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M. Greco, G. Pancheri-Srivastava and Y. Srivastava: WEAK  
AND RADIATIVE ASYMMETRIES.

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ABSTRACT.

It is shown that the weak amplitude must also be radiatively corrected for a satisfactory analysis of the neutral current in  $e^+e^- \rightarrow \mu^+\mu^-$ . Radiative asymmetry can be of possible help in isolating the weak effect provided the symmetric cross-section can be measured to  $\approx 1\%$  accuracy at PEP/PETRA energies.

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Various proposals<sup>(1)</sup> have been made to study the neutral weak effects in the colliding beam process  $e^+e^- \rightarrow \mu^+\mu^-$ . For machines presently under construction (PETRA and PEP), the energy per beam  $E$  is in the range 15-20 GeV. Since in the Weinberg-Salam<sup>(2)</sup> model the mass of the  $Z^0$ -meson is supposed to be in the vicinity of 80-100 GeV, investigation has mainly focused upon the asymmetry caused by the weak-EM interference. The expected effect is, however, masked by the radiative asymmetry which is opposite in sign and depends crucially upon the energy resolution ( $\Delta\omega$ ) of the muons. This poses contradictory demands. On the one hand, we would like to reduce the radiative asymmetry and thus keep  $\frac{\Delta\omega}{E}$  as large as possible. But then hard photon corrections<sup>(3,4)</sup>, about which there is much calculational uncertainty, become important. On the other hand, for the soft photon region ( $\frac{\Delta\omega}{E} \ll 1$ ) where we have very accurate theoretical formulae<sup>(5)</sup>, the weak effect becomes relatively smaller.

Due to the above reasons, we would like to present an alternative way which uses the radiative corrections to our advantage. The method envisages a rather accurate experiment,  $\frac{\Delta\omega}{E} \simeq (10^{-2} - 10^{-3})$ , for which the well known radiative correction factor enhances the EM-weak interference in the symmetric cross-section. The role of radiative asymmetry is reversed between the symmetric and the antisymmetric parts of the cross-section: whereas in the latter it enhances the purely EM term at the expense of the interference, in the former it is just the opposite (see Eqs. (9) below). Our method should be particularly useful if, as all experiments done so far seem to indicate,  $\sin^2\theta_w \simeq 0.25$  so that the  $(e - \mu)$  neutral current is almost purely axial.

Without radiative corrections, the differential cross-section in the Weinberg-Salam model is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4S} \left\{ \left[ 1 - \frac{1}{8} (3\tan\theta_w - \cot\theta_w)^2 \frac{s}{M_Z^2 - s} \right] (1 + \cos^2\theta - P_+ P_- \sin^2\theta \cos 2\Phi) - \frac{1}{\sin^2 2\theta_w} \left( \frac{s}{M_Z^2 - s} \right) \cos\theta \right\} \quad (1)$$

where  $P_{\pm}$  are the transverse polarizations of the  $e^{\pm}$  beam. In the above, only the EM and the EM-weak interference terms have been kept. For simplicity, in the following we shall assume that the Weinberg angle  $\theta_w = 30^\circ$ , which is consistent with recent experiments<sup>(6)</sup>. This leads to  $M_Z \simeq 87$  GeV.

Using the soft photon formalism, which should be accurate for  $\frac{\Delta\omega}{E} \simeq (10^{-2} - 10^{-3})$ , the radiatively corrected symmetric cross-section becomes<sup>(5,7)</sup>

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{\text{sym}} &\equiv \frac{1}{2} \left[ \frac{d\sigma}{d\Omega}(\theta, \Phi) + \frac{d\sigma}{d\Omega}(\pi - \theta, \Phi) \right] \simeq \\ &\simeq \frac{\alpha^2}{4S} \left(\frac{\Delta\omega}{E}\right)^{\beta_e + \beta_\mu} \left\{ (1 + \cos^2\theta - P_+ P_- \sin^2\theta \cos 2\Phi) (1 + C_{\text{sym}}^{\text{QED}}) - \right. \\ &\quad \left. - \frac{4}{3} \left( \frac{s}{M_Z^2 - s} \right) \cos\theta \left[ 2\beta_{\text{int}} \ln\left(\frac{\Delta\omega}{E}\right) (1 + C_{\text{sym}}^{\text{INT}}) + C_{\text{asy}}^{\text{INT}} \right] \right\} \quad (2) \end{aligned}$$

where

$$\begin{aligned} \beta_{e,\mu} &= \frac{4\alpha}{\pi} \left( \ln \frac{2E}{m_{e,\mu}} - \frac{1}{2} \right), & \beta_{\text{int}} &= \frac{4\alpha}{\pi} \ln \left( \tan \frac{\theta}{2} \right) \\ C_{\text{sym}}^{\text{QED}} &\simeq \frac{13}{12} (\beta_e + \beta_\mu), & C_{\text{sym}}^{\text{INT}} &\simeq \frac{11}{12} (\beta_e + \beta_\mu), \end{aligned} \quad (3)$$

and we have linearized the  $\beta_{\text{int}}$  dependence.

The finite antisymmetric correction  $C_{\text{asy}}^{\text{INT}}$  has not been calculated completely, but it is expected to be of the order of  $C_{\text{asy}}^{\text{QED}} \equiv X(\theta)$  of reference (7). In any case, this contribution, being independent of  $\beta_{e,\mu}$  as well as of  $\frac{\Delta\omega}{E}$ , should be quite small ( $\lesssim 10\%$ ) compared to  $2\beta_{\text{int}} \ln(\frac{\Delta\omega}{E})$ . Henceforth,  $C_{\text{asy}}^{\text{INT}}$  will be ignored.

As can be seen from Eq. (2), the weak effect is substantially enhanced by the factor  $\tau(\theta, \Delta\omega) \equiv \frac{8\alpha}{\pi} \ln(\tan \frac{\theta}{2}) \ln \frac{\Delta\omega}{E}$ , leading us to suggest very accurate experiments near the forward direction. Notice that both the requirements of good collinearity and very forward scattering are opposite to those required for cleaner asymmetry measurement.

We now present specific results for three cases:

- 1) maximum transverse polarization, i. e.  $P_+ = P_- = .924$  and  $\Phi = 0$ .

The  $\theta$ -integrated cross-section for  $\frac{\Delta\omega}{E} = 10^{-3}$  is given by

$$\left(\frac{d\sigma}{d\Phi}\right)_{\text{max. Pol.}}^{\Phi=0} \simeq \frac{\alpha^2}{4s} (1 + \beta_e + \beta_\mu) \left(\frac{\Delta\omega}{E}\right)^{\beta_e + \beta_\mu} (1.53) \left[ 1 - 0.11 \left(\frac{s}{M_z^2 - s}\right) \right] \quad (4)$$

For  $\sqrt{s} = 30$  GeV, this gives a (-1.5%) effect and a (-3%) effect at  $\sqrt{s} = 40$  GeV.

- 2) No polarization, i. e.  $P_+ = P_- = 0$ . Then

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{\text{sym}} \simeq \left(\frac{\pi\alpha^2}{2s}\right) (1 + \beta_e + \beta_\mu) \left(\frac{\Delta\omega}{E}\right)^{\beta_e + \beta_\mu} (1 + z^2) \left[ 1 - \frac{32\alpha}{3\pi} \left(\frac{z}{1+z^2}\right) \left(\frac{s}{M_z^2 - s}\right) \ln(\tan \frac{\theta}{2}) \ln\left(\frac{\Delta\omega}{E}\right) \right] \quad (5)$$

where  $z = \cos \theta$ .

We define

$$\Delta(\theta) \equiv \frac{32\alpha}{3\pi} \left( \frac{z}{1+z} \right) \left( \frac{s}{M_z^2 - s} \right) \ln(\tan \frac{\theta}{2}) \ln\left( \frac{\Delta\omega}{E} \right) \quad (6)$$

and evaluate it as a function of  $\theta$ . For  $\sqrt{s} = 30$  GeV,  $\frac{\Delta\omega}{E} = 10^{-3}$ ,  $\Delta(\theta)$  becomes 5.5%, 3.6% and 1.5% for  $\theta=1^\circ, 5^\circ$  and  $30^\circ$  respectively. At  $\sqrt{s} = 40$  GeV the effect is doubled. This clearly shows that beam pipe experiments, which measure essentially forward muons, are best suited for investigating this effect.

- 2) If we integrate over  $\theta$  as well as  $\Phi$ , we get for the radiatively corrected cross-section (for  $\frac{\Delta\omega}{E} = 10^{-3}$ )

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) \simeq \left( \frac{4\pi\alpha^2}{3s} \right) \left( \frac{\Delta\omega}{E} \right)^{\beta_e + \beta_\mu} (1 + \beta_e + \beta_\mu) \left[ 1 - 0.06 \left( \frac{s}{M_z^2 - s} \right) \right] \quad (7)$$

This produces (-0.9%) and (-1.7%) effects at  $\sqrt{s} = 30$  GeV and 40 GeV respectively.

Now we compare briefly the type of measurement we propose (e. g.  $\Delta(\theta)$ ) with the usual one of charge asymmetry A. The radiatively corrected anti-symmetric cross-section is given by (5, 7)

$$\begin{aligned} \left( \frac{d\sigma}{d\Omega} \right)_{\text{asy}} &\equiv \frac{1}{2} \left[ \frac{d\sigma}{d\Omega}(\theta, \Phi) - \frac{d\sigma}{d\Omega}(\pi - \theta, \Phi) \right] = \\ &= \frac{\alpha^2}{4s} \left( \frac{\Delta\omega}{E} \right)^{\beta_e + \beta_\mu} \left\{ (1 + \cos^2 \theta - P_+ P_- \sin^2 \theta \cos 2\Phi) \left[ \tau(\theta, \Delta\omega) (1 + C_{\text{sym}}^{\text{QED}}) + \right. \right. \\ &\left. \left. + X_{\text{pol}}(\theta, \Phi) \right] - \frac{4}{3} \left( \frac{s}{M_z^2 - s} \right) \cos \theta \left[ 1 + C_{\text{sym}}^{\text{INT}} \right] \right\} \quad (8) \end{aligned}$$

where  $X_{\text{pol}}(\theta, \Phi)$  is defined in reference (7).

Under the same approximations, the  $\Phi$ -integrated charge asymmetry  $A(\theta) \equiv (\frac{d\sigma}{d\cos\theta})_{asy} / (\frac{d\sigma}{d\cos\theta})_{sym}$  and  $\Delta(\theta)$  are given by

$$A(\theta) \simeq \tau(\theta, \Delta\omega) - \frac{z}{1+z} B + X(\theta) \quad (9a)$$

$$A(\theta) \simeq \tau(\theta, \Delta\omega) - \frac{z}{1+z} B, \quad (9b)$$

where  $B = \frac{4}{3} \frac{s}{M_z^2 - s}$  and  $X(\theta) = X_{pol}(\theta, \Phi = \pi/4)$ .

The above formulae exhibit succinctly the problem associated with the asymmetry measurement: a large back-ground subtraction due to the EM asymmetry has to be made. On the contrary  $\Delta(\theta)$ , being proportional to the weak effect, gives us the desired result directly. Notice that, in the very forward direction,  $\Delta(\theta)$  and  $A(\theta) - A_{EM}(\theta)$  are roughly of the same order of magnitude, i. e. a few percent. Thus, a simultaneous determination of  $A(\theta)$  and  $\Delta(\theta)$  under the same experimental conditions should provide a better knowledge of the weak effect.

A more complete analysis, valid also at higher energies, is under preparation and shall be presented elsewhere.

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