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LNF-78/44(R)
12 Ottobre 1978

S. Guiducci, G. Martinelli and M. Preger: ALFA. FEASIBILITY STUDY OF AN ELECTRON PULSE STRETCHER TO INCREASE THE DUTY FACTOR OF THE FRASCATI LINAC.

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1. - INTRODUCTION.

In July 1976 a pulse stretcher for the Linac of the Frascati National Laboratories (LNF) was first proposed (ALFA project). Two solutions were considered: a modification of the storage ring Adone and the construction of an entirely new machine. The first solution has been the object of a previous paper [(1)] and will not be further discussed here.

In this report the preliminary design of an electron pulse stretcher with an average current of the order of 100 μA, duty-cycle near 1, relative energy spread of the order of 10^{-3} and energy between 200 and 500 MeV is presented. These parameters can be achieved with a storage ring of ~120 m circumference and 5m bending radius; the maximum energy of the LNF Linac has to be increased to 500 MeV at an electron peak current of 100 mA and with the present duty-cycle. An "energy compressor" is also required.

2. - OUTLINES OF THE RESONANT EXTRACTION THEORY.

The theory of resonant extraction by sextupolar magnetic field has been extensively developed [(1, 3, 4)]. We shall recall here some formulas which will be useful for the definition of our machine parameters.

The perturbation introduced by nonlinear magnetic fields (sextupoles) in a circular accelerator optical structure divides the particle phase space into stable and unstable regions. If the sextupolar magnetic fields are:

$$B_x = 2S\,x\,z \quad , \qquad\qquad B_z = S\,(x^2 - z^2) \quad , \tag{1}$$

x and z being the coordinates in the plane perpendicular to the equilibrium orbit, and if the horizontal betatron frequency lies near an m/3 resonance (m integer), the stability region in the $(x, \beta_x x')$ phase space is bounded by a triangle (see figure 1). The triangle vertex coordinates at a point s along the equilibrium orbit are given by:

$$\vec{x}_i = (\Delta E/E)\,\vec{\psi} + M\vec{x}_{oi} \quad , \qquad (i = 1, 2, 3) \tag{2}$$

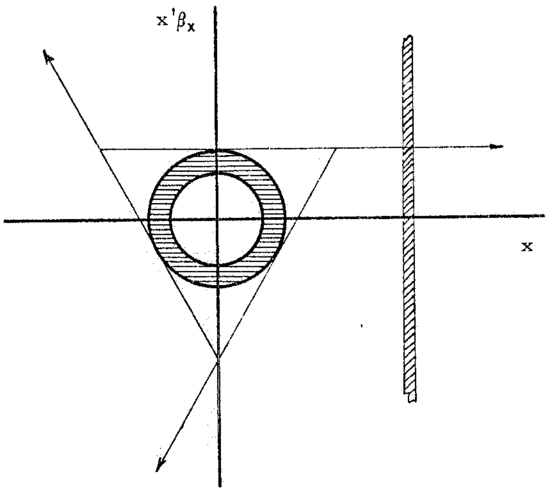

FIG. 1 - Stability triangle, arms of the separatrix and septum shadow in phase space $(x, \beta_x x')$. The beam occupies the shaded area.

where $\vec{x}_i$ is a vector with components (x, x') (o), and the vector $\vec{\psi}$, defined in the same way, represents the displacement of the closed orbit and its derivative for a particle of unit relative energy deviation from the nominal energy of the machine. M is a 2 x 2 matrix which depends on the sextupole position with respect to s and on the particle energy, and leads to a rotation in the $(x, \beta_x x')$ phase space. The components of the vectors $\vec{x}_{oi}$ at a machine symmetry point:

$$x_{o1,2} = \pm \varrho \left(\frac{\beta(s)}{R}\right)^{1/2} \frac{\Delta\nu}{2\sqrt{3}\, H_{33}} ,$$

$$x_{o3} = 0 ,$$

$$x'_{o1,2} = \varrho \frac{1}{(\beta(s)R)^{1/2}} \frac{\Delta\nu}{6\, H_{33}} , \qquad (3)$$

$$x'_{o3} = -\varrho \frac{1}{(\beta(s)R)^{1/2}} \frac{\Delta\nu}{3\, H_{33}} ,$$

where ϱ is the bending radius, R is the average ring radius, $\beta(s)$ is the horizontal betatron function, $\Delta\nu$ is the distance between the horizontal betatron frequency and the m/3 resonance and H_{33} gives the equivalent intensity of the sextupole system [3, 5]. $\Delta\nu$ and H_{33} are functions of the particle energy.

The particles which are outside the stability triangle tend to move away along the three arms of the separatrix. It is possible to extract the stored beam in a given time interval by progressively shrinking the stability triangle until all the particles find themselves outside of it.

There are two extraction methods, according to how the stability triangle area is made to shrink: achromatic and monochromatic extraction.

In the first method [5] the ratio $\Delta\nu/H_{33}$, which determines (apart from second order terms in $\Delta E/E$) the stability triangle area, must be made independent of the particle energy. The triangle area is made to vary decreasing $\Delta\nu$. This means the quadrupole fields of the ring must be varied accordingly; at least part of the quadrupoles has therefore to be pulsed. All particles having a given emittance are extracted at the same time, independent of their energy: the energy spread of the extracted beam will then be, to first approximation, the initial Linac beam energy spread plus the energy lost by radiation between injection and extraction. The condition that $\Delta\nu$ is energy independent is obtained by canceling the machine natural chromaticity with appropriate sextupoles. Furthermore by locating the sextupoles at appropriate places along the circumference, H_{33} can also be made independent of energy.

For monochromatic extraction $\Delta\nu/H_{33}$ is made to depend on the particle energy, in such a way that its value decreases as particles lose energy by synchrotron radiation. Therefore the particles, by losing energy, find themselves outside the stability triangle and are extracted. This method allows the energy spread of the extracted beam to be greatly reduced ($\sim 10^{-3}$).

(o) Apex indicates derivation with respect to s.

3. - EXTRACTED BEAM: DUTY-FACTOR, ENERGY SPREAD, EMITTANCE.

Let us define the stretcher duty - factor as:

$$D = T_e / T_L \quad , \qquad (4)$$

where T_e is the time needed to extract a stored beam, and T_L is the inverse of the Linac repetition rate. The average energy loss per unit time in a circular machine is given by:

$$\frac{dE}{dt} = \frac{K E^4}{\varrho L} \quad , \qquad (5)$$

where $K = 2.65 \times 10^4 \text{ GeV}^{-3} \text{ m}^2 \text{ s}^{-1}$, and L is the machine circumference ($L = 2\pi R$).

If extraction is achromatic the duty - factor is determined by the time variation of $\Delta\nu / H_{33}$; the energy spread of the extracted beam is:

$$(\Delta E / E)_{ex} = (\Delta E / E)_L + K E^3 T_L D / (\varrho L) \quad , \qquad (6)$$

$(\Delta E/E)_L$ being the energy spread of the injected beam. The second r. h. s. term accounts for the radiated energy.

It extraction is monochromatic and for the ideal case of vanishing emittance of the injected beam the duty - factor is given by:

$$D = \frac{\varrho L (\Delta E / E)_L}{K E^3 T_L} \quad , \qquad (7)$$

Ideally, the extracted beam has no energy spread. In the real case, however, a finite spread $(\Delta E/E)_{ex}$ in the extracted beam has to be accounted for so that (7) should actually be corrected as follows:

$$D = \frac{\varrho L}{K E^3 T_L} \left[(\Delta E / E)_L + (\Delta E / E)_{ex} \right] \quad , \qquad (8)$$

$(\Delta E/E)_{ex}$ can be computed from the distribution of the injected beam in phase space and is given by:

$$(\Delta E / E)_{ex} = \frac{6}{\varrho} \sqrt{R} \frac{W_M^{1/2} - W_m^{1/2}}{\left[\frac{\partial}{\partial (\Delta E / E)} \frac{\Delta \nu}{H_{33}} \right]_{\frac{\Delta E}{E} = 0}} \quad , \qquad (9)$$

where W_M and W_m are the maximum and mininum horizontal emittance of the stored beam respectively.

If injection is in the vertical plane and extraction in the horizontal plane (see for instance EROS[6] and ALIS [7] designs), the extracted beam vertical emittance is determined by injection only.

The horizontal emittance is instead given by:

$$W_r = \frac{\Delta x \cdot \Delta x'}{\pi} \quad , \qquad (10)$$

$$\Delta x' = \frac{W_M^{1/2} - W_m^{1/2}}{\sqrt{\beta_x^{ex}}} \qquad , \qquad (11)$$

β_x^{ex} = horizontal betatron function at the extraction point ,

$$\Delta x' = \frac{X_s^2 - X_o^2}{X_o \operatorname{cotgh}(3\sqrt{3}\pi\Delta\nu) - X_s} \qquad , \qquad (12)$$

$$X_o = \varrho \left(\frac{\beta_x^{ex}}{R}\right)^{1/2} \frac{\Delta\nu}{2\sqrt{3}\,H_{33}} \qquad , \qquad (13)$$

X_s = horizontal coordinate of extraction septum.

Combined extraction: the two extraction methods can be combined by letting the triangle area change due to the radiation losses and at the same time varying the machine quadrupole fields. In this way a duty - factor approaching 1 over a large energy range can be obtained, at the expense of an increased energy spread, but the extracted beam horizontal emittance is increased, because of the stability triangle rotation at the extraction point due to the M matrix, which is a function of the particle extraction energy.

4. - DESIGN CRITERIA.

The machine must be designed for high injection and extraction efficiencies and for good performance (duty-cycle, emittance, average current and energy spread) over the largest possible energy range. Since furthermore the energy spread of the extracted beam should be the smallest possible it seems obvious to resort to monochromatic extraction. Achromatic extraction could be useful if the energy compression scheme is not realized: the proposed design allows for it (with a suitable arrangement of pulsed quadrupoles) at the expense of a slight decrease in duty - factor.

4. 1. - Injector Linac.

Optimum performance over the required energy range (1) requires some important modifications to the present LNF Linac, namely:

a) Increase of the maximum energy (at 10 mA peak current) to 500 MeV with the same duty-cycle.
b) Possibility of operating at various repetition rates and pulse widths.
c) Addition of an energy compressor to bring the energy spread down to $\sim 10^{-3}$ from the present $\pm$. 5%.

a) and b) require the installation of two new accelerating sections and of all klystrons replacement by higher efficiency ones (65%). The klystron modulators would also have to be modified.

Assuming the present peak current of 100 mA can still be obtained at 500 MeV, the peak current as a function of energy would be given by:

$$I_p(E) = \frac{0.574 - E_{GeV}}{0.74} \text{ Ampere.} \qquad (14)$$

On the low energy side peak current is limited to $\sim$ 250 mA by Linac parameters. The RF pulse length is given by the waveguide filling time (t_F = 1. 2 μs for the high energy sections) plus the steady state beam length t_B (8). The maximum Linac duty-cycle is limited by the klystrons average power. Assuming the latter is mantained at the present value (~3 KW) the klystron duty-cycle K would be given by:

$$K = f\,(t_B + t_F) = 1.1 \times 10^{-3} \quad , \qquad (15)$$

where f is the repetition rate. The average current would be:

$$\bar{I} = I_p\,(E)\;\frac{K\,t_B}{t_B + t_F} \quad . \qquad (16)$$

We also assume that the energy compressor [12] is capable of decreasing the Linac relative beam energy spread from the present $\sim 10^{-2}$ to $\sim 10^{-3}$. Figg. 2 show the parameters of the modified Linac beam as a function of repetition rate, at three different energies.

4. 2. - The pulse stretcher: choice of ϱL and of the extracted beam parameters.

Taking into account the Linac characteristics described in 4. 1, the product ϱL of the ring circumference by the bending radius (see formula (8)) has been chosen so as to be able to extract monochromatically with unit duty-cycle up to about 500 MeV [11] ($\varrho L \simeq 590$ m).

The vertical emittance of the extracted beam depends on the lenght of the Linac pulse, on $(\Delta E'/E)_L$ and on the ring chromaticity (see chapter 6).

The average extracted current is given by the average Linac current (Fig. 2), multiplied by the overall efficiency η (η is the product of injection, extraction and transport efficiencies).

Since up to now no machine has been operated with similar injection and extraction characteristics, no experimental data for a reliable estimate of η are available. Numerical estimates can be obtained by computer simulations, tracking the path of a number of particles over many turns. The average extracted current presented in this report assumes unit efficiency so that is directly comparable with that of other existing projects [6, 7]; it should however be borne in mind that it will eventually have to be scaled by the actual value of η. It is reasonable to assume that the overall efficiency can be made $\gtrsim 50\%$.

Given the ring chromaticity and the Linac beam emittance, the relative energy spread achievable by monochromatic extraction is of the order of 10^{-3}.

In Figg. 3 the average current ($\eta = 1$), duty-cycle and vertical beam emittance are shown as functions of energy for various Linac repetition rates. Horizontal emittance does not depend on energy and can be varied by means of the sextupoles (see Chapter 7).

Fig. 3a shows the extracted beam parameters at a Linac repetition rate of 500 Hz: it can be seen that at this frequency a good vertical emittance can be achieved (since injection takes less than 3 turns, see Chapter 6), but the current is limited by the Linac duty-cycle K. At energies below 280 MeV, it is not possible to decrease further the injected beam energy spread, and therefore fast extraction of the remaining particles may be necessary. The average current decreases as E^3 for energies below 280 MeV.

Fig. 3c shows the same quantities for a Linac repetition rate of 200 Hz. Average current is limited at high energy by beam loading in the Linac (14), and at low energy by the peak current limitation at 250 mA [10]. At this frequency the average current achievable at any energy is about twice that achievable at 500 Hz. On the other hand the vertical emittance of the extracted beam is worse, because the length of the injection pulse corresponds to about 10 revolutions in the ring. Of course it is always possible to improve the extracted beam emittance, by shortening the injection pulse, with a corresponding decrease in average current. At energies above 350 MeV, assuming the relative energy spread of the injected beam is kept below ~1%, the extracted beam duty-cycle decreases like E^{-3}.

Fig. 3b refers to a repetition rate of 350 Hz, and is an intermediate case between the previous two.

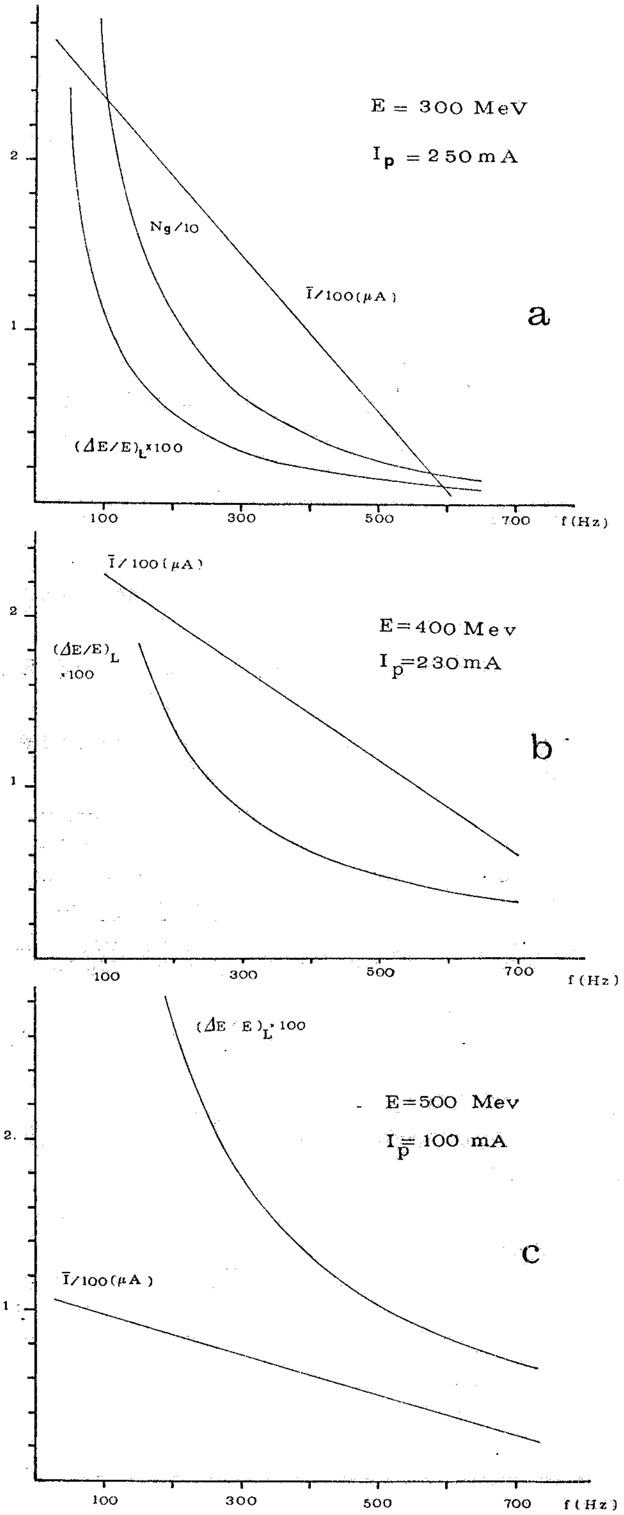

FIG. 2 - Parameters of the injected beam as a function of the Linac repetition rate, required to obtain D = 1. $\bar{I}$ = average current. Ng = Linac pulse length measured in units of revolutions in the ring.

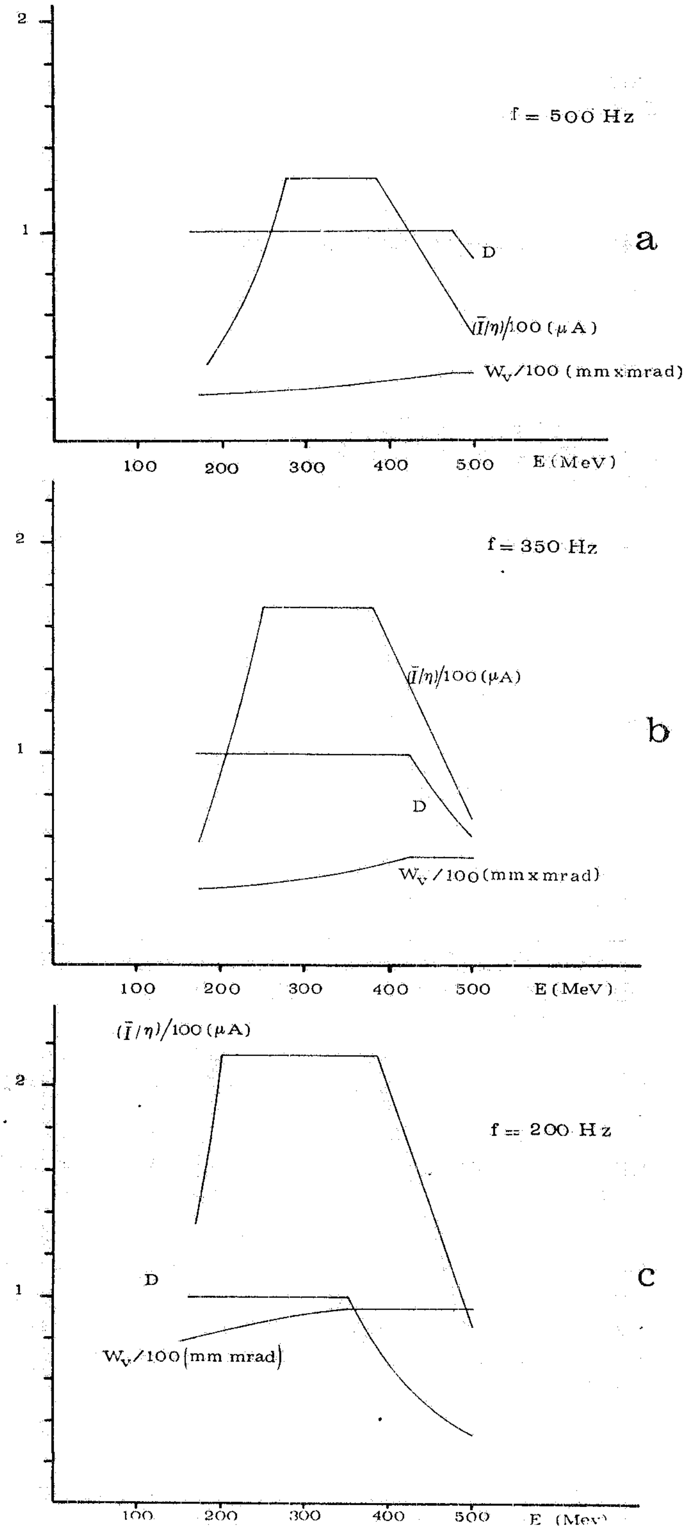

FIG. 3 - Parameters of the extracted beam as a function of the electron energy, at fixed repetition rate, per unit efficiency η. D= duty-cycle; W_V = vertical emittance; $\bar{I}$ = average current per unit efficiency.

It is possible to improve the vertical emittance of the extracted beam, at the expense of average current, and viceversa. In Table I the parameters of the extracted beam are given as a function of energy, in the extreme cases of maximum average current and minimum vertical emittance.

TABLE I

	E (MeV)	$\bar{I}/\eta$ (μA)	W_v(mmxmrad)	f (Hz)	D
Maximum average current	200	215	83	200	1
	300	215	92	200	1
	400	200	95	200	0.67
	500	86	95	200	0.34
Minimum vertical emittance	200	46	22	500	1
	300	125	25	500	1
	400	115	28	500	1
	500	50	32	500	0.86

4.3. - General remarks on the optical structure.

The optical structure has been designed to fulfill the following conditions:

a) Because of the particular features of injection and extraction, the pulsed bending elements pose a hard technical problem (see Chapter 6 and 7). Their design becomes easier when the length of the dedicated straight sections increases. This led us to choose a 3 m length for the injection and extraction straight sections.

b) $\psi = 0$ at the injection and extraction points: dispersion suppression at injection causes all the particles to be injected with the same horizontal closed orbit displacement, independent of their energy. This minimizes the injected beam horizontal emittance. Moreover, in achromatic extraction, non zero dispersion causes an increase in the extracted beam emittance and horizontal aperture, due to the drift of the stability triangle along the horizontal axis, when the energy of the beam decreases because of radiation loss (see formula (2)).

c) The need to have as small an aperture as possible in the magnetic elements requires a small ψ ($\lesssim 2$ m) all along the ring, and in particular in the bending magnets.

d) One of the three arms of the separatrix, limiting the stable region in phase space, must be parallel to the x axis (the exit angle must be independent of position): this requirement leads to maximum extraction efficiency.

e) The ratio $R_\beta = \beta_x^{max}/\beta_x^{inj}$ (β_x^{max} being the maximum value of the horizontal betatron function in the ring and β_x^{inj} its value at injection point) must be made as small as possible, compatible with the condition of having the maximum value of the vertical betatron function at the injection point: the maximum amplitude of the horizontal betatron oscillations and therefore the necessary aperture, increases with R_β.

5. - OPTICAL STRUCTURE.

The optical structure is built of two six-cell arcs, and two straight sections dedicated to injection and extraction. The layout of the ring is shown in fig. 4.

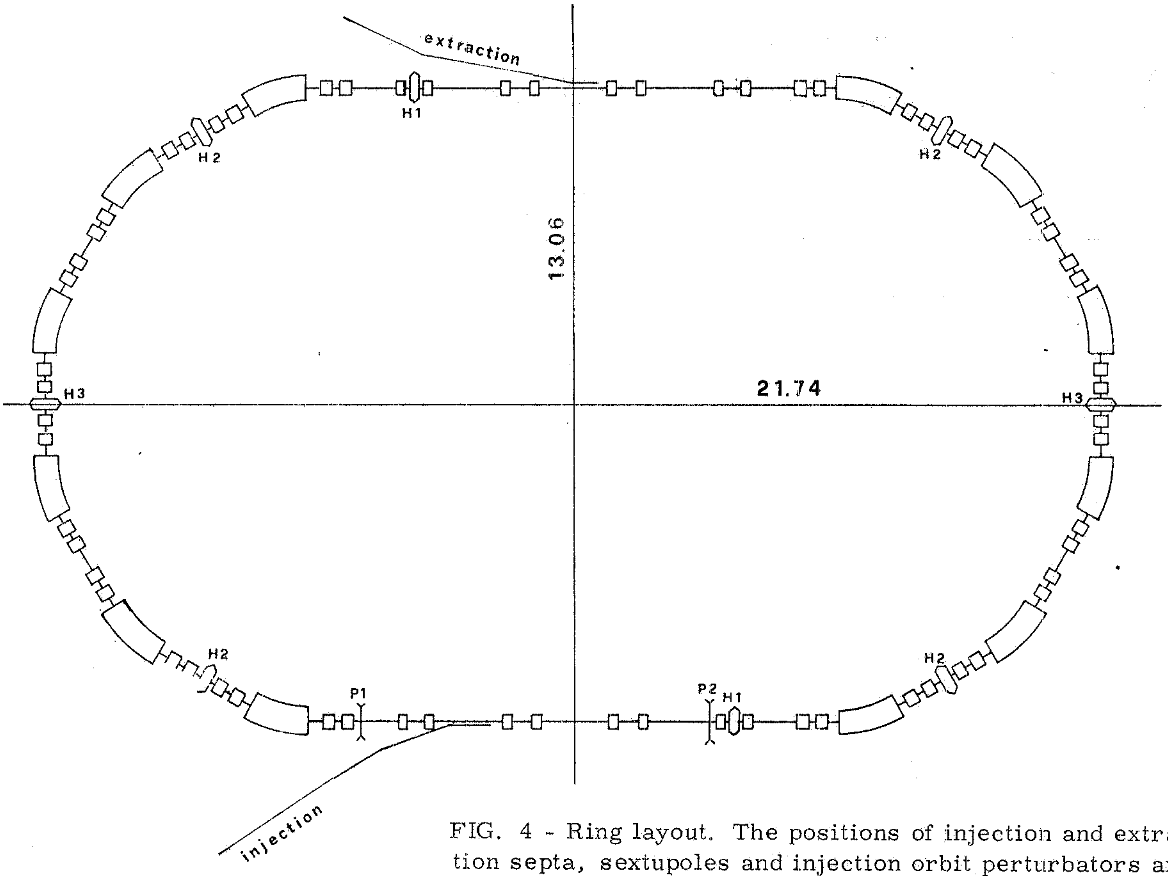

FIG. 4 - Ring layout. The positions of injection and extraction septa, sextupoles and injection orbit perturbators are shown.

The condition of a vanishing dispersion in the straight sections can be satisfied by the requirement that the horizontal transfer matrix T of a period of the bending section satisfy the condition $T^6 = I$ (6).

The maximum value of ψ in the bending section is determined by the magnet bending radius and the horizontal betatron phase advance per period ($\psi_{max} \propto \varrho / \mu_x^2$). Following the choice of 5 m bending radius, in order to decrease ψ_{max}, a total betatron phase advance in the bending sections $\Delta \mu_x = 4 \times (2\pi)$ has been chosen, corresponding to a $2\pi/3$ phase advance per period.

Fig. 5 shows the optical functions β_x, β_z and ψ over one fourth of the circumference. The main parameters of the structure are listed in Table II.

The vertical aperture is determined by the injected beam emittance, closed orbit errors and injection closed orbit perturbation ("bump"). Since this orbit perturbation is localized near the injection straight (see fig. 6), it is possible to define two regions, the first inside the injection bump, and the second outside the bump, with different vertical aperture requirements: for the inside region, it is necessary to add the actual orbit perturbation amplitude, while for the outside region it is sufficient to add the residual orbit (if any) due to bump errors. In order to find the quadrupole and sextupole inside diameters and the magnet gap height, vacuum chamber thickness and other space requirements (donut bake-out, etc.) must be taken into account.

The horizontal aperture is determined by extraction conditions: it is given by the extraction septum position plus the value of the function ψ ($\Delta E/E$) plus the "jump" effected by the particles when they cross the extraction septum (see Chapter 7).

The horizontal and vertical aperture values are summarized in Table III.

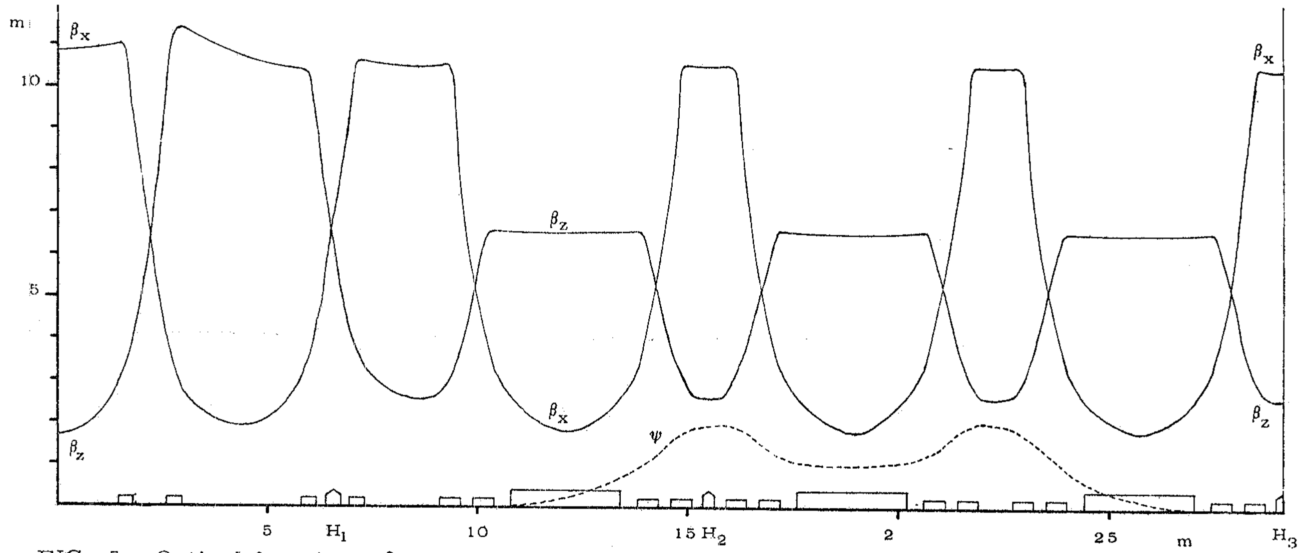

FIG. 5 · Optical functions β_x, β_z, ψ over one fourth of the ring, starting from the extraction straight section. The positions of sextupoles H_1, H_2, H_3 are also shown.

TABLE II

Number of periods in the arcs	12
Number of magnets	12
Number of quadrupoles	64
Number of independent quadrupole power supplies	6
Number of sextupoles	10
Number of independent sextupole power supplies	3
Circumference	L = 118 m
Bending radius	ϱ = 5 m
Horizontal betatron wavenumber	$Q_x = 5.33$
Vertical betatron wavenumber	$Q_z = 4.125$
Maximum β_x in the ring	$\beta_x^{max} = 10.77$ m
Minimum β_x in the ring	$\beta_x^{min} = 1.77$ m
Maximum β_z in the ring	$\beta_z^{max} = 11.44$ m
Minimum β_z in the ring	$\beta_z^{min} = 1.66$ m
Maximum ψ in the ring	$\psi^{max} = 1.99$ m
Maximum ψ in bending magnets	$\psi^{mag} = 1.13$ m
Natural horizontal chromaticity	$C_x = -7.5$
Natural vertical chromaticity	$C_z = -5.0$
Maximum field in bending magnets	B = 0.333 T
Maximum gradient in quadrupoles	G = 2.2 T/m
Maximum sextupole intensity (a = free radius, B_p = pole field)	$S = B_p/a^2 = 5 T/m^2$

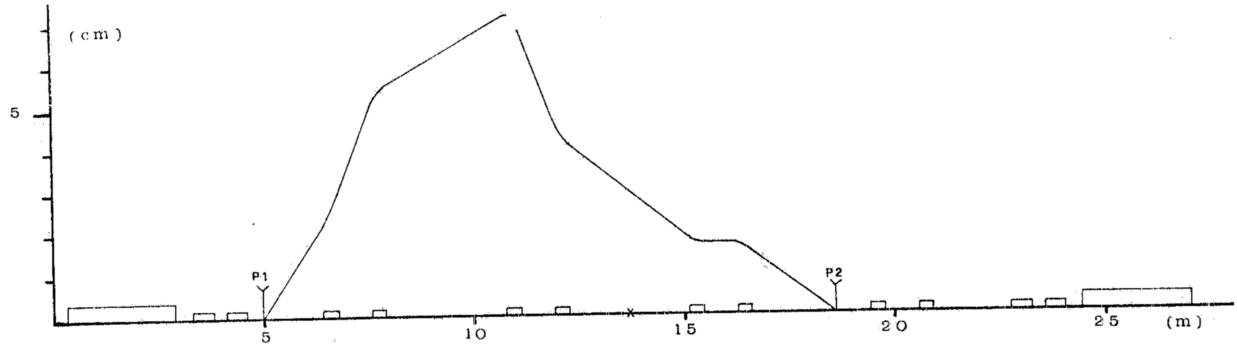

FIG. 6 - Injection bump closed orbit. The perturbation vanishes (apart from errors) in the region outside perturbators P 1 and P2.

TABLE III

	Region inside bump	Region outside bump
Total horizontal aperture (mm)	180	180
Total vertical aperture (mm)	180	85

6. - INJECTION.

The extracted beam average current and beam emittance are mostly determined by injection; more precisely, since extraction takes place in the horizontal plane the horizontal emittance of the extracted beam depends on the horizontal parameters of the injected beam and on the extraction procedure. On the contrary the vertical emittance of the stored beam does not change during extraction; it is therefore necessary to design the injection procedure so as to minimize the stored beam vertical emittance, while optimizing extraction parameters.

The proposed solution is a mixed horizontal/vertical injection (6, 7). The Linac beam is injected off the equilibrium orbit, in the horizontal plane, while the equilibrium orbit in the vertical plane is distorted by means of a suitable local bump. The solution optimizes injection efficiency while keeping the vertical emittance reasonably small.

The inflector, beam and perturbed orbit positions at the injection point are shown schematically in Fig. 7.

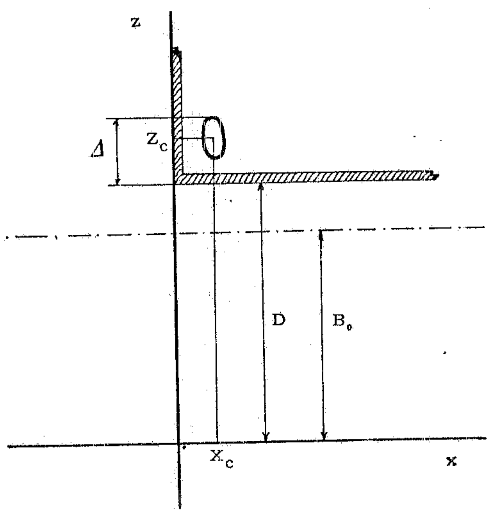

FIG. 7 - Inflector and electron beam positions with respect to the equilibrium orbit: (X_c, Z_c) are the coordinates of the beam center, B_o is the position of the closed orbit bump,

$$\Delta = \sqrt{\beta_z^{inj}}\; W_z + Z_c - D.$$

Let:

X_c = horizontal coordinate of the beam center at the inflector;

Z_z = vertical coordinate of the beam center at the inflector;

D = vertical distance between inflector and equilibrium orbit;

B_o = distance between perturbed orbit and equilibrium orbit at the time when the Linac pulse starts to be injected (t = 0).

It the betatron frequencies Q_x and Q_z are fixed, the position of a particle at the inflector at the N-th turn will be given by:

$$\begin{aligned} x(t_i + NT) &= X_o \cos(2\pi N Q_x + \varphi_x) \quad , \\ z(t_i + NT) &= Z_o \cos(2\pi N Q_z + \varphi_z) + B(t_i + NT) \quad , \end{aligned} \qquad (17)$$

where:

$$\begin{aligned} X_o &= \left[x^2(t_i) + (\beta_x^{inj}\, x'(t_i))^2 \right]^{1/2} \quad ; \qquad \varphi_x = tg^{-1} \frac{\beta_x^{inj}\, x'(t_i)}{x(t_i)} \\ Z_o &= \left[(z(t_i) - B(t_i))^2 + (\beta_z^{inj}\, z'(t_i))^2 \right]^{1/2} \quad ; \qquad \varphi_z = tg^{-1} \frac{\beta_z^{inj}\, z'(t_i)}{\left[z(t_i) - B(t_i) \right]} \end{aligned} \qquad (18)$$

T = revolution period

t_i = time at which the particle enters the ring

B(t) = perturbed orbit displacement from the equilibrium orbit at time t.

If the perturbation B(t) does not change with time (B(t) = D), and the betatron wave numbers are Q_x = 5.33 and Q_z = 4.125, the position of the beam in phase space $(x, \beta_x^{inj} x')$ is shown in Fig. 8a as a function of number of turns, Figures 8b and 8c show the same quantities in the vertical phase space $(z, \beta_z^{inj} z')$ and the physical transverse plane (x, z) respectively.

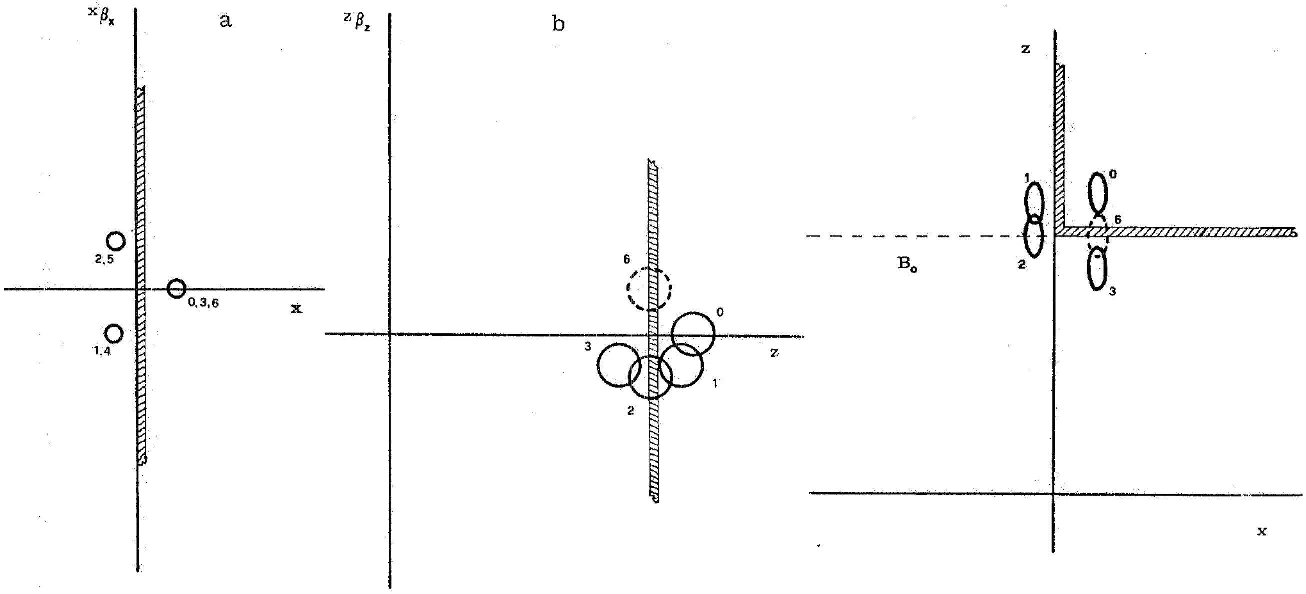

FIG. 8 - Beam position at the inflector as a function of the number of revolutions for a fixed bump (B_o = D); the area occupied by the beam is shown in phase space $(x, \beta_x x')$, $(z, \beta_z z')$ and in the physical transverse plane (x, z).

As it can be seen from Fig. 8c, the beam would hit the inflector at the sixth turn: it is therefore necessary to let B change with time so the perturbed orbit amplitude is that shown in fig. 9.

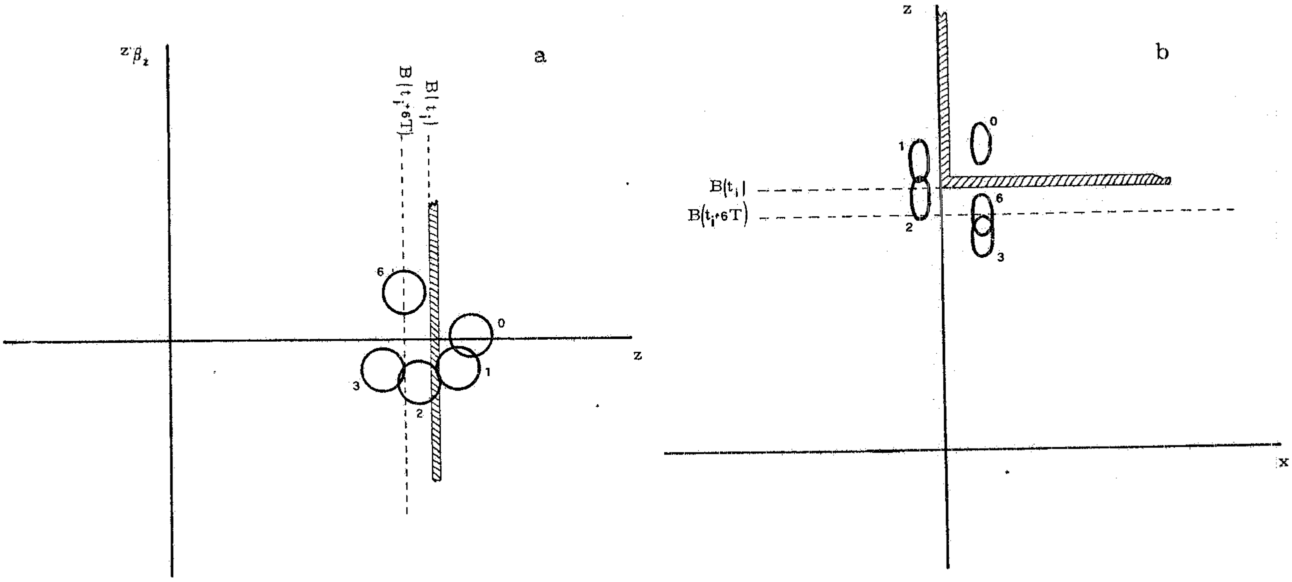

FIG. 9 - Beam position at the inflector as a function of the number of revolutions, with time-variable bump: $B(t_i + 6T)$ is the distance between the closed orbit and equilibrium orbit at 6 th revolution.

Assuming B(t) is linear with time:

$$B(t) = B_o - (ct/L)\ \Delta B_o \tag{19}$$

the values of B_o and ΔB_o must be chosen so that for any N, when $\cos(2\pi N Q_x + \varphi_x) > 0$, the constraint $Z(t_i + NT) < D$ is satisfied, namely:

$$Z_o \cos(2\pi N Q_z + \varphi_z) + B_o - (N + ct_i/L)\ \ \Delta B_o < D \quad . \tag{20}$$

It should be pointed out that if (20) holds for particles injected at t= 0, it will also hold for all particles injected later.

As it can be seen from fig. 9, (20) holds for all particles in the beam if when

$$\sqrt{\beta_x^{inj} W_x} + X_c \cos(2\pi N Q_x) > 0 \quad , \tag{21}$$

then:

$$\sqrt{\beta_z^{inj} W_z} + (Z_c - B_o) \cos(2\pi N Q_z) + B_o - N\Delta B_o \angle D \ , \tag{22}$$

W_x and W_z are the horizontal and vertical emittances of the Linac beam; the l. h. s. of (22) gives the maximum vertical displacement z at the N-th turn experienced by particles injected at t = 0.

Q_x and Q_z depend on the chromaticities C_x and C_z and on the displacement $\Delta E/E$ of the particles from the nominal energy: it is therefore necessary for (22) to hold for any pair of Q values Q_x, Q_z and for all energies in between E and $E + \Delta E_L$.

It is furthermore necessary that the last injected particle sees a bump amplitude such that it misses the inflector. This leads to the condition:

$$B_o - \Delta B_o (ct^*/L) > \Delta \tag{23}$$

t^* = time at which the last particle enters the ring

$$\Delta = Z_c - D + \sqrt{\beta_z^{inj} W_z} \quad , \tag{24}$$

When D is fixed, inequalities (22) and (23) define a region A in the plane (B_o, ΔB_o), inside which it is possible to inject all particles without hitting the inflector; a particular case is shown in Fig. 10a.

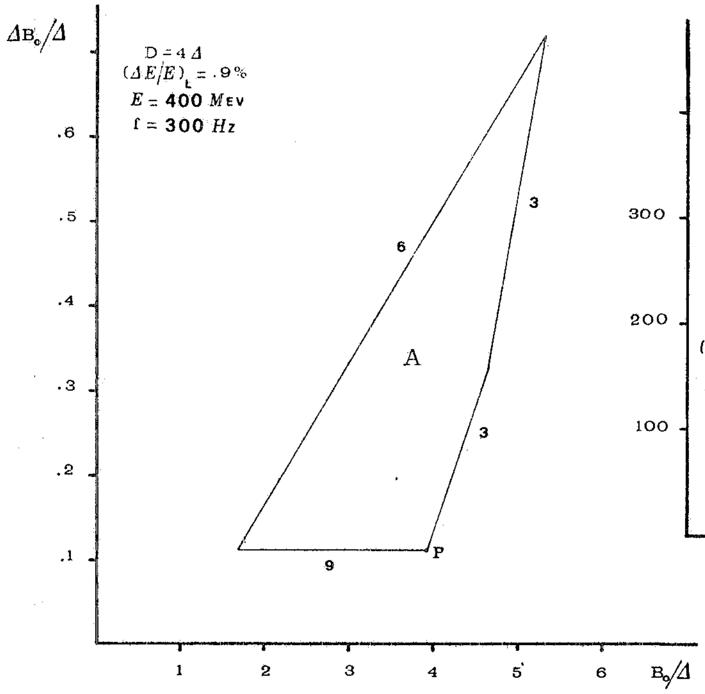

FIG. 10a - Injection diagram. The values (B_o, ΔB_o) inside region A give the best injection efficiency. Near each line is written the number referred to in formula (22) and the last injection turn is $c/L\ t^* = 6$. P is the piont which corresponds to minimum vertical emittance.

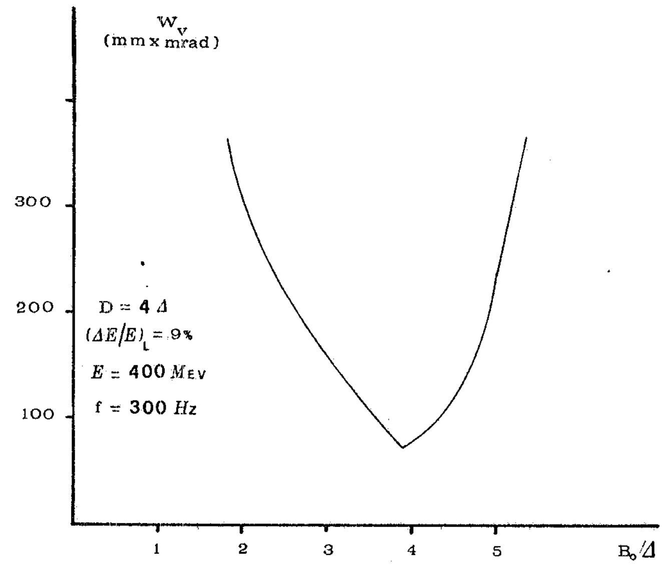

FIG. 10b - Minimum vertical emittance W_v as a function of B_o/Δ . (E = 400 MeV, f = 300 H_z).

The injected beam vertical emittance (which is conserved during horizontal extraction), is given by:

$$W_v = \left(\left| Z_c - B_o + \Delta B_o \frac{ct^*}{L} \right| + \sqrt{\beta_z^{inj} W_z} \right)^2 / \beta_z^{inj} \quad , \tag{25}$$

which holds at a symmetry point ($\alpha = 0$), and shows that vertical emittance increases with t^* and ΔB_o (at fixed B_o). The point inside region A giving the minimum vertical emittance should be selected; Fig. 10b shows the behaviour of the minimum vertical emittance as a function of B_o/Δ , for the same configuration considered in Fig. 10a.

As it has been said earlier, the vertical emittance of the extracted beam depends on the length of the Linac pulse and on the injected beam energy spread $(\Delta E/E)_L$ (which changes with injection energy): Fig. 3 shows (together with the other beam parameters) the vertical emittance achievable in different modes of operation (D is assumed to be 6.15 cm).

All above formulae hold if the sextupoles can be pulsed off at injection [12]; a reduction of the order of 10% in the duty factor follows. Solutions with d. c. sextupoles require some more study.

7. - EXTRACTION.

The sextupole arrangement around the ring should allow control of the horizontal chromaticity and define a stability triangle at the extraction point that has one of its three arms parallel to the x- axis in phase space.

From Fig. 5 it can be seen that sextupoles H_2 are placed where the dispersion ψ is not zero, and therefore can be used to change the chromaticity. Sextupoles H_3 change the extracted beam horizontal emittance and energy spread, while sextupoles H_1 rotate the stability triangle in phase space.

In the monochromatic extraction method matrix M defined in formula (2) produces a rotation by an angle

$$\psi = \frac{1}{3}(\pi - \eta_{33}) \tag{26}$$

where η_{33} is the solution of

$$\begin{cases} H_{33} \cos \eta_{33} = \Sigma_j H_j \cos (3\mu_j + \frac{\pi}{2}) & (27) \\ H_{33} \sin \eta_{33} = \Sigma_j H_j \sin (3\mu_j + \frac{\pi}{2}) & (27') \end{cases}$$

and where, for each sextupole H_j, the equivalent intensity is given by:

$$H_j = \left[\frac{2 B_H L_H}{B_o \varrho a^2}\right] \frac{R\varrho}{48\pi} \left(\frac{\beta_H}{R}\right)^{3/2} \tag{28}$$

B_H = magnetic field at pole,

a = sextupole inner radius,

μ_j = horizontal betatron phase advance from the extraction septum to sextupole location.

In order to fulfill the condition on the direction of the arm of the separatrix, it is necessary for the r. h. s. of (27') to vanish ($\eta_{33} = \pi$). With the arrangement shown in Fig. 5:

$$H_1 = -\frac{N_2 H_2 + N_3 H_3}{N_1 \sin (3\mu_1 + \frac{\pi}{2})}, \tag{29}$$

$$H_{33} = (N_2 H_2 + N_3 H_3) \operatorname{cotg} (3\mu_1 + \frac{\pi}{2}), \tag{29'}$$

$$N_1 = 2, \qquad N_2 = 4, \qquad N_3 = 2,$$

where N_1, N_2, N_3 are the number of sextupoles of kind H_1, H_2, H_3 respectively.

The overall ring chromaticity is given by:

$$C_T = C_R + D_R N_2 H_2 \tag{30}$$

C_R = natural ring chromaticity without sextupoles

$$D_R = 12 \frac{\psi_{H_2}}{\varrho} \left(\frac{R}{\beta_{H_2}}\right)^{1/2} . \tag{31}$$

The horizontal emittance of the extracted beam is determined by the horizontal emittance of the Linac beam, by the overall chromaticity (through $\Delta\nu$), by X_c (distance between the beam center and the equilibrium orbit at injection) and by the position (X_s) of the extraction septum.

$$W_R = \frac{2}{\pi} \sqrt{\frac{W_x}{\beta_x^{ex}}} \; \Delta x \quad , \tag{32}$$

where Δx is the horizontal spread of the extracted beam, given by:

$$\Delta x = (X_s^2 - X_o^2) \,/\, (X_o \operatorname{cotgh}(3\sqrt{3}\pi \left|\Delta\nu_o\right|) - X_s) \tag{33}$$

$$X_o = \sqrt{3}\,(X_c + \sqrt{\beta_x^{inj} W_x})\, R_\beta^{1/2} \tag{34}$$

$R_\beta = (\beta_x^{ex} / \beta_x^{inj})$ has been defined in Chapter 4 and $\Delta\nu_o$ is given by:

$$\Delta\nu_o = 6 \left[1 + \frac{N_3 H_3}{N_2 H_2}\right] \frac{\sqrt{R}}{\varrho} \operatorname{cotg}(3\mu_1 + \frac{\pi}{2}) \left(\frac{C_T - C_R}{D_R}\right) \left(\frac{X_c}{\sqrt{\beta_x^{inj}}} + \sqrt{W_x}\right) \tag{35}$$

The extracted beam energy spread depends on the horizontal emittance of the Linac beam and on the chromaticity:

$$\left(\frac{\Delta E}{E}\right)_{ex} = 12 \left[1 + \frac{N_3 H_3}{N_2 H_2}\right] \frac{\sqrt{R}}{\varrho} \operatorname{cotg}(3\mu_1 + \frac{\pi}{2}) \left(\frac{C_T - C_R}{D_R}\right) \frac{\sqrt{W_x}}{C_T} \tag{36}$$

The extraction efficiency η_x depends on the septum thickness:

$$\eta_{ex} \sim (1 - s/\Delta x) \tag{37}$$

where s is the thickness of the extraction septum and Δx is the "jump" defined by formula (33): extraction efficiency increases with the distance X_s of the septum from the equilibrium orbit: on the other hand it can be seen from (32) that the horizontal emittance is proportional to Δx: the choice of the septum position should allow a high extraction efficiency, together with a reasonable horizontal emittance. In Table IV the extraction parameters are listed as a function of total horizontal chromaticity for X_s = 6.5 cm.

Electrostatic septa can be much thinner that magnetic septa, although angular deflection per unit length is smaller. In order to obtain the deflection required by the physical dimensions of the magnetic lattice elements, an extraction system consisting of an electrostatic septum followed by a magnetic septum is proposed; it is shown schematically in Fig. 11. Electrostatic deflection must get the beam across the thickness of the magnetic septum coil (of the order of 1 cm). The required electric field, for a ~3m long straight section is of the order of 100 KV/cm.

TABLE IV

C_T	$\Delta E/E$ x 1000	Wr (mm x mrad)	Δx (cm)	$\Delta \nu_o$ x 100	K_1 (m^{-2})	K_2 (m^{-2})	K_3 (m^{-2})
- 4	0.99	2.39	0.78	- 1.11	0.65	0.27	- 0.33
	0.90	2.12	0.69	- 1.01	0.59		- 0.35
	0.81	1.87	0.61	- 0.91	0.53		- 0.37
	0.72	1.62	0.53	- 0.81	0.48		- 0.39
- 4.5	1.04	2.97	0.97	- 1.31	0.77	0.23	- 0.21
	0.96	2.67	0.87	- 1.21	0.71		- 0.23
	0.88	2.39	0.78	- 1.11	0.65		- 0.25
	0.80	2.12	0.69	- 1.01	0.59		- 0.27
	0.72	1.87	0.61	- 0.91	0.53		- 0.29
	0.64	1.62	0.53	- 0.81	0.47		- 0.31
- 5	1.01	3.28	1.07	- 1.41	0.83	0.19	- 0.12
	0.93	2.97	0.97	- 1.31	0.77		- 0.14
	0.86	2.67	0.87	- 1.21	0.71		- 0.15
	0.79	2.39	0.78	- 1.11	0.65		- 0.17
	0.72	2.12	0.69	- 1.01	0.59		- 0.19
	0.65	1.86	0.61	- 0.91	0.53		- 0.21
	0.57	1.62	0.53	- 0.81	0.47		- 0.23
- 5.5	1.04	3.94	1.28	- 1.61	0.95	0.16	- 0
	0.98	3.60	1.17	- 1.51	0.89		- 0.02
	0.91	3.27	1.07	- 1.41	0.83		- 0.04
	0.85	2.96	0.97	- 1.31	0.77		- 0.06
	0.78	2.67	0.87	- 1.21	0.71		- 0.08
	0.72	2.39	0.78	- 1.11	0.65		- 0.10
	0.65	2.12	0.69	- 1.01	0.59		- 0.12
	0.59	1.86	0.61	- 0.91	0.53		- 0.14
	0.52	1.62	0.53	- 0.81	0.47		- 0.15

$K = \frac{24\pi H}{R\varrho}\left(\frac{R}{\beta}\right)^{3/2}$; $\qquad S = \frac{B_H}{a^2} = \frac{K B_o \varrho}{L_H}$;

S ($T\ m^{-2}$) = sextupole gradient

B_H(T) = magnetic field at pole

ϱ(m) = magnet bending radius

a(m) = sextupole free radius

L_H(m) = sextupole length

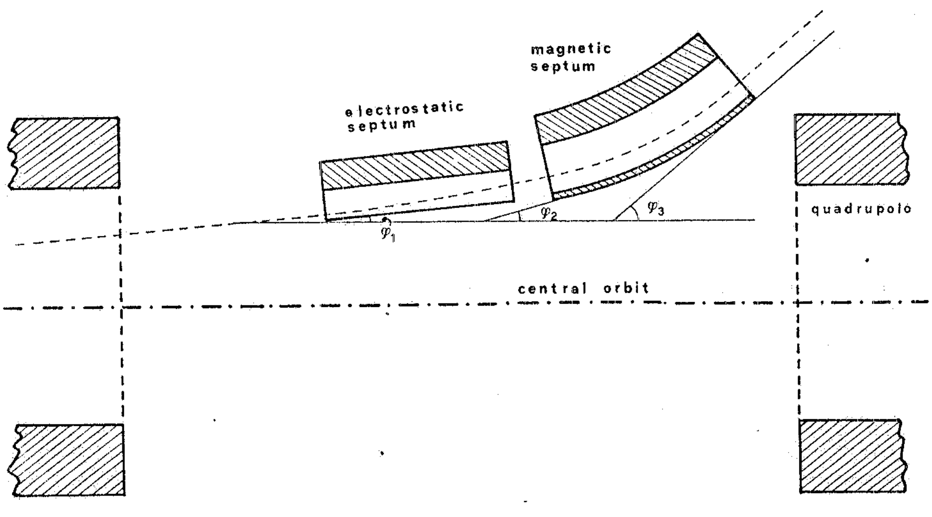

FIG. 11 - Schematic layout of the extraction section.

In this way it is possible to keep the extraction efficiency near unity. More precise injection and extraction efficiency figures will have to be obtained by means of tracking programs taking into account all contributing factors.

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