed. ω and Φ resonances are introduced as δ-functions. From 0.9 to 3.45 GeV, all available data (1) from ACO, VEPP-2M, DCI-M3N, ADONE- $\gamma\gamma$2, ADONE-MEA and SPEAR-MARK I have been averaged over 100 ÷ 200 MeV intervals as shown in Fig. (1a). In the interval 3.45 to 7.8 GeV, we have utilized data (1) from DELCO, DASP, PLUTO and SPEAR - MARK I, whenever available. The heavy lepton contribution has been subtracted. Also, radiative corrections have been applied (7). The band shown in Fig. (1a) is mainly due to systematic discrepancies between experiments in the interval 15 ÷ 25 GeV^2 and to statistical errors above 25 GeV^2. Using the above inputs, we plot $M(\bar{s})$ in Fig. (1b). Here the band reflects only the systematic discrepancies between various experiments, the statistical error on the integral being negligible.

Clearly, the local structures present in $R_{exp}(s)$ have almost disappeared in $M(\bar{s})$. This, therefore, allows a much cleaner separation of various quark contributions. In fact, these data naturally divide into

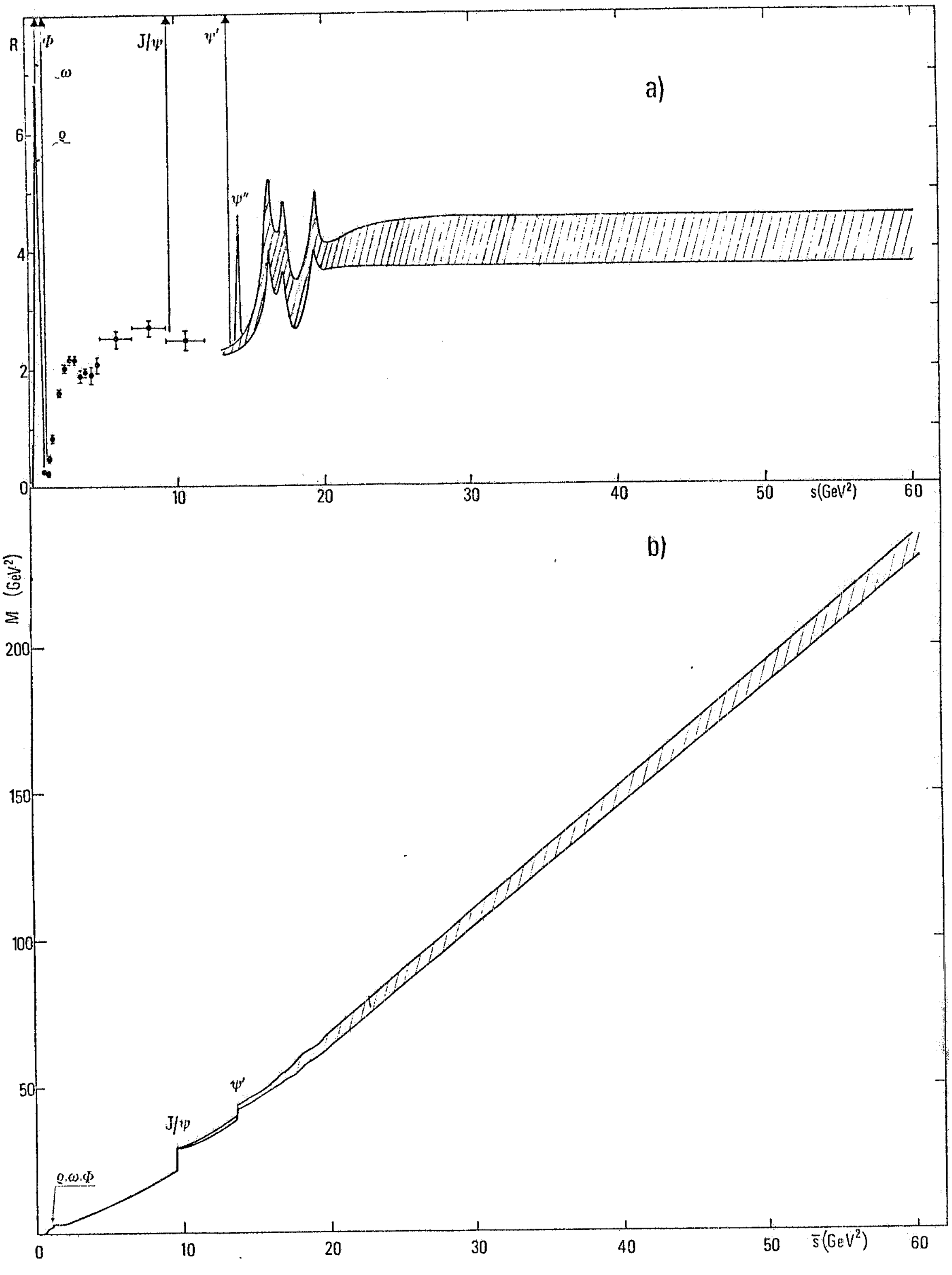

Fig. 1 - (a) $R_{exp}(s)$ vs. s as explained in the text. (b) $M(\bar{s})$ vs. $\bar{s}$.

three regions with effective slopes: 2 ($s \simeq 0 \div 5$ GeV^2); 2·5 ($5 \div 9$ GeV^2) and 4·2 ($9 \div 60$ GeV^2).

For clarity, we present seperately in Fig. (2), the low energy data alone on a bigger scale. This figure confirms that the average value of R between $0 \div 9$ GeV^2 agrees with the value of R averaged over the ρ-ω Φ - resonances. This gives us confidence in the idea of semi-local duality.

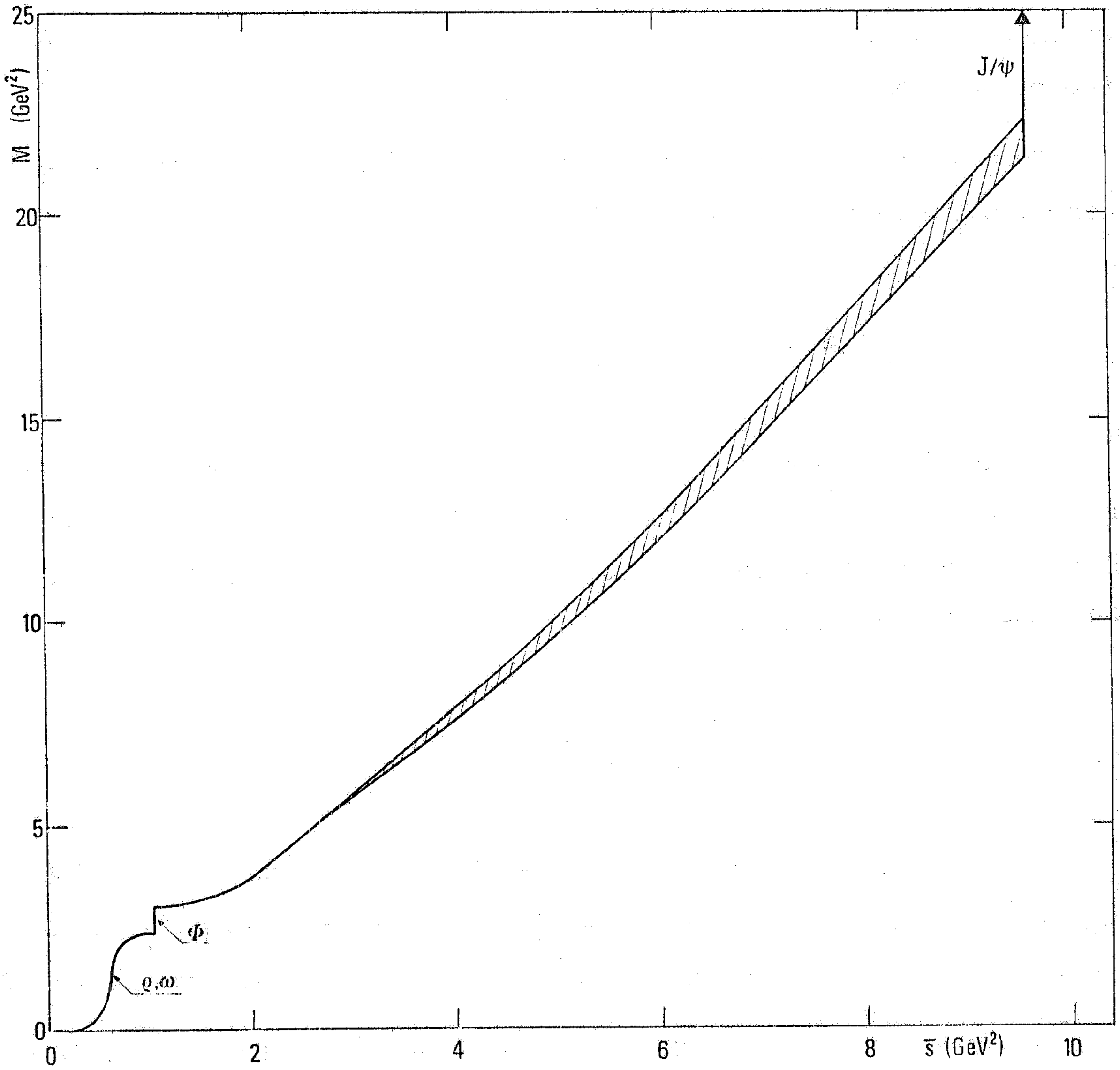

Fig. 2 - M($\bar{s}$) vs. $\bar{s}$ for the energy region $\bar{s} \lesssim 10$ GeV^2.

A more accurate test of duality in this region can not be made at present because of the lack of seperation of s-quark contribution from those of u and d-quarks. This also shows up in the Φ continuum which apparently sets in later (around $s \gtrsim 5$ GeV^2) and which may be responsible for the change in the slope from 2 to 2. 5. The above conclusions rely on the experimental assumption that at low energies ($s \simeq 2 \div 6$ GeV^2) all produced particles are pions. Indeed, the number of kaons has been observed $^{(1)}$ to increase with energy and due to the smaller detection efficiency for kaons (relative to pions) at low energies, the true $R_{exp}(s)$ may be higher than the value shown in Fig. (1a). In turn, this might further reduce the difference between the two slopes (i.e. 2 and 2. 5).

At this point we can make a simple consistency check with first order correction expected from QCD:

$$R(s) \simeq 3 \sum_{i=u,d,s} Q_i^2 \left(1 + \frac{\alpha(s)}{\pi}\right)$$

Taking 3 massless quarks only, we have $\alpha(s) \simeq \dfrac{4\pi}{9 \ln(s/\Lambda^2)}$. Thus, for a value of $\Lambda^2 \simeq 0.5$ GeV^2 we find $R(\sim 9$ GeV$^2) \simeq 2.30$. (The inclusion of the c-quark changes this result by $\lesssim 1\%$). We compare with a smoothed value, R_{av}, defined as

$$R_{av}(\bar{s}) = \frac{1}{\bar{s}} \int_{4 m_\pi^2}^{\bar{s}} R_{exp}(s)\, ds$$

From figure (2), we find $R_{av}(\sim 9$ GeV$^2) = 2.27 \pm 0.05$ (statistical error only) in good agreement with the above value. Notice that due to the summation over R_{exp}, the statistical error on R_{av} is considerably reduced.

Let us now discuss the contribution to R of charm quark alone. Due to the distance of this threshold from those of all other quarks, it is possible to isolate this quark's contribution $M_c(\bar{s})$. This is acheived by sub tracting from $M(\bar{s})$ for $\bar{s} > 9\ \mathrm{GeV}^2$ - the contribution of light quarks evaluated using the mean value $R_{\mathrm{light}}(s) \simeq 2.4 \pm 0.1$. The result is plotted in fig. (3). This figure provides a precise test of duality since the straight line behaviour of $M_c(\bar{s})$ when extrapolated down to $(8 \div 9)\ \mathrm{GeV}^2$ averages quite accurately over the low-lying resonances J/ψ and ψ'. This aspect of duality has been noticed previously $^{(2)}$.

We evaluate the effective charm threshold and deduce therefrom the c-quark mass using two different parametrizations. First, we use the naive formula

$$R_c^{(1)}(s) = \bar{R}_c\ \theta(s - s_c) , \tag{4}$$

and then the QCD prediction$^{(5)}$

$$R_c^{(2)}(s) = (3\,Q_c^2)\ \frac{1}{2}\ \beta(3 - \beta^2)\left[1 + \frac{4}{3}\ f(\beta)\,\alpha(s)\right] , \tag{5}$$

where $Q_c = 2/3$ is the charge of the c-quark, $\beta = \sqrt{1 - 4m_c^2/s}$

$$f(\beta) = \frac{\pi}{2\beta} - \frac{(\beta + 3)}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right) , \tag{6}$$

and$^{(8)}$

$$\alpha(s) \gtrsim \frac{12\pi}{27\ \ln(s/\Lambda^2) - 2\ \ln\left(\dfrac{s + 5m_c^2}{\Lambda^2 + 5m_c^2}\right)} \tag{7}$$

Using eq. (4) and the data, we find $\frac{1}{2}\sqrt{s_c} \doteq \tilde{m}_c \simeq (1.3 \div 1.4)$ GeV and $\bar{R}_c \simeq (1.6 \div 1.8)$. If we interpret $\bar{R}_c$ to be $(3\,Q_c^2)$ then the above value for the charge Q_c is too high. As we show now, the QCD correction amelio-

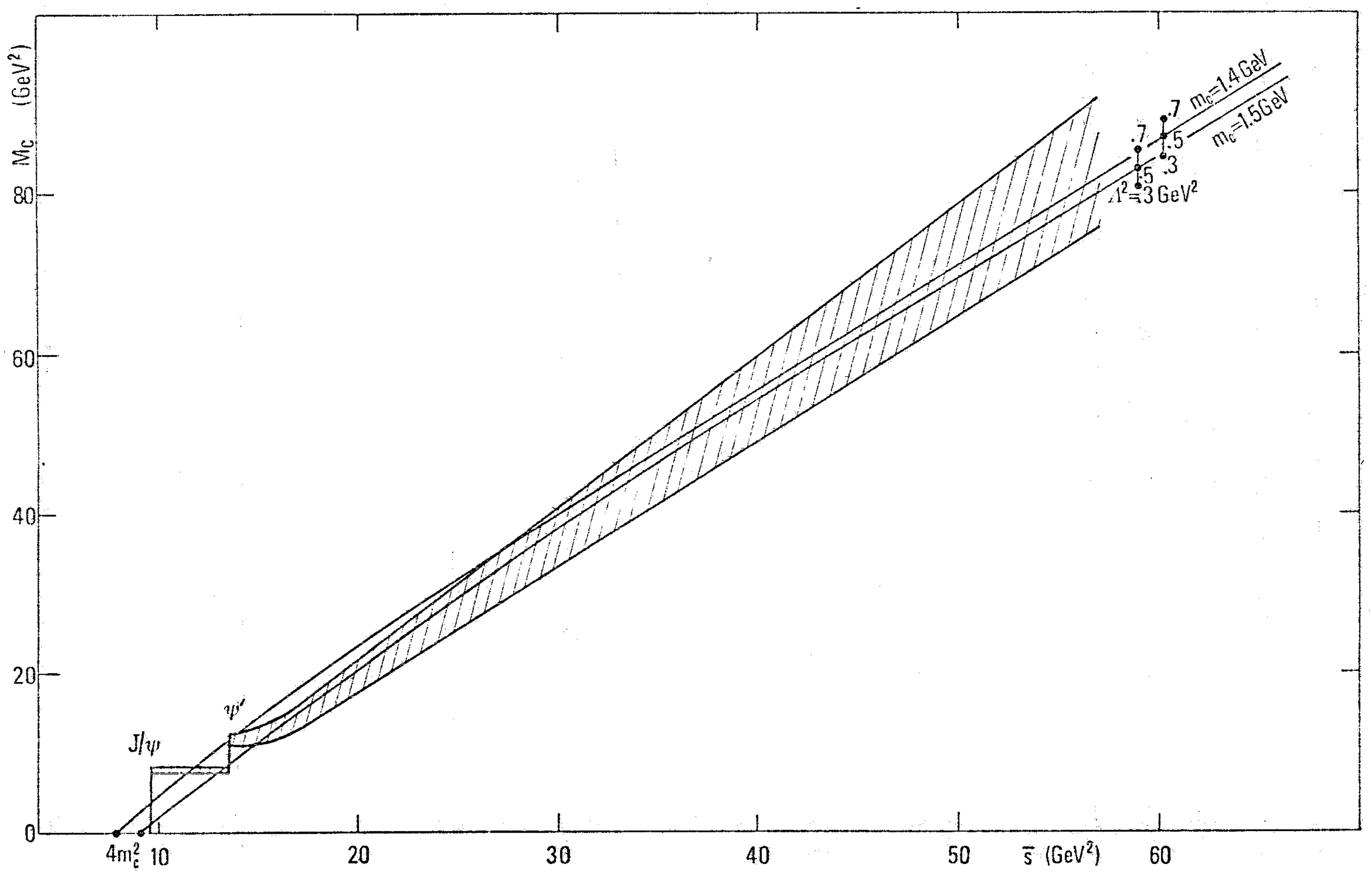

Fig. 3 - $M_c(\bar{s})$ vs. $\bar{s}$. The band in the figure takes into account both the experimental error associated with $R_{exp}(s)$ as well as the error in the subtraction. The theoretical curves are for m_c = 1.4 (1.5) GeV and Λ^2 = 0.5 GeV^2. For each mass value, a representative point for Λ^2 = 0.3 and 0.7 GeV^2 is also shown.

rates this situation. In figure (3), the QCD result as given by eq. (5) is plotted for different values of m_c = 1.4 GeV and 1.5 GeV and Λ^2 = 0.3, 0.5 and 0.7 GeV^2. It is clearly seen from the figure that the QCD expression with the correct value of the charge, a mass value (1.45 ± 0.05) GeV and Λ^2 = (0.5 ÷ 0.6) GeV^2 reproduces the data quite well.

We consider the above as a success of QCD and our smoothing procedure. We stress again that our method uses all data, including narrow resonances like J/ψ and ψ'. In fact, the amount of resonance contributions allows us to verify the basic hypothesis of semi-local duality. On the contrary, the smearing procedure of reference (5) discards such narrow structures.

Using the result of the above analysis we can also discuss the charm contribution to the function D(Q^2) in the space like region. From eqs. (1) and (5) and the obtained values of m_c = 1.45 GeV and Λ^2 = 0.5 GeV^2, we plot in figure (4), $D^c_{th}(Q^2)$. For comparison with experimental data, we also plot $D^c_{exp}(Q^2)$ constructed in the following way. The contribution

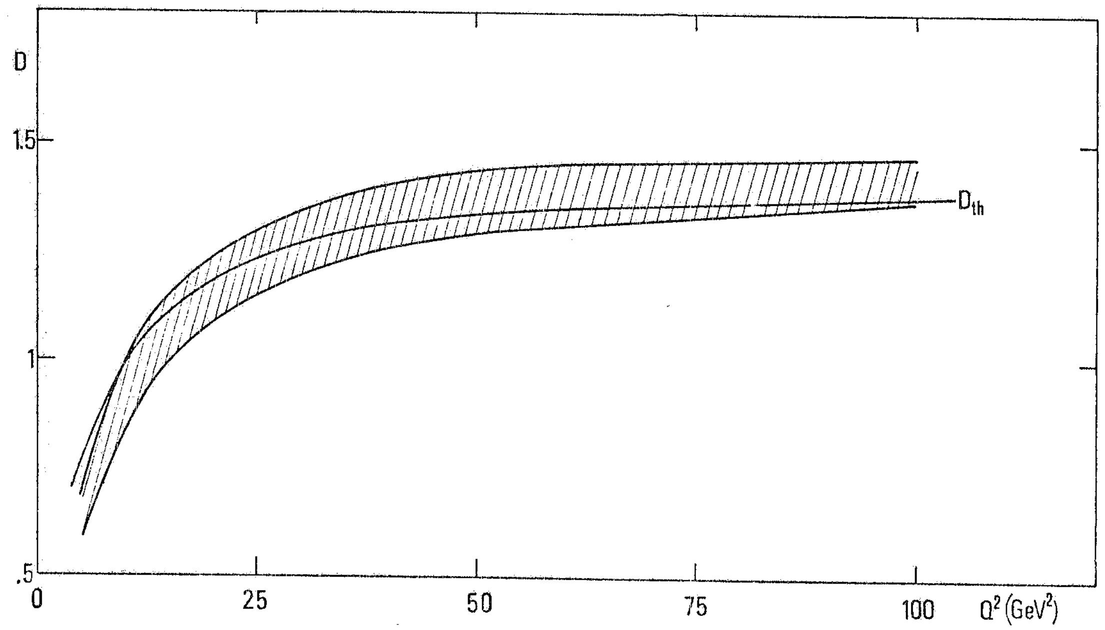

Fig. 4 - $D^c(Q^2)$ vs. Q^2. The solid line is $D^c_{th}(Q^2)$ for m_c= 1.45 GeV and Λ^2 = 0.5 GeV^2. The experimental band for D^c exp (Q^2) is obtained as in Fig. 3.

of the u, d and s quarks to $R_{\exp}(s)$ is subtracted as before, in the range $4m_c^2 \leq s \leq 60\ \mathrm{GeV}^2$. For $s \geq 60\ \mathrm{GeV}^2$, $R_c(s)$ has been extrapolated using the QCD formula (5). Once again the agreement is quite reasonable.

In conclusion, we have presented a duality-based smoothing procedure for R(s), which led us to make a rather direct comparison with the QCD predictions. The method allows us to use all the experimental data down to threshold and gives excellent support for semi-local duality. Isolating the charm contribution, we find $m_c = (1.45 \pm 0.05)$ GeV. All our analysis in space and time-like region is consistent with the QCD predictions for a value of $\Lambda^2 = (0.5 \div 0.6)\ \mathrm{GeV}^2$. Obviously, a similar analysis for b (bottom) and higher mass quarks can be attempted when higher energy data become available.

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FOOTNOTES AND REFERENCES.

1. J. E. Augustin et al. Phys. Rev. Letters 34, 764 (1975); V. A. Sidorov presented at XVIII Int. Conf. on High Energy Physics, Tbilisi 1976; G. J. Feldman, presented at XIX Int. Conf. on High Physics, Tokyo 1978; G. P. Murtas, ibid; J. P. Perez-Y-Jorba, ibid; R. Baldini-Celio et al. , Lett. Nuovo Cimento, to be published; C. Bacci et al. , Phys. Lett. to be published; B. Esposito et al. , Lett. Nuovo Cimento 19, 21 (1977); B. H. Wiik and G. Wolf, Desy Report 78/23 (1978), and references therein.
2. For earlier applications of duality in annihilation, see for instance: A. Bramon, E. Etim and M. Greco, Phys. Lett. 41B, 609 (1972); M. Greco, Nucl. Phys. 63B, 398 (1973); J. J. Sakurai, Phys. Lett. 46B, 207 (1973); E. Etim and M. Greco, Nuovo Cimento Lett. 12, 91 (1975); M. Greco, Phys. Lett. 77B, 84 (1978).
3. S. Adler: Phys. Rev. D10, 3714 (1974).
4. A. De Rujula and H. Georgi: Phys. Rev. D13, 1296 (1976).
5. E. Poggio, H. Quinn and S. Weinberg: Phys. Rev. D14, 1829 (1976).
6. G. Gounaris and J. Sakurai: Phys. Rev. Lett. 21, 244 (1968).
7. M. Greco, G. Pancheri - Srivastava and Y. Srivastava: Nucl. Physics B101, 234 (1975); F. Yndurain: Nucl. Phys. B136, 533 (1978).
8. A. slightly different expression is obtained by O. Nachtmann and W. Wetzel, Heidelberg preprint HD - THEP -78 - June 1978, and verified by R. Barbieri and G. Curci. However, the numerical change is very little for our purposes. Thus, we continue to use formula (7). We thank G. Curci for communicating to us these results.

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