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TRANSVERSE MOMENTUM OF JETS IN ELECTROPRODUCTION FROM QUANTUM CHROMODYNAMICS

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Detailed predictions of QCD on transverse momentum distributions of jets in leptoproduction are presented. The average $p_\perp^2$ is found to be asymptotically proportional to W^2 (the hadronic invariant mass squared) with a coefficient function that depends little on x and y. Contributions from gluons in the nucleon are relatively smaller than for σ_L/σ_T.

In the framework of quantum chromodynamics (QCD) the problem of extending the theory of deep inelastic processes beyond the level of totally inclusive cross sections [1] has recently received renewed attention. In this paper we make a quantitative study of the transverse momentum distribution of jets produced in deep inelastic electron (or muon) production. Predictions along the same lines have already been given for the $p_\perp$ distributions of muon pairs produced in hadronic collisions [2–4] and for the jet structure in e^+e^- annihilations [5]. Hard gluon bremsstrahlung and other parton processes of order α_s (the QCD coupling $\alpha_s \equiv g_s^2/4\pi$) induce logarithmic scaling violations and also produce hadronic jets in the final state with transverse momentum that increases with $\sqrt{Q^2}$. Renormalization group techniques applied to coefficients in the light cone expansion teach us how to reconstruct the asymptotic form of structure functions by a judicious use of the lowest-order parton diagrams [6]. We are therefore led to expect that further more detailed but closely related predictions from parton dynamics are also asymptotically satisfied.

To order α_s the two parton processes relevant for events with large $p_\perp$ in leptoproduction are:

$$q_i(\bar{q}_i) + \gamma^* \to q_f(\bar{q}_f) + G, \tag{1}$$

$$G + \gamma^* \to q + \bar{q}, \tag{2}$$

where q ($\bar{q}$) is a quark (antiquark), G a gluon and γ^* the virtual photon of mass squared Q^2. Events arising from the parton processes (1) and (2) appear as three jet events: the nucleon fragments with small $p_\perp$ and the two final parton jets with nearly opposite and generally large $p_\perp$. We are interested in the hard component (i.e. of order $\sqrt{Q^2}$) of the $p_\perp$ spectrum of the final partons. We therefore neglect the $p_\perp$ of partons inside the nucleon and identify the momentum of each final parton with that of the corresponding hadronic jet.

We define as usual

$$x = Q^2/2(Pq) = Q^2/2M\nu, \quad y = \nu/E_\ell, \tag{3}$$

where P and q are the nucleon and the photon momenta and E_ℓ is the lab energy of the incoming lepton. We work in the frame where γ^* has zero energy. In this frame we define:

$$z = Q^2/2(Kq), \quad x \leqslant z \leqslant 1, \tag{4}$$

$$c \equiv \cos\theta, \tag{5}$$

where K is the incoming parton momentum, and θ is the angle between q_i ($\bar{q}_i$) and G in process (1) or between G and $\bar{q}$ in process (2). For both processes we evaluate the cross sections off transverse and longitudinal photons. The normalization of the related quantities:

$$\sigma^{T,L}(z) = \int_{-1}^{1} \frac{d\sigma^{T,L}(z,c)}{dc}\, dc, \tag{6}$$

are fixed by specifying the contributions of process (1) and (2) to the structure functions of electro-production:

$$2F_1(x) = \int_x^1 \frac{du}{u}\left\{\sum_{i\equiv q,\bar{q}} q_i(u)e_i^2 \times\left[\delta\left(\frac{x}{u}-1\right)+\frac{\alpha_s}{2\pi}\sigma^T_{q\gamma^*}\left(\frac{x}{u}\right)\right] + G(u)\left(\sum_{i\equiv q} e_i^2\right)\frac{\alpha_s}{2\pi}\sigma^T_{G\gamma^*}\left(\frac{x}{u}\right)\right\}, \tag{7}$$

$$\frac{F_2(x)}{x} - 2F_1(x) = \int_x^1 \frac{du}{u}\left\{\sum_{i\equiv q,\bar{q}} q_i(u)e_i^2 \frac{\alpha_s}{2\pi}\sigma^L_{q\gamma^*}\left(\frac{x}{u}\right) + G(u)\left(\sum_{i\equiv q} e_i^2\right)\frac{\alpha_s}{2\pi}\sigma^L_{G\gamma^*}\left(\frac{x}{u}\right)\right\}, \tag{8}$$

where q_i and G are the quark (or antiquark) and gluon parton densities and e_i is the charge of q_i in units of the proton charge. With these conventions we find:

$$\frac{d\sigma^T_{q\gamma^*}}{dc} = \frac{4}{3}\frac{2z}{[1+(2z-1)c]^2}\left\{\frac{z(1+z^2)}{1-z}\frac{1+c}{1-c} + (1-z)^2 + \frac{(1-z)[1-c+2z^2(1+c)]}{1+(2z-1)c}\right\}, \tag{9}$$

$$\frac{d\sigma^T_{G\gamma^*}}{dc} = \frac{z(1-z)}{[1+(2z-1)c]^2}\left\{\frac{z}{1-z}\frac{1+c}{1-c} + \frac{1-z}{z}\frac{1-c}{1+c} - \frac{2[1+(2z-1)c]^2}{1-c^2} + 4z(1-z)\right\}, \tag{10}$$

$$\frac{d\sigma^L_{q\gamma^*}}{dc} = \frac{4}{3}\frac{8z^3(1-z)(1+c)}{[1+(2z-1)c]^3} \quad \left(\sigma^L_{q\gamma^*}(z) = \tfrac{8}{3}z\right), \tag{11}$$

$$\frac{d\sigma^L_{G\gamma^*}}{dc} = \frac{8z^2(1-z)^2}{[1+(2z-1)c]^2} \quad \left(\sigma^L_{G\gamma^*}(z) = 4z(1-z)\right), \tag{12}$$

where the integrals for σ^L (see eq. (6)) have also been written down. Note that the corresponding integrals for σ^T that appear in eq. (7) would diverge logarithmically without a suitable regularization of the singularities at $c = \pm 1$ in eqs. (8) and (9) (for example by giving a mass to the quark). This well-known feature which is related to the appearance of scaling violations, is of no importance when moments in $p_\perp$ are considered. In fact the transverse momentum of the final partons is given by:

$$p_\perp^2 = Q^2(1-z)^2(1-c^2)/[1+(2z-1)c]^2, \tag{13}$$

so that moments of $p_\perp$ are independent of the regularization mass. Note in particular that:

$$(p_\perp^2)_{max} = Q^2(1-z)/4z = \tfrac{1}{4}W_z^2, \tag{14}$$

where W_z is the invariant mass of the two final partons.

It is instructive to consider the longitudinal structure function $F_L(x) \equiv F_2 - 2xF_1$. From eqs. (8), (11), (12) one finds:

$$\begin{aligned} F_L(x,Q^2) &= \frac{\alpha_s(Q^2)}{2\pi}x^2\left\{\int_x^1 \frac{du}{u^3}\left[\frac{8}{3}\sum_{i\equiv q,\bar{q}}(uq_i(u,Q^2)e_i^2) + 4\left(\sum_{i\equiv q}e_i^2\right)(uG(u,Q^2))\left(1-\frac{x}{u}\right)\right]\right\} \\ &= \frac{\alpha_s(Q^2)}{2\pi}x^2\left\{\int_x^1 \frac{du}{u^3}\left[\tfrac{8}{3}F_2(u,Q^2) + 4\left(\sum_{i\equiv q}e_i^2\right)(uG(u,Q^2))\left(1-\frac{x}{u}\right)\right]\right\} \end{aligned} \tag{15}$$

($\Sigma_{i\equiv q}e_i^2 = \frac{10}{9}$ for 4 quarks). This is the exact asymptotic limit of F_L (up to terms of order $\alpha_s^2(Q^2)$) as proved by renormalization group techniques and the light cone operator expansion [7,8] [‡1], and is obtained from the lowest-order calculation by replacing α_s with $\alpha_s(Q^2)$ and folding with the Q^2 dependent parton densities. It is exactly the same procedure that we shall use in the following in order to evaluate the aver-

[‡1] In ref. [7] a Casimir operator $C_2(G)$ has to be replaced by $T(R)$. See ref. [8] eqs. (11), (13).

age value of $p_\perp^2$ as function of x and y. Other moments in $p_\perp$ could similarly be obtained from the angular distributions in eqs. (8)–(12).

The validity of this approach has been recently supported by computations in second-order perturbation theory [9]. From eqs. (9) to (13) we obtain:

$$p^{\mathrm{T}}_{\mathrm{q}\gamma^*}(z) \equiv \int \frac{p_\perp^2}{Q^2}\frac{\mathrm{d}\sigma^{\mathrm{T}}_{\mathrm{q}\gamma^*}}{\mathrm{d}p_\perp^2}\,\mathrm{d}p_\perp^2 = \frac{4}{3}\,\frac{4z^2-2z+7}{12z}, \tag{16}$$

and similarly:

$$p^{\mathrm{T}}_{\mathrm{G}\gamma^*}(z) = \frac{1-z}{3z}\,[z^2+(1-z)^2], \tag{17}$$

$$p^{\mathrm{L}}_{\mathrm{q}\gamma^*}(z) = \tfrac{4}{9}(1-z), \quad p^{\mathrm{L}}_{\mathrm{G}\gamma^*}(z) = \tfrac{2}{3}(1-z)^2. \tag{18}$$

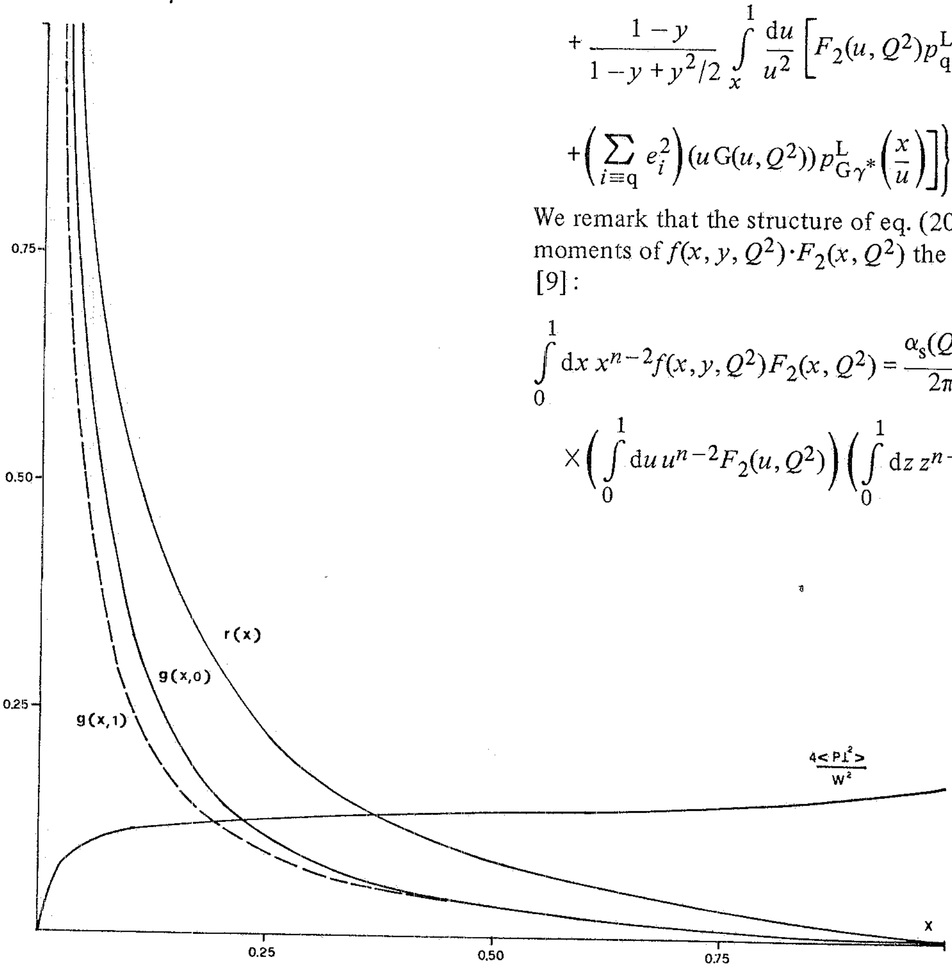

Fig. 1. Plots of $\langle p_\perp^2\rangle/Q^2 \approx \alpha_s(Q^2)g(x,y)$ for $y=0$ (——), $y=1$ (– – –), of $F_{\mathrm{L}}(x,Q^2)/F_2(x,Q^2) \approx \alpha_s(Q^2)r(x)$ and of $4\langle p_\perp^2\rangle/W^2$ for $y=1$ (and with $\alpha_s(Q^2)=1$).

Recalling that in electroproduction:

$$\mathrm{d}\sigma/\mathrm{d}x\,\mathrm{d}y \sim [(1-y)F_2(x)+\tfrac{1}{2}y^2(2xF_1(x))], \tag{19}$$

we are finally led to (up to terms of higher order in $\alpha_s(Q^2)$):

$$\begin{aligned}\frac{\langle p_\perp^2\rangle}{Q^2} \equiv f(x,y,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\,\frac{x}{F_2(x,Q^2)}\Bigg\{ &\int_x^1 \frac{\mathrm{d}u}{u^2} \times \Bigg[F_2(u,Q^2)p^{\mathrm{T}}_{\mathrm{q}\gamma^*}\left(\frac{x}{u}\right) + \left(\sum_{i\equiv\mathrm{q}} e_i^2\right)(u\mathrm{G}(u,Q^2))\,p^{\mathrm{T}}_{\mathrm{G}\gamma^*}\left(\frac{x}{u}\right)\Bigg] \\ &+ \frac{1-y}{1-y+y^2/2}\int_x^1\frac{\mathrm{d}u}{u^2}\Bigg[F_2(u,Q^2)p^{\mathrm{L}}_{\mathrm{q}\gamma^*}\left(\frac{x}{u}\right) + \left(\sum_{i\equiv\mathrm{q}} e_i^2\right)(u\mathrm{G}(u,Q^2))p^{\mathrm{L}}_{\mathrm{G}\gamma^*}\left(\frac{x}{u}\right)\Bigg]\Bigg\}.\end{aligned} \tag{20}$$

We remark that the structure of eq. (20) implies for x moments of $f(x,y,Q^2)\cdot F_2(x,Q^2)$ the simple form [9]:

$$\int_0^1 \mathrm{d}x\, x^{n-2} f(x,y,Q^2)F_2(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \times \left(\int_0^1 \mathrm{d}u\, u^{n-2}F_2(u,Q^2)\right)\left(\int_0^1 \mathrm{d}z\, z^{n-1}p^{\mathrm{T}}_{\mathrm{q}\gamma^*}(z)\right)+\ldots,$$

where the dots stand for all other similar terms besides the contribution of $p^{\mathrm{T}}_{\mathrm{q}\gamma^*}(z)$.

Note also that near $x = 1$, if $F_2(x, Q^2) \sim (1-x)^A$ and $x\mathrm{G}(x, Q^2) \sim (1-x)^B$ (with $B \geqslant A$) then $f(x,y,Q^2) \sim (1-x)$ consistent with eq. (14). The leading behaviour near $x = 1$ arises from $p^{\mathrm{T}}_{\mathrm{q}\gamma^*}$ as is clear from eqs. (16)–(18). On the other hand near $x = 0$ if $F_2(x, Q^2)$ and $x\mathrm{G}(x, Q^2)$ approach constant values, then $f(x, y, Q^2) \sim \ln(1/x)$ also consistent with eq. (14) and the leading terms are due to $p^{\mathrm{T}}_{\mathrm{q}\gamma^*}$ and $p^{\mathrm{T}}_{\mathrm{G}\gamma^*}$. The dominance of the transverse over longitudinal contributions is actually approximately valid over the whole range of x, so that the dependence of $f(x,y,Q^2)$ on y is rather weak as is confirmed by the numerical computations below.

The magnitude and the form of $f(x, y, Q^2)$ can be studied by neglecting as a first approximation the Q^2 dependence of parton densities and factoring out $\alpha_s(Q^2)$:

$$\langle p_\perp^2\rangle/Q^2 \equiv f(x,y,Q^2) \approx \alpha_s(Q^2) g(x,y). \qquad (21)$$

We shall come back later on the validity of this approximation. We take for $F_2(x)$ a reasonable fit to the data in the SLAC region:

$$F_2(x) = (1-x)^3\,[1.274 + 0.5989(1-x) - 1.675(1-x)^2]\,, \qquad (22)$$

while for the gluons we tentatively set:

$$x\mathrm{G}(x) = \tfrac{1}{2}(1+\eta)(1-x)^\eta, \qquad (23)$$

which corresponds to half of the nucleon momentum being carried by gluons.

In fig. 1 we have plotted $g(x,y)$ for $y = 0$ and 1. For comparison the function $r(x)$ defined as:

$$\frac{F_\mathrm{L}(x,Q^2)}{F_2(x,Q^2)} = \alpha_s(Q^2) r(x,y,Q^2) \approx \alpha_s(Q^2) r(x), \qquad (24)$$

is also shown. Finally $4\langle p_\perp^2\rangle/W^2$ ($W^2 = (Q^2/x)(1-x)$) is displayed for $y = 1$. As anticipated above the y dependence of $g(x,y)$ is almost negligible. The marked x dependence of $g(x,y)$ is mostly related to the variation of the invariant mass squared of the hadrons, as is clear from the fact that $\langle p_\perp^2\rangle/W^2$ is nearly a constant for x not too small. We also note that the naive parton model relation between the longitudinal structure function and $\langle p_\perp^2\rangle$: $F_\mathrm{L}/F_2 \approx 4\langle p_\perp^2\rangle/Q^2$ is not reproduced even approximately in QCD. The relative importance

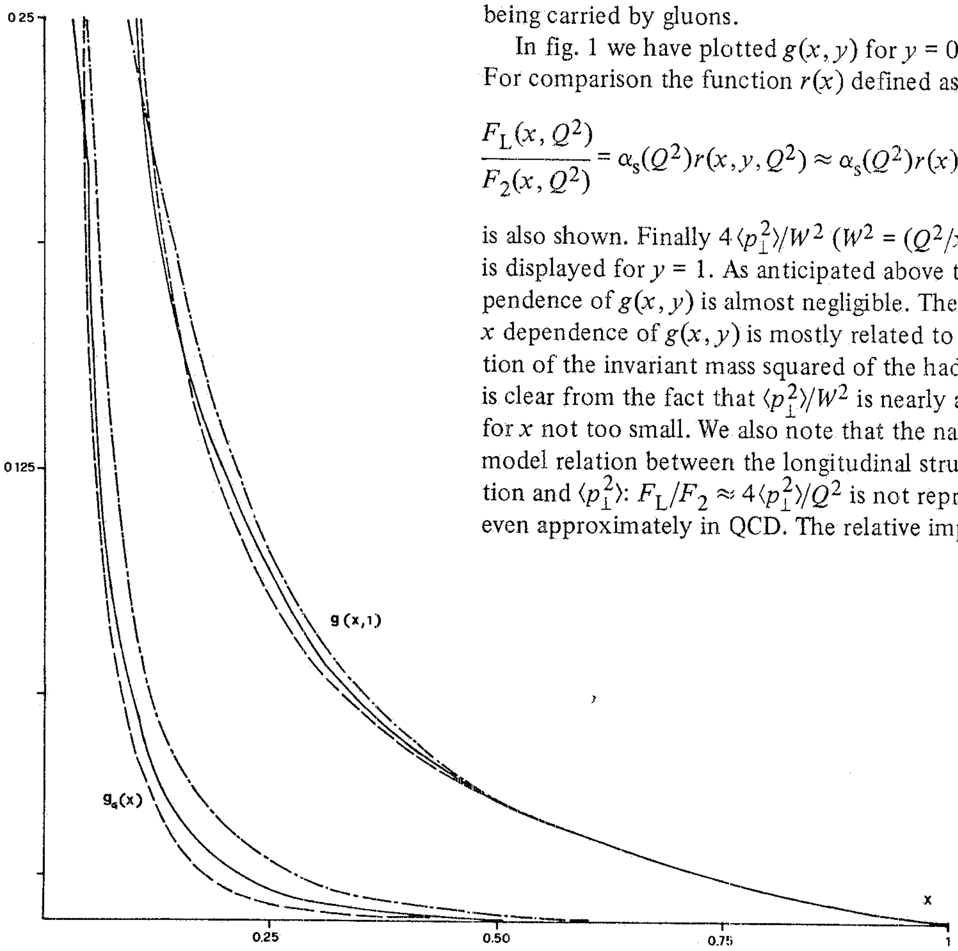

Fig. 2. With $\langle p_\perp^2\rangle/Q^2 \approx \alpha_s(Q^2) g(x,y)$ we separate at $y = 1$ the contributions of quarks and gluons, $g_\mathrm{G}(x)$, in the nucleon. The different curves for g_G and $g(x, 1)$ are for $\eta = 4$ (——), $\eta = 6$ (– – –) in eq. (23) and for $x\mathrm{G}(x)$ given by eq. (25) (–·–·–).

of quarks and gluons and the sensitivity to the badly known gluon x distribution can be estimated from fig. 2 where $g(x, 1)$ is plotted for $\eta = 4$ and 6. It is remarkable that $g(x, y)$ is less sensitive to $G(x)$ than the longitudinal structure function. In fact because of the different behaviour near $x = 0$ it turns out that the relatively large value of $xG(x)$ at $x = 0$ makes the gluons to dominate the quarks in $g(x, y)$ only at very small x, while in $r(x)$ they are already important at $x \approx 0.25$ (see fig. 3). This is true in spite of the fact that the form of $xG(x)$ in eq. (23) probably overestimates the gluons in the small x range. For example, we also plotted in figs. 2 and 3 $g(x, 1)$ and $r(x)$ for a different gluon parametrization:

$$xG(x) = aF_2(x)(1-x), \qquad (25)$$

with F_2 given by eq. (22) and the constant a fixed to make the total fraction of gluon momentum equal to 1/2.

In conclusion, from the very weak y dependence and the dominance of quarks over gluons we see that the bulk of $g(x, y)$ is due to $p^{\mathrm{T}}_{\mathrm{q}\gamma^*}$. The dominance of the $p^{\mathrm{T}}_{\mathrm{q}\gamma^*}$ term implies that the Q^2 dependence of $F_2(x, Q^2)$ is in part canceled between numerator and denominator in the expression of $f(x, y, Q^2)$ in eq. (20). By taking the empirical fits of $F_2(x, Q^2)$ at different Q^2 suggested in ref. [10] we have in fact checked that in the Q^2 range of current interest, for $x \gtrsim 0.1$ most of the Q^2 dependence of $f(x, y, Q^2)$ is through the factor $\alpha_s(Q^2)$.

Present experimental data on exclusive leptoproduction only give information on the $p_\perp$ and rapidity distributions of one given hadron in the final state. The theoretical analysis in QCD [11] of these one particle cross sections is complicated by our ignorance on the parton fragmentation functions into hadrons. A simpler theoretical description, as given here, is obtained by integrating over the fragmentation functions, i.e. by considering bulk properties of the hadronic jet. The identification of three jet events and the study of their $p_\perp$ distribution would be particularly important, as in the case for the $1 + \cos^2\theta$ distribution of jets in e^+e^-. Since jets with largest $p_\perp^2$ are most easily detected, the above discussion suggests to take at fixed E_ℓ the largest possible W^2, i.e. large y and relatively small x.

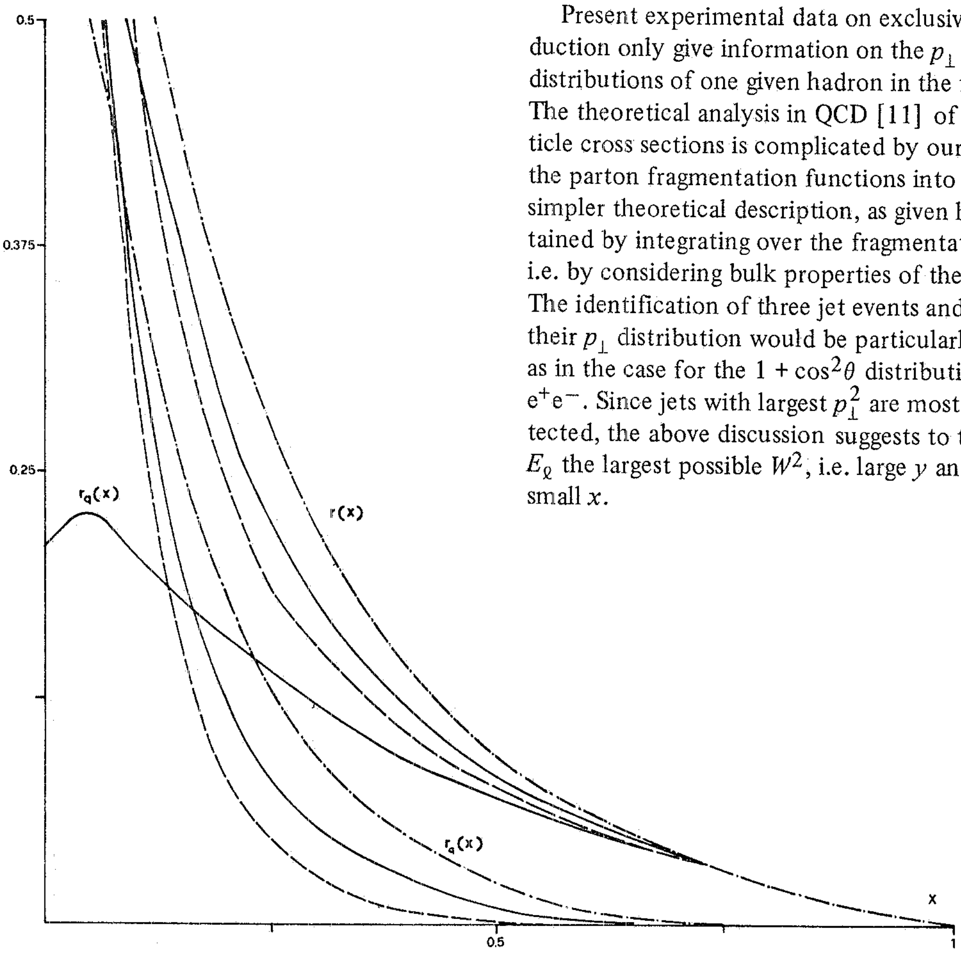

Fig. 3. With $F_L(x, Q^2)/F_2(x, Q^2) \approx \alpha_s(Q^2)r(x)$ we plot $r(x)$ and the separate contributions $r_{q,G}(x)$ of quarks and gluons for different parametrization of gluons: for $\eta = 4$ (——) and $\eta = 6$ (– – –) in eq. (23) and for $xG(x)$ given by eq. (25) (—·—·—).

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