ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-78/37

G. Dattoli, R. Mignani and D. Prosperi: MORE ABOUT A NEW APPROACH TO THE MAGNETIC MOMENTS OF CHARMED BARYONS

Estratto da:
Lett. Nuovo Cimento 22, 639 (1978)

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

LETTERE AL NUOVO CIMENTO VOL. 22, N. 16 19 Agosto 1978

More about a New Approach to the Magnetic Moments of Charmed Baryons.

G. DATTOLI
CNEN, Divisione Nuove Attività, Centro di Frascati - Frascati
Istituto di Fisica dell'Università - Roma

R. MIGNANI
Istituto di Fisica dell'Università - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

D. PROSPERI
Istituto di Fisica Sperimentale dell'Università - Napoli
Istituto Nazionale di Fisica Nucleare - Laboratori Nazionali di Frascati

(ricevuto l'8 Maggio 1978)

It has been realized very recently [1,2] that an useful approach to theoretically investigating the electromagnetic properties of charmed hadrons can be obtained by introducing a new U_3 symmetry [2], built up by including u, d, and c quarks. In particular, it has been shown [2] that, in this scheme, one is able to recover the right magnetic moments of (nonstrange) charmed baryons, previously calculated in the framework of SU_4 [3,4]. However, such a formalism does not allow, obviously, us to derive the magnetic moments of hadrons with both charm and strangeness. It is the purpose of this letter to show that suitable changes in our previous formalism can provide a straightforward way to derive also the magnetic moments of the strange charmed baryons.

Let us briefly review the basic features of the new U_3 scheme discussed by us in ref. [2]. We start from the « planar representation » (I_3, $\tilde{Y} \equiv Y + C$)—introduced in ref. [4]—of the SU_4 fundamental quartet (see fig. 1). After having got rid of the s-quark and suitably shifted the $\tilde{Y}$-axis ($\tilde{Y} \to Y' \equiv \tilde{Y} - \frac{2}{3}$), one finds the u-, d- and c-quark weight diagram of fig. 2, which has to be identified with the $[\bar{3}]$-representation of a new U_3 group [2].

(1) S. IWAO: Kanazawa University preprint HPICK-042 (1977); *Lett. Nuovo Cimento*, **20**, 522 (1977).
(2) G. DATTOLI, R. MIGNANI and D. PROSPERI: *Lett. Nuovo Cimento*, **22**, 147 (1978).
(3) L. CHOUDARY and V. JOSHI: *Phys. Rev. D*, **13**, 3115, 3120 (1976); D. B. LICHTENBERG: *Phys. Rev. D*, **15**, 345 (1977).
(4) G. DATTOLI, G. MATONE and D. PROSPERI: *Nuovo Cimento*, **45 A**, 187 (1978).

The Gell-Mann-Nakano-Nishijima formula now reads [1,2]

$$Q = \frac{1}{2}\lambda_3 - \frac{1}{2\sqrt{3}}\lambda_8 + \frac{1}{\sqrt{6}}\lambda_0\,, \tag{1}$$

where λ_3, λ_8 are the usual Gell-Mann–Ne'eman matrices and $\lambda_0 \equiv \sqrt{\frac{2}{3}}\mathbf{1}$.

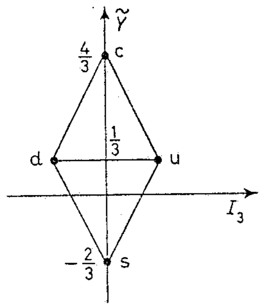

Fig. 1.

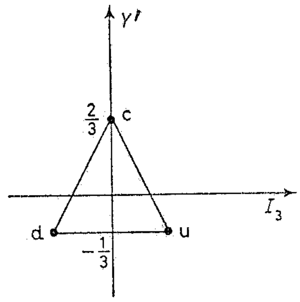

Fig. 2.

Fig. 1. – Planar weight diagram of the SU_4 fundamental representation in the $(I_3, \tilde{Y})$-plane.

Fig. 2. – Fundamental representation of the new U_3 group built up with the u, d and c quarks in the (I_3, Y')-plane.

It is now straightforward to define an U'-spin [2] (also called L-spin [5]), acting on the (u, c)-doublet, in analogy with the old U-spin [6-8]. By the U'-spin formalism one is able to derive the following general formulae for the magnetic moments of the charmed (nonstrange) baryons belonging to the $[\overline{8}]$- and $[\overline{10}]$-multiplets [2] [*]:

a) Baryon octet:

$$\frac{\mu}{\mu_p} = -\frac{2}{3}\left\{\frac{(Q-3)^2}{4} - u'(u'+1) - \frac{1}{2}\right\}. \tag{2}$$

b) Baryon decuplet:

$$\frac{\mu}{\mu_p} = Q\,. \tag{3}$$

Let us now apply a technique similar to the previous one to the quark sector (c, d, s). First of all, we define

$$\tilde{I} \equiv 2I_3 + B\,, \qquad Z_3 \equiv (\tilde{Y} - B)/2 \tag{4}$$

---

[6] K. T. Mahantappa and D. G. Unger: *Phys. Rev. D*, **16**, 3284 (1977).
[7] S. Meshkov, C. A. Levinson and H. J. Lipkin: *Phys. Rev. Lett.*, **10**, 361 (1963).
[8] S. P. Rosen: *Phys. Rev. Lett.*, **11**, 100 (1963); R. J. Oakes: *Phys. Rev.*, **132**, 2349 (1963).
[*] M. A. Beg, B. W. Lee and A. Pais: *Phys. Rev. Lett.*, **13**, 514 (1964).
[5] See fig. 4, 5 and tables I*a*), *b*) of ref. [2].

so that $Q = Z_3 + \frac{1}{2}\tilde{I}$. The notation reflects the fact that $\tilde{I}$ and Z_3 can be considered as an hypercharge and a spin third component, respectively. The basic quartet of quarks in terms of the new variables is shown in fig. 3, whereas the corresponding values of Z, Z_3 and $\tilde{I}$ are listed in table I.

TABLE I. – *Values of Z, Z_3 and $\tilde{I}$ for the basic quartet of quarks.*

State	Z	Z_3	$\tilde{I}$
d	0	0	$-\frac{2}{3}$
u	0	0	$\frac{4}{3}$
s	$\frac{1}{2}$	$-\frac{1}{2}$	0
c	$\frac{1}{2}$	$\frac{1}{2}$	0

By removing the u-quark in fig. 3, we arrive to the weight diagram of fig. 4. It is easily seen that it coincides with the [3]-representation of a SU_3 group, whose commuting operators are $\tilde{I}$ and Z_3. Clearly, the c and s quarks constitute a Z-spin doublet, while the d-quark is a singlet and, within the new SU_3 group, plays the role of the « strange » quark.

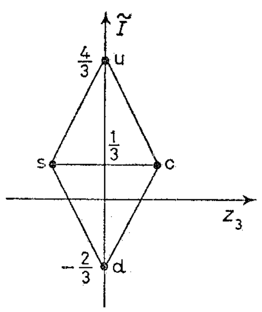

Fig. 3.

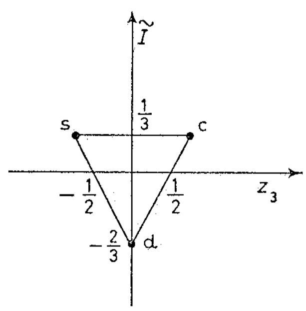

Fig. 4.

Fig. 3. – Fundamental quartet of quarks in the $(Z_3, \tilde{I})$-plane.

Fig. 4. – Fundamental representation of the SU_3 group built up with the c, d and s quarks in the $(Z_3, \tilde{I})$-plane.

Next, we can define an M-spin (similar to the U'-spin, but acting in the (c, d, s)-sector), whose third component, M_3, is given by

$$M_3 = \frac{1}{2}(2\tilde{I} - Q) = \frac{1}{4}[3(B + 2I_3) - (S + C)]\,. \tag{5}$$

The weight diagrams of the baryons belonging to the (c, d, s)-sector and to the [8] and [10] representations in terms of the new variables I and Z_3 are depicted in figs. 5*a*) and *b*); the corresponding Q, M and M_3 values are reported in tables I*a*) and *b*).

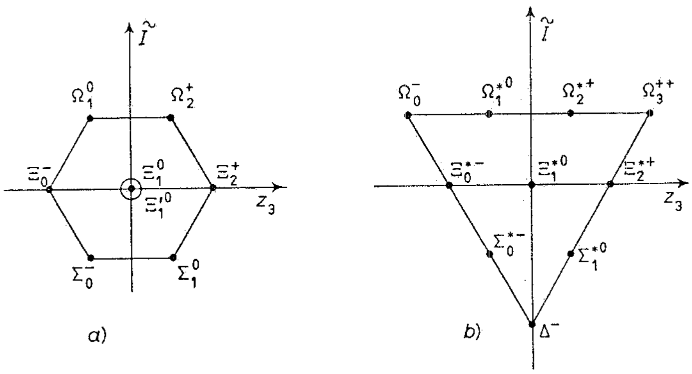

Fig. 5. – Baryon octet (*a*) and decuplet (*b*) of the (c, d, s) quark sector in the $(Z_3, \tilde{I})$-plane.

From the standard definitions of the physical magnetic moment μ, and by taking into account the results of ref. (8), one finds, for the baryon octet:

$$\frac{\mu_8}{\mu_p} = \frac{2}{3}\left\{\left(\frac{Q}{2}+1\right)^2 - M(M+1)\right\} \tag{6}$$

and, therefore (9) (*)

$$\begin{cases} \mu(\Omega_2^+) = \mu(\Xi_2^+) = \mu_p\,, \\ \mu(\Omega_1^0) = \mu(\Sigma_1^0) = \mu_n = -\frac{2}{3}\mu_p\,, \\ \mu(\Xi_0^-) = \mu(\Sigma_0^-) = -\frac{1}{3}\mu_p\,, \\ \mu(\Xi_1^0) + \mu(\Xi_1'^0) = 0\,. \end{cases} \tag{7}$$

By the way, let us notice that Ξ_1^0 and $\Xi_1'^0$ are not eigenstates of the M-spin (the same happens to Λ_0^0 and Σ_0^0 for the U-spin: See *e.g.* ref. (10)).

As to the « baryon decuplet », we have (8):

$$\frac{\mu_{10}}{\mu_p} = Q\,, \tag{8}$$

and

$$\begin{cases} \mu(\Delta^-) = \mu(\Sigma_0^{*-}) = \mu(\Xi_0^{*-}) = \mu(\Omega_0^-) = -\mu_p\,, \\ \mu(\Omega_1^{*0}) = \mu(\Xi_1^{*0}) = \mu(\Sigma_1^{*0}) = 0\,, \\ \mu(\Omega_2^{*+}) = \mu(\Xi_2^{*+}) = \mu_p\,, \\ \mu(\Omega_3^{++}) = 2\mu_p\,. \end{cases} \tag{9}$$

(9) A. W. HENDRY and D. B. LICHTENBERG: *Phys. Rev. D*, **12**, 2756 (1975); D. B. LICHTENBERG: *Lett. Nuovo Cimento*, **13**, 346 (1975); J. FRANKLIN: *Phys. Rev. D*, **12**, 2077 (1975).

(*) In the following, we adopt, for the charmed baryon states, the nomenclature introduced in ref. (9), which, in our opinion, is the most clear and consistent with the standard SU_3 notation.

(10) H. J. LIPKIN: *Lie Groups for Pedestrians* (Amsterdam, 1965); D. B. LICHTENBERG: *Unitary Symmetry and Elementary Particles* (New York, N. Y., 1970).

It can be immediately checked that the above results coincide with those previously derived by different approaches [3,4] in the limit of an exact SU_4 symmetry.

Lastly, let us consider the quark sector (u, c, s) of fig. 1. By applying the procedure of ref. [2] (*), we can construct the weight diagrams of fig. 6, 7*a*) and 7*b*) (where we defined $I' \equiv \tilde{I} - \frac{2}{3}$), for the fundamental triplet, and the baryons of the (u, c, s)-sector belonging to the usual octet and decuplet. Now, the u-quark plays the role of the « charmed » quark in the U_3-scheme of ref. [2].

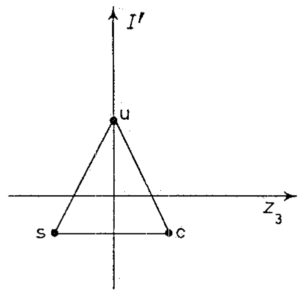

Fig. 6. – Fundamental representation of the U_3 group built up with the u, c and s quarks in the (Z_3, I')-plane.

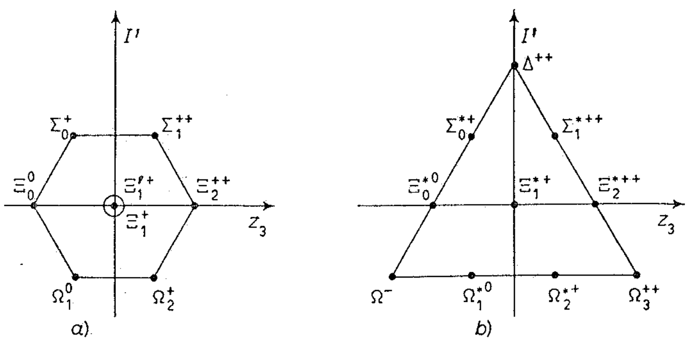

Fig. 7. – Baryon octet (*a*) and decuplet (*b*) of the (u, c, s) sector in the (Z_3, I')-plane.

We can introduce an N-spin (acting on the (u, c)-doublet), with third component given by (it is $Q' = Q - \lambda_0/\sqrt{6}$: see ref. [2]):

$$N_3 = \tfrac{1}{2}(2I' - Q')\,. \tag{10}$$

The values of Q', N, N_3 for the baryons of the $[\bar{8}]$ and $[\overline{10}]$ representations are listed in tables II*a*), II*b*). The formulae for the magnetic moments of baryons are quite

(*) Let us stress that, in this case, the quarks u, c, s belong to a $[\bar{3}]$ representation of U_3 (rather than SU_3). See ref. [2] for further details.

TABLE IIa). – *Values of Q, M, M_3 for the baryons belonging to the* [illegible] *SU_3 in the* (c, d, s)*-sector.*

State	Q	M	M_3
Ω_2^+	1	$\frac{1}{2}$	$\frac{1}{2}$
Ξ_2^+	1	$\frac{1}{2}$	$-\frac{1}{2}$
Ω_1^0	0	1	1
χ_1^0 ([a])	0	1	0
φ_1^0 ([b])	0	0	0
Σ_1^0	0	1	-1
Σ_0^-	-1	$\frac{1}{2}$	$-\frac{1}{2}$
Ξ_0^-	-1	$\frac{1}{2}$	$\frac{1}{2}$

(*a*) $|\chi_1^0\rangle = \frac{1}{2}(-|\Xi_1^0\rangle + \sqrt{3}|\Xi_1'^0\rangle)$ $(M = 1)$.
(*b*) $|\varphi_1^0\rangle = \frac{1}{2}(\sqrt{3}|\Xi_1^0\rangle + |\Xi_1'^0\rangle)$ $(M = 0)$.

TABLE II *b*). – *Values of Q, M, M_3 for the baryons belonging to the* [10]*-representation of SU_3 in the* (c, d, s)*-sector.*

State	Q	M	M_3
Ω_0^-	-1	$\frac{3}{2}$	$\frac{3}{2}$
Ξ_0^{*-}	-1	$\frac{3}{2}$	$\frac{1}{2}$
Σ_0^{*-}	-1	$\frac{3}{2}$	$-\frac{1}{2}$
Δ^-	-1	$\frac{3}{2}$	$-\frac{3}{2}$
Ω_1^0	0	1	1
Ξ_1^{*0}	0	1	0
Σ_1^{*0}	0	1	-1
Ω_2^+	1	$\frac{1}{2}$	$\frac{1}{2}$
Ξ_2^{*+}	1	$\frac{1}{2}$	$-\frac{1}{2}$
Ω_3^{++}	2	0	0

analogous to those valid in the (u, d, c)- quark sector ([2]) (eqs. (2), (3) of ref. ([2])) (*). It is therefore very easy to derive the following results:

$$(11) \qquad \begin{cases} \mu(\Sigma_1^{++}) = \mu(\Xi_2^{++}) = \frac{2}{3}\mu_p\,, \\ \mu(\Sigma_0^+) \;\;= \mu(\Omega_2^+) \;\;= \mu_p\,, \\ \mu(\Xi_0^0) \;\;= \mu(\Omega_1^0) \;\;= -\frac{2}{3}\mu_p\,, \\ \mu(\Xi_1^+) \;\;+ \mu(\Xi_1'^+) = \frac{2}{3}\mu_p \end{cases}$$

(*) For instance, for the baryon octet eq. (2) must be used, with u' replaced by N. Moreover, one has to introduce a «fictitious» magnetic moment. See ref. ([2]).

TABLE III*a*). – *Values of* Q', N, N_3 *for the baryons belonging to the* $[\bar{8}]$-*representation of* U_3 *in the* (u, c, s)-*sector.*

State	Q'	N	N_3
Σ_1^{++}	1	$\frac{1}{2}$	$-\frac{1}{2}$
Ξ_2^{++}	1	$\frac{1}{2}$	$\frac{1}{2}$
Σ_0^{+}	0	1	-1
χ_1^{+} (a)	0	1	0
φ_1^{+} (b)	0	0	0
Ω_2^{+}	0	1	1
Ξ_0^{0}	-1	$\frac{1}{2}$	$\frac{1}{2}$
Ω_1^{0}	0	$\frac{1}{2}$	$-\frac{1}{2}$

(*a*) $|\chi_1^+\rangle = \frac{1}{2}(-|\Xi_1^+\rangle + \sqrt{3}|\Xi_1'^+\rangle)$.
(*b*) $|\varphi_1^+\rangle = \frac{1}{2}(\sqrt{3}|\Xi_1^+\rangle + |\Xi_1'^+\rangle)$.

TABLE III*b*). – *Values of* Q', N, N_3 *for the baryons belonging to the* $[\overline{10}]$-*representation of* SU_3 *in the* (u, c, s)-*sector.*

State	Q'	N	N_3
Δ^{++}	1	$\frac{3}{2}$	$\frac{3}{2}$
Σ_1^{*++}	1	$\frac{3}{2}$	$\frac{1}{2}$
Ξ_2^{*++}	1	$\frac{3}{2}$	$-\frac{1}{2}$
Ω_3^{++}	1	$\frac{3}{2}$	$-\frac{3}{2}$
Σ_0^{*+}	0	1	1
Ξ_1^{*+}	0	1	0
Ω_2^{*+}	0	1	-1
Ξ_0^{*0}	-1	$\frac{1}{2}$	$\frac{1}{2}$
Ω_1^{*0}	-1	$\frac{1}{2}$	$-\frac{1}{2}$
Ω^{-}	-2	0	0

for the octet (*), and

$$\text{(12)} \qquad \begin{cases} \mu(\Delta^{++}) = \mu(\Sigma_1^{*++}) = \mu(\Xi_2^{*++}) = \mu(\Omega_3^{++}) = 2\mu_p\,, \\ \mu(\Sigma_0^{*+}) = \mu(\Xi_1^{*+}) \;\;= \mu(\Omega_2^{*+}) \;\;= \mu_p\,, \\ \mu(\Xi_0^{*0}) = \mu(\Omega_1^{*0}) \;\;= 0\,, \\ \mu(\Omega^{-}) \;\;= -\mu_p \end{cases}$$

for the decuplet [8].

(*) In this case, too, Ξ_1^+ and $\Xi_1'^+$ are not eigenstates of the N-spin. See ref. [10].

TABLE IV. – *The four subgroups of SU_4 and their additive quantum numbers.*

	quark sector	group structure	additive quantum numbers
I	u, d, s	SU_3	S
II	u, d, c	U_3	C
III	d, s, c	SU_3	$\tilde{I} - B = 2I_3 = S'$
IV	u, s, c	U_3	$\tilde{I} - B = 2I_3 = C'$

In conclusion, let us make some final remarks, aimed to further clarify our formalism. In general, given a SU_n subgroup, one can find n SU_{n-1} (or U_{n-1}) subgroups (*e.g.*, in the case of SU_3, we have 3 SU_2 subgroups, leading to the usual definitions (6,10) of I-, U- and V-spin). In our case, we obtain four subgroups (corresponding to different quark sectors): See table IV. The additive quantum numbers for $SU_{3\mathrm{I}}$ and $U_{3\mathrm{II}}$ are strangeness and charm, respectively, while, for $SU_{3\mathrm{III}}$ and $U_{3\mathrm{IV}}$, the following quantities:

$$(\tilde{I} - B)_{\mathrm{III}} = 2I_3(\mathrm{III}) = S' \tag{13}$$

and

$$(\tilde{I} - B)_{\mathrm{IV}} = 2I_3(\mathrm{IV}) = C' \tag{14}$$

play the role of quantum numbers « strangenesslike » and « charmlike », respectively.

Eventually, let us mention that, in a very recent paper (11), use has been made of the restriction to the (c, d, s)- quark sector. However, the magnetic moments of the charmed baryons have been calculated by using an approach different from the present one, and the whole philosophy is rather unlike ours.

* * *

Thanks are due to P. CAMIZ for kind interest and useful discussions.

(11) S. IWAO: *Lett. Nuovo Cimento*, **21**, 239, 245 (1978).