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G. Pancheri-Srivastava and Y. Srivastava:
ENERGY-MOMENTUM DISTRIBUTION IN e^+e^- ANNIHILATION.

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G. Pancheri-Srivastava and Y. Srivastava(o) : ENERGY-MOMENTUM DISTRIBUTION IN e^+e^- ANNIHILATION(x).

ABSTRACT.

Analytic formulae are derived for the energy, longitudinal and transverse momentum dostributions of "soft" radiation in QED. A corresponding result is obtained for $e^+e^- \rightarrow q\bar{q}$ + gluons via QCD. Lastly, a model for hadronic longitudinal and transverse momentum distribution is proposed and compared with the SPEAR data.

The measurement of total cross-section and discovery of jets in e^+e^- annihilation has given a great boost to the parton model and its underlying theory QCD. The existence and some general features of the jets in (massless quark) QCD has been demonstrated in perturbation theory by Sterman and Weinberg[1]. In the present work, we discuss a model which borrows some techniques from an earlier work of ours in QED, where we were able to obtain a transverse momentum damping[2]. That analysis is extended to energy as well as the longitudinal momentum distribution of the radia-

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tion. The resulting formulae are also valid for colorless states of QCD with only minor changes. It is shown how the Sterman-Weinberg result is obtained from our formulae. Under certain extra assumptions, we compute the hadronic longitudinal and transverse momentum distribution in e^+e^- annihilation and compare it successfully to the SPEAR data. In a certain limit, our $k_\perp$ distribution becomes identical to Hagedorn's result in the thermodynamical model. Lastly, it is briefly pointed out that everything else being the same, gluon jets are expected to be twice as spread out as quark jets.

We start with an expression derived by Etim, Pancheri and Touschek[3] for the 4-momentum distribution $d^4P(K)$ of the emitted radiation in an arbitrary QED process $i \to f + \Gamma(K)$, using the Bloch-Nordsieck theorem and energy-momentum conservation:

$$d^4P(K) = (d^4K) \int \frac{(d^4x)}{(2\pi)^4}\, e^{iK\cdot x - h(x,\epsilon)} \tag{1}$$

where
$$h(x,\epsilon) = \int_0^\epsilon d^3\bar{n}(k)\left[1 - e^{-ik\cdot x}\right] \tag{2}$$

and the average number $d^3\bar{n}(k)$ of real photons emitted in (d^3k) is given by

$$d^3\bar{n}(k) = \beta\left(\frac{dk}{k}\right) f(\underline{\hat{n}})\,(d\Omega_n). \tag{3}$$

Here β is the spectrum and $f(\underline{\hat{n}})$ the angular distribution for single photon emission[3]:

$$\beta = -\int (d^2n)\, j_\mu(n)\, j^{\mu *}(n) \tag{4.1}$$

$$f(\underline{\hat{n}}) = -\frac{k^2}{2}\, j_\mu(k)\, j^{\mu *}(k)\ ; \qquad \int f(\underline{\hat{n}})\,(d\Omega_n) = 1 \tag{4.2}$$

$$j_\mu(k) = \frac{ie}{(2\pi)^{3/2}} \sum_i \eta_i \frac{p_{i\mu}}{(p_i\cdot k)}\ . \tag{4.3}$$

$\eta_i = +1\ (-1)$ for incoming (outgoing) particle/antiparticle, k is the single photon momentum such that $k^2 = 0$ but $K^2 \neq 0$. The upper limit ϵ in eq. (2) is the maximum frequency allowed for a single photon emission in a given process.

Clearly, the above integrals are very complicated so we aim for an approximate analytic form for it[2]. Consider, first the energy distribution $dP(\omega)$ alone, which is obtained upon integrating over $(d^3\underset{\sim}{K})$ where

$K^\mu = (\omega, \underset{\sim}{K})$:

$$\frac{dP(\omega)}{d\omega} = \int (d^3\underset{\sim}{K}) \int \frac{(d^4x)}{(2\pi)^4}\, e^{iK\cdot x - h(x,\epsilon)} = \int_{-\infty}^{\infty} \frac{dt}{(2\pi)}\, e^{i\omega t - h(\underset{\sim}{x}=0,t,\epsilon)}, \tag{5}$$

where

$$h(t,\epsilon) = \beta \int_0^{\epsilon} \left(\frac{dk}{k}\right)\left[1 - e^{-ikt}\right]. \tag{6}$$

Now, $h(t,\epsilon) \xrightarrow[t\to 0]{} \beta(i\epsilon t)$ and $\xrightarrow[|t|\to\infty]{} \beta \ln(i\epsilon t)$. Thus, we approximate $h(t)$ by $\tilde{h}(t) = \beta \ln(1 + i\epsilon t)$, and eq. (5) is replaced by

$$d\tilde{P}(\omega) = \frac{d\omega}{(2\pi)} \int_{-\infty}^{\infty} \frac{dt\, e^{i\omega t}}{\left[1 + i\epsilon t\right]^{\beta}} = \frac{\beta}{\Gamma(1+\beta)} \left(\frac{d\omega}{\epsilon}\right)\left(\frac{\omega}{\epsilon}\right)^{\beta-1} e^{-\omega/\epsilon}. \tag{7}$$

Eq. (7) is our proposed analytic approximation to the energy distribution. It is easily checked that the first two moments $\langle\omega^0\rangle$ and $\langle\omega'\rangle$ computed using the exact $dP(\omega)$ given by eq. (5) and our approximation for it viz. $d\tilde{P}(\omega)$ are identical. This gives us confidence about the obtained exponential energy cut-off. Notice that the cut-off is a truely cooperative effect (it needs atleast n photons to get a total energy $\omega = n\epsilon$) and cannot be obtained in any finite order in perturbation theory. (For a discussion see ref. 2).

A similar trick can be devised for $K_\parallel$ as well as $K_\perp$ distribution. For $K_\parallel$-distribution, we propose

$$d\tilde{P}(K_{\parallel}) = \frac{dK_{\parallel}}{(2\pi)} \int_{-\infty}^{\infty} dx_3 \, e^{-iK_{\parallel}x_3 - \tilde{h}(x_3,\epsilon)} , \tag{8}$$

with $h(x_3,\epsilon)$ approximated by $\tilde{h}(x_3,\epsilon) = \frac{\beta}{2} \ln\left[1 + x_3^2 \frac{\epsilon^2}{2}(1-4A)\right]$, (9)

and $A = \frac{1}{4}\int f(\hat{\underline{n}})\, n_\perp^2 \,(d^2n) = \frac{\langle n_\perp^2 \rangle}{4}$. (10)

Thus, (4A) is the <u>average</u> transverse spread of a <u>single</u> radiation. Calling $B = \frac{1}{2}(1-4A)$, we find

$$d\tilde{P}(K_{\parallel}) = \frac{\beta/2}{\sqrt{\pi}\,\Gamma(1+\beta/2)} \frac{dK_{\parallel}}{(\epsilon\sqrt{B})} \left(\left|\frac{K_{\parallel}}{2\sqrt{B}\epsilon}\right|\right)^{\frac{\beta-1}{2}} K_{\frac{\beta-1}{2}}\left(\frac{K_{\parallel}}{\sqrt{B}\epsilon}\right) , \tag{11}$$

where $K_\nu(z)$ is the Bessel function of the IIIrd kind. Eq. (11) has very interesting, small and large $K_{\parallel}$, limits.

For $K_{\parallel} \to 0$, $\frac{dP(K_{\parallel})}{dK_{\parallel}} \underset{\beta<1}{\sim} K_{\parallel}^{\beta-1} \to \infty$ (12)

$\underset{\beta>1}{\sim}$ constant . (13)

Eq. (12) is the limit relevant for QED, while eq. (13) will be seen relevant for QCD. For $K_{\parallel} \to \infty$,

$$\frac{dP(K_{\parallel})}{dK_{\parallel}} \underset{K_{\parallel}\to\infty}{\sim} e^{-K_{\parallel}/\epsilon\sqrt{B}} . \tag{14}$$

It turns out[2] that as the incident energy $W \to \infty$, $A \to 0$ thus $B \to 1/2$ and we find an exponential damping in $K_{\parallel}$ for $K_{\parallel} \gg \epsilon$.

A completely analogous analysis can be carried through for the $K_\perp$ distribution. The result is

$$\frac{d\tilde{P}(K_\perp)}{dK_\perp} = \frac{\beta/2}{\Gamma(1+\beta/2)} \frac{K_\perp}{(\epsilon^2 A)} \left(\frac{K_\perp}{2\epsilon\sqrt{A}}\right)^{-1+\frac{\beta}{2}} K_{1-\frac{\beta}{2}}\left(\frac{K_\perp}{\epsilon\sqrt{A}}\right) . \tag{15}$$

Again, for $K_\perp \to 0$, $$\frac{d\tilde{P}}{dK_\perp} \underset{\beta<1}{\sim} K_\perp^{-1+\beta} \to \infty \tag{16}$$

$$\underset{\beta>1}{\sim} K_\perp \to 0 \,. \tag{17}$$

As before, eq. (16) is appropriate for QED and eq. (17) for QCD.

For $K_\perp \to \infty$, $$\frac{dP(K_\perp)}{dK_\perp} \sim e^{-K_\perp/\epsilon\sqrt{A}} \,. \tag{18}$$

We notice in passing that for $\beta = 4$, eq. (15) reduces to Hagedorn's $K_\perp$--distribution (for massless particles) obtained in the thermodynamic model. We intend to pursue this correspondence in detail elsewhere.

In QED, $e^+e^- \to e^+e^-$ scattering, for example, A can be explicitly calculated[2] and one finds that for large s and fixed t scattering

$$\epsilon\sqrt{A} \to m\sqrt{\left(\frac{3}{8}\ln\frac{s}{m^2} - 1\right)}$$

so that $K_\parallel$ scales with $\epsilon\sqrt{B} \sim \epsilon/\sqrt{2}$, i.e. the <u>energy</u>, whereas $K_\perp$ scales with $\epsilon\sqrt{A} \sim m$ (upto logarithms). This is the genesis of transverse momentum damping in our approach[4].

Now let us discuss the β-factor. In $e^+e^- \to a\bar{a}$, for the e^+e^- state alone, it is given by (see eqs. (4.1) and (4.3))

$$\beta_e = \frac{2\alpha}{\pi}\int_{-1}^{+1} dz \,\frac{(1-z^2)}{(1-v^2z^2)^2} \tag{19}$$

$$\underset{v = p/E \to 1}{\overset{\text{rel. limit}}{\sim}} \left(\frac{4\alpha}{\pi}\right)\left(\ln\frac{2E}{m}\right) \,. \tag{20}$$

What happens when $m \to 0$? Standard QED for massless electron does not exist. However, jet-like cross-sections can nevertheless be defined. In our language, there is a simple recipe. Consider a cone of half-angle δ about the e^+ and e^- axes. Then, eq. (19) for $m = 0$ ($v = 1$) is replaced by

$$\beta \Rightarrow \frac{2\alpha}{\pi}\int_{-z_0}^{z_0} \frac{dz(1-z^2)}{(1-z^2)^2} = \frac{2\alpha}{\pi}\ln\left(\frac{1+z_0}{1-z_0}\right) \,,$$

where $z_o = \cos 2\delta$. For small δ, $\beta(\delta) \simeq (\frac{4\alpha}{\pi}) \ln(\frac{1}{\delta})$. (21)

Let us turn now to QCD. Various authors have shown that in the leading logarithmic approximation, exponentiation similar to QED occurs here as well[5,6,7,8]. For $e^+e^- \to q\bar{q} \to$ hadrons, we have $\alpha \Rightarrow C_F \bar{\alpha}$ where $C_F = 4/3$ for SU(3) color and

$$\bar{\alpha} = \frac{g_E^2}{4\pi} \simeq \frac{6\pi}{(33 - 2n_F)(\ln W/\Lambda)} , \qquad (22)$$

where n_F is the number of flavors.

Following Sterman and Weinberg, for massless quarks, let us define f as the fraction of events carrying a certain portion $(1-\varepsilon)$ of total energy $W = 2\mathcal{E}$ in a half-cone δ. How do we obtain it ? Take eq. (7) for $d\tilde{P}(\omega)$ and integrate upto $(\Delta\omega)$:

$$f(\varepsilon,\delta) = \int_0^{\Delta\omega} d\tilde{P}(\omega) = \frac{1}{\Gamma(\beta)} \int_0^{\Delta\omega/\mathcal{E}} dx\, x^{\beta-1} e^{-x} = (\frac{\Delta\omega}{\mathcal{E}})^{\beta} \gamma^{*}(\beta, \frac{\Delta\omega}{\mathcal{E}}) , \qquad (23)$$

where γ^* is the incomplete gamma function. For $\frac{\Delta\omega}{\mathcal{E}} = 2\varepsilon \ll 1$, it becomes

$$f(\varepsilon,\delta) \simeq 1 + \beta(\delta)\ln(2\varepsilon) \simeq 1 - (\frac{32}{25 \ln W/\Lambda}) \ln(\frac{1}{\delta}) \ln(\frac{1}{2\varepsilon}) , \qquad (24)$$

using eqs. (21) and (22) with $n_F = 4$ flavors. Eq. (24) is the singular part of the Sterman-Weinberg expression. Clearly, eq. (24) is valid only when the 2^{nd} term is small compared to 1, whereas our earlier expression eq. (23) should be valid more generally.

We discuss now an application of our formulae to hadronic momentum distributions. Implicitly or otherwise, the whole 2-jet picture in QCD rests on the assumption that the produced hadrons are roughly alligned to the quark direction with a spread defined by the gluon radiation. We would like to boldly extend this concept to its limit, which is to say that the hadronic 4-momentum distribution in a 2-jet process would be given by the <u>same</u> $d^4P(K)$ as that of the radiation. This hypothesis seems to work rather well as we show below.

To analyse the very good SPEAR data at W = 7.4 GeV[9], we use 4-flavors with quark masses : m_u = 4 MeV , m_d = 7.5 MeV, m_s = 150 MeV and m_c = 1.5 GeV[10]. The parameter $\epsilon\sqrt{A}$ can be eliminated in favor of the mean $\langle k_\perp \rangle$ - which we take from the data

$$\langle k_\perp \rangle = (\epsilon\sqrt{A})\sqrt{\pi}\,\frac{\Gamma(1/2+\beta/2)}{\Gamma(\beta/2)}$$

$$\langle k_\perp^2 \rangle = 2\beta(\epsilon^2 A)\ .$$

Figure 1 shows the theoretical prediction (normalized to give the same

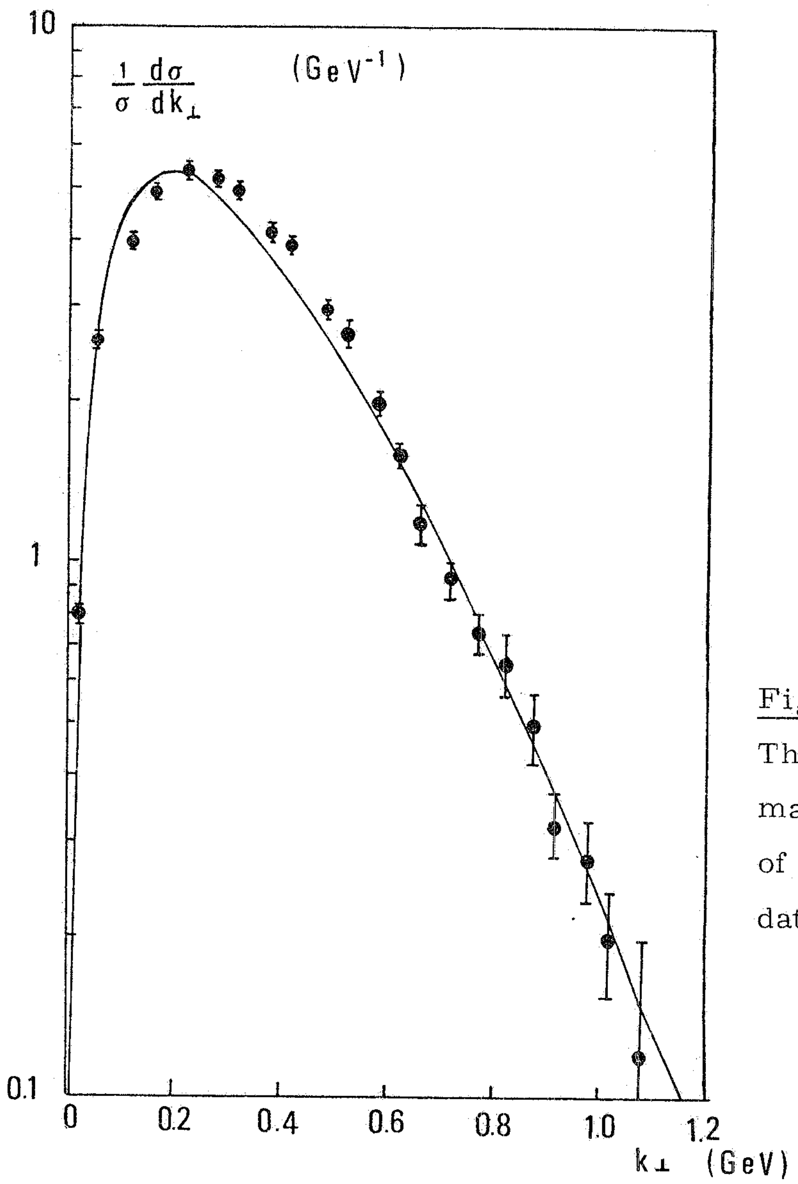

Fig. 1 - $\frac{1}{\sigma}(\frac{d\sigma}{dk_\perp})$ versus $k_\perp$. The theoretical curve is normalized to the same number of produced particles. The data (ɸ) are from ref. 9.

number of particles) as well as the experimental result for the $K_\perp$ distribution at 7.4 GeV. The agreement is quite satisfactory. Figure 2 shows the corresponding comparison for the $x_\parallel = 2K_\parallel/W$ distribution, where we have deliberately normalized to the high $x_\parallel$ tail to emphasize the discrepancy (near $x_\parallel = 0$) between our model and the data. It highlights the neglect of the pion mass - remember our formulae are for massless objects ! Optimistically, we hope that a reasonable mass correction will ameliorate the situation. The other probable source of discord near $x_\parallel \simeq 0$ has to do with

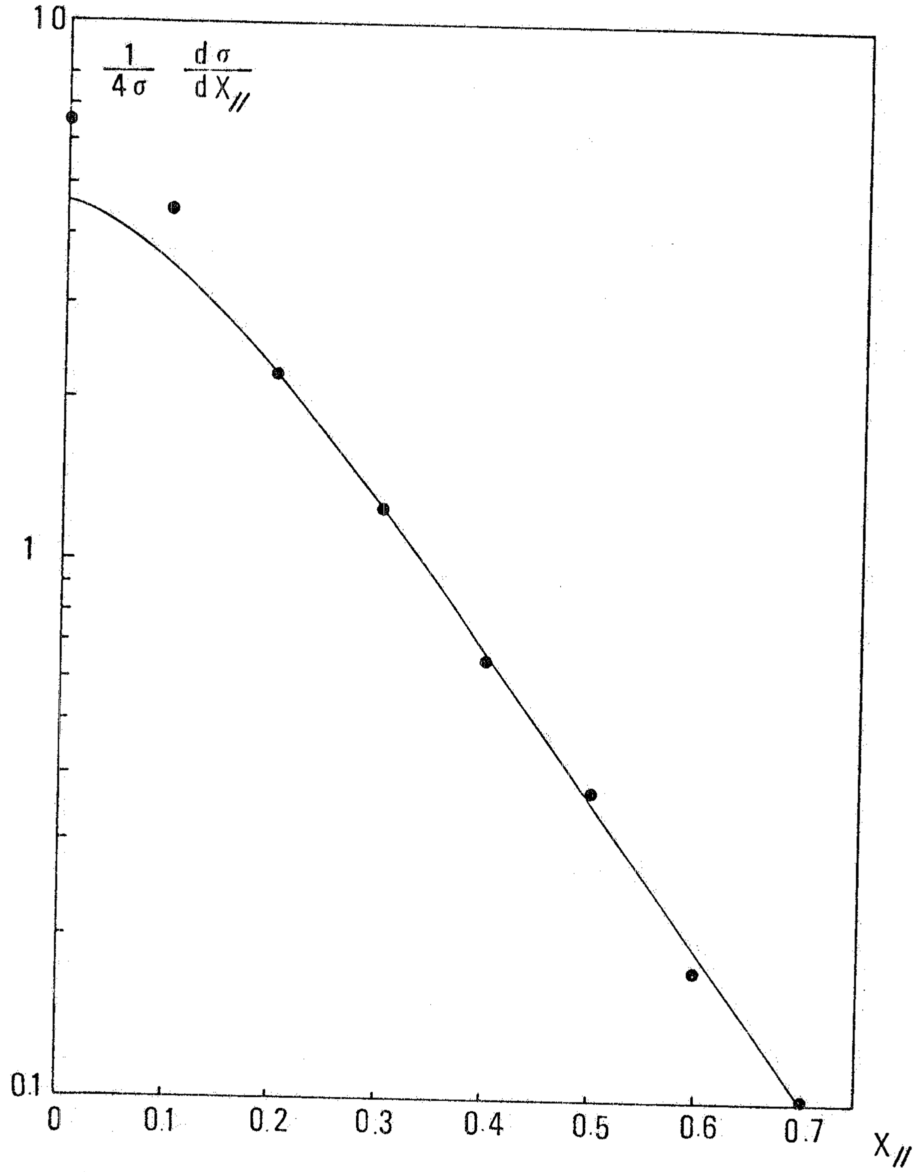

Fig. 2 $\frac{1}{4\sigma}\left(\frac{d\sigma}{dx_\parallel}\right)$ versus $x_\parallel$. The theoretical curve is normalized to the high $x_\parallel$ tail. The data (●) are from ref. 9.

charm production. (For a charmed quark of mass ~ 1.5 GeV, the distribution at 7.4 GeV deviates from $1+\cos^2\vartheta$ quite a bit).

Inspite of these reservations, it is safe to conclude that the general shape of the data are well reproduced. The present approach therefore may be a viable alternative to the quark fragmentation models.

We postpone a discussion of gluon jets to another work and for now record only some qualitative remarks. In the massless quark jet picture, if we assume that A is the same for quark and gluon jets, then very approximately

$$\frac{\langle K_\perp \rangle_{glue}}{\langle K_\perp \rangle_{quark}} \simeq \left(\frac{9}{4}\right) \frac{\langle N_q \rangle}{\langle N_g \rangle} .$$

Thus, if multiplicity due to quark and gluons is roughly the same, then $\langle K_\perp \rangle_{glue} \simeq 2 \langle K_\perp \rangle_{quark}$ leading us to conclude that hadrons from gluons are more spread out. Preliminary reports from DESY on Y-resonance decay seem to bear this out.

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FOOTNOTES AND REFERENCES

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[2] G. Pancheri-Srivastava and Y. Srivastava, Phys. Rev. D15, 2915 (1977).

[3] E. Etim, G. Pancheri and B. Touschek, Nuovo Cimento 51, 276 (1967). It was further developed and elaborated in a variety of papers. See, e. g. G. Pancheri, Nuovo Cimento 60, 321 (1969); G. Pancheri-Srivastava, Phys. Letters 44B, 109 (1973); M. Greco, G. Pancheri-Srivastava and Y. Srivastava, Phys. Letters 56B, 367 (1975); Nuclear Phys. B101, 234 (1975).

4 Another interesting case is the fixed-angle scattering (i. e. s, t, u large and of same order). Then, we find the remarkable result that (without any logs) $A \xrightarrow[(z \neq 1)]{(z\ \text{fixed})} \frac{1}{4}(1 - z^2) = p_\perp^2/16s$, where $p_\perp$ is the transverse momentum of the emitting legs (leading particles). Thus, for fixed-angle scattering, the argument of the K-function never gets big so that we lose the exponential damping and are left with a mere power law damping in $K_\perp$ for large $K_\perp$. This result also suggests a new type of scaling. Consider particles (radiation) emitted with moderate leading particle transverse momentum $p_\perp$. The transverse momentum $K_\perp$ for such particle production should scale with $(K_\perp/p_\perp)$.

5 See, for example, J. M. Cornwall and G. Tiktopoulos, Phys. Rev. D13, 3370 (1976); D15, 2937 (1977).

6 Using the coherent state formalism, infra-red finite, exponential expressions have been obtained by M. Greco, F. Palumbo, G. Pancheri-Srivastava and Y. Srivastava, Phys. Letters 77B, 282 (1978). An alternative scheme has been proposed by D. Butler and C. Nelson, SUNY-BING 2/2/78.

7 G. Curci and M. Greco (CERN TH 2526, 1978) have obtained exponentiated expressions also for massless QCD using the coherent state formalism.

8 For a general analysis of mass singularities in QCD, see D. Amati, R. Petronzio and G. Veneziano, CERN TH 2470 (1978); G. Sterman, Stony Book preprints ITP-SB-77-69, 72, 77.

9 All data are taken from G. Hanson, SLAC-PUB-2118 (1978).

10 S. Weinberg, Harvard preprint 1977; H. Leutwyler, Phys. Letters 48B, 45 (1974); H. Fritzsch, CERN TH 2483 (1978).