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N. Lo Iudice and F. Palumbo: POSITIVE PARITY ISOVECTOR COLLECTIVE STATES IN DEFORMED NUCLEI.

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ABSTRACT.

A model is developed for deformed nuclei in which their constituents protons and neutrons are described as axially symmetric rigid bodies. These two rotors, being free to rotate separately and interacting through a harmonic potential, perform intrinsic rotational oscillations with opposite phases about a common axis. The dynamical system so composed does not possess axial symmetry. Its lowest lying excited states which can be excited by electromagnetic radiation are two in number and have both an excitation energy of $\sim$ 10 MeV in heavy deformed nuclei. One of them is strongly excited by isovector magnetic dipole radiation, the other is weakly excited by electric quadrupole radiation.

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1. - INTRODUCTION.

The "giant resonance" is an electric dipole mode of excitation occurring in all nuclei. Its first and simplest explaination was given by Goldhaber and Teller[(1)] who developed a macroscopic model in which the excitation is generated by a collective translational oscillation of neutrons against protons.

This two fluid picture of the nucleus suggests the possible existence of new collective modes of excitation in deformed nuclei. Protons and neutrons can be assumed to form two separate rigid bodies of spheroidal shape. The two bodies are then allowed to rotate around a common axis with opposite angular velocities. The restoring force generated by the displacement of protons against neutrons might give rise either to relative rotational oscillations[(2)] or to a configuration in which the nucleus rotates while the proton-neutron symmetry axes stay at a fixed angle.

The purpose of this paper is to study such modes. We assume the nucleus to be composed of an equal number of protons and neutrons. These are described as axially symmetric rigid rotors interacting through a harmonic potential depending on the angle between their symmetry axes. Starting from these assumptions in Sect. 2 we develop a dynamical model for the nucleus so composed, which results to be no longer axially symmetric. In Section 3 we solve the eigenvalue equation for the simplest lowest lying states which come out to be also the most interesting ones. In Section 4 the physical nature of these states is studied by evaluating the electromagnetic (e. m.) transition probabilities.

In the concluding Section the effect of our simplifying assumptions is briefly discussed.

2. - THE MODEL.

The nucleus is described as a system composed of two identical axially symmetric rigid bodies, one consisting of protons the other of neutrons.

We denote the versors of the principal axes of the two rotors by $\hat{\xi}_p$, $\hat{\eta}_p$, $\hat{\zeta}_p$ for protons and by $\hat{\xi}_n$, $\hat{\eta}_n$, $\hat{\zeta}_n$ for neutrons and the Euler angles specifying their orientation by α_p, β_p, γ_p and α_n, β_n, γ_n. Because of the axial symmetry of protons and neutrons the motion of the whole system is fully described by only four variables, namely the Euler angles α_p, β_p and α_n, β_n specifying the orientation of the symmetry axes $\hat{\zeta}_p$ and $\hat{\zeta}_n$ of the two rotors.

If relative traslational motion is excluded, the kinetic energy of the nucleus about its center of mass is

$$T = \frac{1}{2\mathcal{J}_o}\left\{(I_{\xi_p}^{(p)2} + I_{\eta_p}^{(p)2}) + (I_{\xi_n}^{(n)2} + I_{\eta_n}^{(n)2})\right\} , \qquad (1)$$

where $I_{\xi_p}^{(p)}$, $I_{\eta_p}^{(p)}$ and $I_{\xi_n}^{(n)}$, $I_{\eta_n}^{(n)}$ are the components of the angular momenta of protons and neutrons along the axes $\hat{\xi}_p$, $\hat{\eta}_p$ and $\hat{\xi}_n$, $\hat{\eta}_n$ and $\mathcal{J}_o$ their common moment of inertia. Rotations about their own symmetry axes are excluded consistently with a correct quantum mechanical description of the motion of symmetric rigid rotors. We further assume that protons and neutrons interact through a potential which depends on the angle between the symmetry axes $\hat{\zeta}_p$ and $\hat{\zeta}_n$.

Since the two rotors can move separately, the nucleus as a whole, unlike its two components, does not possess axial symmetry. Its principal axes are indeed defined by the relations:

$$\hat{\xi} = \frac{\hat{\zeta}_p \times \hat{\zeta}_n}{\sin(2\theta)} , \qquad \hat{\eta} = \frac{\hat{\zeta}_p - \hat{\zeta}_n}{2\sin\theta} , \qquad \hat{\zeta} = \frac{\hat{\zeta}_p + \hat{\zeta}_n}{2\cos\theta} , \qquad (2)$$

where 2θ is the angle between the symmetry axes $\hat{\zeta}_p$ and $\hat{\zeta}_n$.

The above definitions provide us with a new set of angular coordinates, the Euler angles $\alpha\beta\gamma$ specifying the orientation of the whole nucleus and the angle θ which describes the relative orientation of the two rotors. The correspondence between the new and the old coordinates $\alpha_p\ \beta_p\ \alpha_n\ \beta_n$ is one to one if $\alpha\beta\gamma$ are allowed to vary over their full range and θ is constrained in the interval $0 \lesssim \theta \lesssim \pi/2$. In order to make the new angular coordinates of practical use we must find their conjugate variables. To this purpose we define the quantities:

$$\vec{I} = \vec{I}^{(p)} + \vec{I}^{(n)}, \qquad \vec{S} = \vec{I}^{(p)} - \vec{I}^{(n)}. \tag{3}$$

These are the well known generators of the group O(4). Their intrinsic and laboratory components satisfy the commutation relations listed in Appendix A. These relations define the intrinsic components I_ξ, I_η, I_ζ and S_ξ as the conjugate momenta of $\alpha\beta\gamma$ and θ respectively while show that S_η and S_ζ are not independent variables. All commutation relations are indeed satisfy if we put

$$S_\xi = i\frac{d}{d\theta}, \qquad S_\eta = -\mathrm{ctg}\,\theta\, I_\zeta, \qquad S_\zeta = -\mathrm{tg}\,\theta\, I_\eta. \tag{4}$$

We have now two set of coordinates at our disposal. It is a matter of convenience to use one set or the other. Given the dependence of the interaction potential on θ, the new set $\alpha\beta\gamma\ \theta$ appears to be more suitable for describing the motion of our system. The kinetic energy (1) must therefore be expressed in these new coordinates. This can be achieved by rewriting T as:

$$T = \frac{1}{2\mathcal{J}_o}(\vec{I}^{(p)2} + \vec{I}^{(n)2}) = \frac{1}{4\mathcal{J}_o}(\vec{I}^2 + \vec{S}^2), \tag{5}$$

with the condition

$$I^{(p)}_{\zeta_p} = I^{(n)}_{\zeta_n} = 0.$$

This constraint is automatically fulfilled by expressions (4) of the S-components as one can easily check. We can therefore project I and S on the intrinsic axes and using eqs. (4) obtain for the whole Hamiltonian

$$H = \frac{1}{4\mathcal{J}_o} I_\xi^2 + \frac{1}{4\mathcal{J}_o}(1+\mathrm{tg}^2\theta) I_\eta^2 + \frac{1}{4\mathcal{J}_o}(1+\mathrm{ctg}^2\theta) I_\zeta^2 - \frac{1}{4\mathcal{J}_o}\frac{d^2}{d\theta^2} + V(\theta) \tag{6}$$

where

$$V(\theta) = V(\pi/2 - \theta), \qquad 0 \leq \theta \leq \pi/2, \tag{7}$$

as imposed by the geometry of the system. The Hamiltonian (6) is Hermitian in the domain of wavefunctions ψ satisfying the equation:

$$\lim_{\theta \to 0, \frac{\pi}{2}} \left(\psi \frac{d}{d\theta} \psi' - \psi' \frac{d}{d\theta} \psi\right) = 0 . \tag{8}$$

Its lack of axial symmetry is clearly exhibited by its expression (6). H is invariant however against rotations through the angle π about each principal axis. Its eigenfunctions can then be made to be also simultaneous eigenfunctions of the rotation operators $R_\xi(\pi)$, $R_\eta(\pi)$ and $R_\zeta(\pi)$ with eigenvalues r_ξ, r_η and r_ζ $(r = \pm 1)$ respectively. Further restrictions on the eigenfunctions must be imposed because of the symmetry properties of the system itself. A rotation of protons and (or) neutrons about $\hat{\xi}$ through the angle π leaves the configuration of the system unchanged, namely:

$$\begin{aligned} R_\xi^{(p)}(\pi)\, \psi_{\sigma I M r_\eta r_\zeta} &= \psi_{\sigma I M r_\eta r_\zeta} , \\ R_\xi^{(n)}(\pi)\, \psi_{\sigma I M r_\eta r_\zeta} &= \psi_{\sigma I M r_\eta r_\zeta} , \end{aligned} \tag{9}$$

where I and M are the total angular momentum and its z-component and σ denotes all the additional quantum numbers.

One of eqs. (9) can be replaced by:

$$R_\xi(\pi)\, \psi_{\sigma I M r_\eta r_\zeta} = \psi_{\sigma I M r_\eta r_\zeta} . \tag{10}$$

This latter condition fixes $r_\eta = r_\xi$ so that the eigenfunctions assume the following form :

$$\psi_{\sigma I M r_\eta} = \left[\frac{2I+1}{16\pi^2}\right]^{1/2} \sum_{\substack{K=0,2,\ldots (r_\eta=+1) \\ K=1,3,\ldots (r_\eta=-1)}} \Phi_{\sigma IK}(\theta) \quad \cdot$$

$$\cdot \left[D^I_{M,K}(\alpha,\beta,\gamma) + (-)^I D^I_{M,-K}(\alpha,\beta,\gamma)\right](1+\delta_{K0})^{-1/2} \; . \tag{11}$$

It remains to impose one of conditions (9) for instance the first one. This affects the intrinsic wave function too. A rotation of the protons about $\hat{\xi}$ of the angle π can be indeed obtained as the result of the following operations :

$$R^{(p)}_\xi(\pi)\,\psi_{\sigma I M r_\eta} = R_\xi(\pi/2)\,R_\eta(\pi)R_\theta\,\psi_{\sigma I M r_\eta} \; , \tag{12}$$

where

$$R_\theta\,\Phi_{\sigma IK}(\theta) = \Phi_{\sigma IK}\left(\frac{\pi}{2} - \theta\right) \; . \tag{13}$$

The effects of eq. (12) will be seen in the specific cases we are going to study.

3. - THE EIGENVALUE PROBLEM.

The eigenvalue equation for a wave function of the form given by eq. (11) leads to the following set of coupled equatioms for the intrinsic wavefunctions Φ_{IK} :

$$\left\{\frac{1}{4\mathcal{I}_o}\left[I(I+1) + K^2 \mathrm{tg}^2\theta - \frac{d^2}{d\theta^2}\right] + V(\theta) - E_{\sigma I r_\eta}\right\}\Phi_{\sigma IK} + \frac{1}{4\mathcal{I}_o}\mathrm{tg}^2\theta \quad \cdot$$

$$\cdot \sum_{K' \geq 0} \langle D^I_{MK} | I^2_\eta | D^I_{MK'} + (-)^I D^I_{M-K'} \rangle \left[(1+\delta_{K0})(1+\delta_{K'0})\right]^{-\frac{1}{2}} \Phi_{\sigma IK'} = 0 \tag{14}$$

While posponing a more general discussion at the end of this Section, we solve the above equations for the following set of states : i) $I = 0$, $r_\eta = 1$, ii) $I = 1$, $r_\eta = -1$, iii) $I = 2$, $r_\eta = -1$. In all these cases the system of eqs. (14) reduces to a single equation in $\Phi_{\sigma IK}$. The symmetry constraints (9) relate $\Phi_{\sigma IK}(\theta)$ to $\Phi_{\sigma IK}(\pi/2 - \theta)$ as follows : i) $\Phi_{\sigma 00}(\theta) = \Phi_{\sigma 00}(\pi/2 - \theta)$, ii) $\Phi_{\sigma 11}(\theta) = -\Phi_{\sigma 11}(\pi/2 - \theta)$ and iii) $\Phi_{\sigma 21}(\theta) = \Phi_{\sigma 21}(\pi/2 - \theta)$. The intrinsic wavefunctions can then be written :

$$
\begin{aligned}
&\text{case i)} \quad \Phi_{\sigma 00}(\theta) = \frac{1}{\sqrt{2}}\left[\varphi_{\sigma 00}(\theta) + \varphi_{\sigma 00}(\pi/2 - \theta)\right] ,\\
&\text{case ii)} \quad \Phi_{\sigma 11}(\theta) = \frac{1}{\sqrt{2}}\left[\varphi_{\sigma 11}(\theta) - \varphi_{\sigma 11}(\pi/2 - \theta)\right] ,\\
&\text{case iii)} \quad \Phi_{\sigma 21}(\theta) = \frac{1}{\sqrt{2}}\left[\varphi_{\sigma 21}(\theta) + \varphi_{\sigma 21}(\pi/2 - \theta)\right] ,
\end{aligned}
\tag{15}
$$

where the $\varphi_{\sigma IK}$'s are normalized in $0 \leq \theta \leq \pi/4$.

We must specify at this stage the form of the potential. Our model is physically meaningful for small relative oscillations of protons against neutrons. Their mutual interaction is therefore described appropriately by a harmonic potential satisfying eq. (7), namely :

$$
V(\theta) = \begin{cases} \frac{1}{2} C \theta^2 & 0 \leq \theta \leq \frac{\pi}{4} , \\ \frac{1}{2} C (\frac{\pi}{2} - \theta)^2 & \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} . \end{cases}
\tag{16}
$$

The eigenvalue equation (14) can then be consistently approximated in $0 \leq \theta \leq \frac{\pi}{4}$ by :

$$
\begin{aligned}
\Big\{ -\frac{1}{4\mathcal{J}_o}\frac{d^2}{d\theta^2} + \frac{K^2}{4\mathcal{J}_o\theta^2} + \frac{1}{2}(2\mathcal{J}_o)\,\omega^2(I,K)\,\theta^2 + \\
+ \frac{1}{4\mathcal{J}_o}\left[I(I+1) - \frac{2}{3}K^2\right] - E_{\sigma IK} \Big\}\, \varphi_{\sigma IK} = 0 ,
\end{aligned}
\tag{17}
$$

where:

$$\omega(I,K) = \sqrt{C(I,K)/(2\mathcal{J}_o)} \,,$$
(18)
$$C(I,K) = C + \frac{1}{2\mathcal{J}_o}\left[\frac{K^2}{15} + (1+\delta_{K0})^{-1}\langle D^I_{MK}| I^2_\eta | D^I_{MK} + (-)^I D^I{}_{M-K}\rangle\right]$$

and where we write $E_{\sigma IK}$ instead of $E_{\sigma I r_\eta}$. The relative motion of protons and neutrons is confined within a range of θ of the order:

$$\theta_o(I,K) = \left[2\mathcal{J}_o C(I,K)\right]^{-1/4} . \qquad (19)$$

Numerical estimates made in Appendix B give:

$$\omega(I,K) \sim \omega = \sqrt{\frac{C}{2\mathcal{J}_o}} \simeq 60\, A^{-1/6}\delta \,,$$
(20)
$$\theta_o^2(I,K) \sim \theta_o^2 = \frac{L}{\sqrt{2\mathcal{J}_o C}} \simeq 1.2\, A^{-3/2}\delta^{-1} \,,$$

where δ is the deformation parameter[(3)].

For heavy deformed nuclei ($A = 180$, $\delta = 0.25$) $\omega \simeq 6$ MeV and $\theta_o^2 \simeq 1.5 \times 10^{-3}$ while for light deformed nuclei ($A = 24$, $\delta = 0.5$) $\omega \simeq 18$ MeV and $\theta_o^2 \simeq 2 \times 10^{-2}$. In both cases the value of θ_o^2 is small enough to justify the extension of the range of θ from zero to infinity in the eigenvalue equation (17) which can be written simply

$$\left\{ -\frac{1}{4\mathcal{J}_o}\frac{d^2}{d\theta^2} + \frac{K^2}{4\mathcal{J}_o\theta^2} + \frac{1}{2}(2\mathcal{J}_o)\,\omega^2\theta^2 - \varepsilon_{\sigma K}\right\}\varphi_{\sigma K} = 0 \,, \qquad (21)$$
$$0 \leq \theta \leq \infty$$

where

$$\varepsilon_{\sigma K} = E_{\sigma IK} - \frac{1}{4\mathcal{J}_o}\left[I(I+1) - \frac{2}{3}K^2\right] \qquad (22)$$

represents the intrinsic eigenvalues. Both the intrinsic functions and eigenvalues do not depend on I which therefore has been suppressed.

The $I = 0$, $r_\eta = 1$ $(K = 0)$ eigenfunctions satisfying the Hermiticity condition (8) are the even solutions of the one dimensional harmonic oscillator:

$$\varphi_{\sigma o} = \varphi_{no} = \left[2^{2n-1}(2n)!\sqrt{\pi}\,\theta_o\right]^{-1/2} \exp\left(-\frac{\theta^2}{2\theta_o^2}\right) H_{2n}\left(\frac{\theta}{\theta_o}\right), \qquad (23)$$

with eigenvalues

$$\varepsilon_{\sigma o} = \varepsilon_{no} = \left(2n + \frac{1}{2}\right)\omega \quad . \qquad (24)$$

For $r_\eta = -1$ and $I = 1, 2$, K must be equal to unity. The corresponding eigenfunctions are:

$$\varphi_{\sigma 1} = \varphi_{n1} = \left[\frac{2n!}{\theta_o \Gamma(n+\varrho+1)}\right]^{-1/2} \left(\frac{\theta}{\theta_o}\right)^{\frac{1}{2}+\varrho} \exp\left(-\frac{\theta^2}{2\theta_o^2}\right) L_n^{\varrho}\left(\frac{\theta^2}{\theta_o^2}\right), \qquad (25)$$

where

$$\varrho = \frac{1}{2}(1 + 4K^2)^{1/2}, \qquad (26)$$

with eigenvalues

$$\varepsilon_{\sigma 1} = \varepsilon_{n1} = (2n + \varrho + 1)\,\omega \quad . \qquad (27)$$

L_n^{ϱ} are generalized Laguerre Polynomial[5]. It is important to observe that because of the independence of φ_{nK} on I the intrinsic eigenfunctions Φ_{n11} and Φ_{n21} given by eq. (15) are odd and even combinations of the same functions and are degenerate in (the intrinsic) energy. The present model is reliable for the lowest intrinsic excitations only. Among the states explicitely determined the lowest in energy are Φ_{o1} and Φ_{1o}. Their corresponding excitation energies are :

i) $\varepsilon_{01} - \varepsilon_{00} = \frac{1}{2}\sqrt{5}\,\omega$, which is ~ 10 MeV in heavy nuclei ($A = 180$ and $\delta = 0.25$) and ~ 27 MeV in the light ones ($A = 24$ and $\delta = 0.5$);

ii) $\varepsilon_{10} - \varepsilon_{00} = 2\omega$, which is ~ 12 MeV and ~ 36 MeV for heavy and light nuclei respectively.

It remains to consider the states satisfying coupled differential equations. They are superpositions of intrinsic functions with different K's. The terms responsible for the admixture are corrections of the order $1/2\Upsilon_o$ to the restoring force constant. On the ground of the numerical estimates of Appendix B we expect therefore that the exact eigenvalues do not differ substantially from the ones obtained by neglecting the coupling terms. These would form rotational bands based on intrinsic states of given K. Obviously only the bands constructed on the lowest excited intrinsic states, which are just the ones already determined, are likely to be physically relevant.

4. - TRANSITION PROBABILITIES.

In order to establish the mechanism of excitation of the states determined in the previous Section we need to evaluate their electromagnetic transition probabilities.

4. 1. - Electric quadrupole transitions.

The electric quadrupole moment referred to the intrinsic axes is :

$$\mathcal{M}(E2,\nu) = \frac{e}{2}\left[\mathcal{M}_{\tau=0}(E2,\nu) + \mathcal{M}_{\tau=1}(E2,\nu)\right], \tag{28}$$

where

$$\mathcal{M}_{\tau}(E2,\nu) = \int \varrho_{\tau}(\vec{r})\, r^2 Y_{2\nu}(\vec{r})\, d\vec{r} \;, \tag{29}$$

$$\varrho_{\tau} = \varrho^{(p)}(\vec{r}) + (-1)^{\tau}\varrho^{(n)}(\vec{r}) \;. \tag{30}$$

$\varrho^{(p)}$ and $\varrho^{(n)}$ are the proton and neutron densities which for $\theta = 0$ are assumed to be

$$\varrho_o^{(p)}(\vec{r}) = \varrho_o^{(n)}(\vec{r}) = \frac{1}{2}\,\varrho(\vec{r}) \;. \tag{31}$$

$\varrho(\vec{r})$ being the total nuclear density. This for a spheroidal shape with a sharp surface is :

$$\varrho(r) = \varrho_o \, s\left\{r - R_o\left[1 + \alpha_{20} Y^{*}_{20}(\vec{r})\right]\right\} \quad , \tag{32}$$

where ϱ_o is the constant density of a sphere of radius R_o and s(x) denotes the step function.

As the proton (neutron) rotor rotates through θ ($-\theta$) about the $\vec{\xi}$ axis the proton (neutron) density becomes :

$$\begin{aligned} \varrho^{(p)}_{\theta}(\vec{r}) &= \frac{1}{2}\,\varrho\left[R_{\xi}^{-1}(\theta)\,\vec{r}\right], \\ \varrho^{(n)}_{\theta}(\vec{r}) &= \frac{1}{2}\,\varrho\left[R_{\xi}(\theta)\,\vec{r}\right]. \end{aligned} \tag{33}$$

Inserting these expressions in eq. (29) we get :

$$\mathcal{M}_{\tau}(E2,\nu) = \sum_{\nu'} Q_{2\nu'}\left\{\langle 2\nu'\right|\left[\exp(-i\,\theta\, I_{\xi}) + (-1)^{\tau}\exp(i\,\theta\, I_{\xi})\right]\left|2\nu\rangle \quad , \tag{34}$$

where

$$Q_{2\nu'} = \frac{1}{2}\int \varrho(\vec{r})\, r^2\, Y_{2\nu'}(\vec{r})\, d\vec{r} \quad . \tag{35}$$

To first order in α_{20}

$$Q_{2\nu'} = \delta_{\nu 0}\, Q_{20} = \frac{1}{2}\,\varrho_o R_o^5\, \alpha_{20}\, \delta_{\nu 0} \quad . \tag{36}$$

Consistently with our model we approximate $\mathcal{M}_{\tau}(E2,\nu)$ with the expression obtained by expanding $\exp(\pm i\,\theta\, I_{\xi})$ up to first order in θ and $(\pi/2 - \theta)$:

$$\begin{aligned} \mathcal{M}_{\tau}(E2,\nu) = Q_{20}\Big\{ & s(\theta - \pi/4)\left[\delta_{\nu 0}(1 + (-1)^{\tau}) - i\,\theta\langle 20|I_{\xi}|2\nu\rangle(1 - (-)^{\tau})\right] + \\ & + s(\pi/4 - \theta)\Big[\langle 20|\,(\exp(-i\tfrac{\pi}{2} I_{\xi}) + (-)^{\tau}\exp(i\tfrac{\pi}{2} I_{\xi})\,|2\nu\rangle + \\ & + i(\tfrac{\pi}{2} - \theta)\langle 20|\, I_{\xi}(\exp(-i\tfrac{\pi}{2} I_{\xi}) - (-1)^{\tau}\exp(i\tfrac{\pi}{2} I_{\xi}))\,|2\nu\rangle\Big]\Big\} \quad . \end{aligned} \tag{37}$$

The above expression specializes in the specific cases we need as :

$$\mathcal{M}_0(E2,0) = Q_{20}\left[2s(\theta - \frac{\pi}{4}) - s(\frac{\pi}{4} - \theta)\right] , \tag{38}$$

$$\mathcal{M}_1(E2,1) = -i\sqrt{6}\, Q_{20}\left[\theta\, s(\theta - \frac{\pi}{4}) + (\frac{\pi}{2} - \theta)\, s(\frac{\pi}{4} - \theta)\right] , \tag{39}$$

$$\mathcal{M}_0(E2,1) = \mathcal{M}_1(E2,0) = 0 . \tag{40}$$

The states we determined have the form :

$$\psi_{nIMr\eta} = \psi_{nIMK} = \left[\frac{2I+1}{16\pi^2}\right]^{1/2}(1+\delta_{K0})^{-1/2}\left[D^I_{MK}\Phi_{nK} + (-)^{I+K} D^I_{M-K}\Phi_{n\bar{K}}\right] \tag{41}$$

having put for convenience $\Phi_{n\bar{K}} = (-)^K \Phi_{nK}$.

The E2-transition probabilities from the ground to the excited states are then :

$$\begin{aligned} B(E2) &= \frac{e^2}{4}\left|\langle nIK \| \mathcal{M}(E2) \| 000\rangle\right|^2 = \\ &= \frac{e^2}{4}\,\frac{2}{(1+\delta_{K0})}\left|\langle nK| \mathcal{M}(E2, \nu = K)|00\rangle\right|^2 . \end{aligned} \tag{42}$$

For K = 1 and n = 0 :

$$\begin{aligned} \langle 012 \| \mathcal{M}_1(E2) \| 000\rangle &\simeq -i\,2\sqrt{3}\, Q_{20}\int_0^\infty d\theta\ \varphi^*_{01}(\theta)\,\theta\,\varphi_{00}(\theta) = \\ &= -i\,2\sqrt{3}\,\frac{\theta_o}{\pi^{1/4}}\,\frac{\Gamma(\frac{5+\sqrt{5}}{4})}{\sqrt{\Gamma(\frac{2+\sqrt{5}}{2})}}\, Q_{20} , \end{aligned} \tag{43}$$

while $\langle 012 \| \mathcal{M}_o(E2) \| 000\rangle = 0$. Therefore we have :

$$B(E2) \simeq 0.05\, A^{11/6}\,\delta\, e^2\, fm^4 , \tag{44}$$

having used eqs. (36) and (B. 16). For $A = 180$, $\delta = 0.25$ B(E2) turns out to be three times the Weisskopf estimate, for $A = 24$, $\delta = 0.5$ B(E2) $\sim$ $\sim$ 2.5 W.u. The vanishing of the reduced matrix element of $\mathcal{M}_0$(E2) characterizes the ψ_{02M1} states as describing isovector quadrupole oscillations. The E2-transition probabilities to the other two states obviously vanish.

4.2. - Magnetic transitions.

The intrinsic magnetic dipole moments are:

$$\mathcal{M}(M1,\nu) = \frac{1}{2}\left[\mathcal{M}_0(M1,\nu) + \mathcal{M}_1(M1,\nu)\right] , \tag{45}$$

where

$$\mathcal{M}_\tau(M1,\nu) = -\frac{1}{2}\int d\vec{r}\, \vec{j}_\tau(\vec{r})\, (\vec{r} \times \vec{\nabla})\, r\, Y_{1\nu}(\vec{r}) , \tag{46}$$

$$\begin{aligned} \vec{j}_\tau(\vec{r}) &= \vec{j}^{(p)}(\vec{r}) + (-)^\tau \vec{j}^{(n)}(\vec{r}) = e\left[\varrho^{(p)}\vec{v}^{(p)}(\vec{r}) + (-)^\tau \varrho^{(n)}\vec{v}^{(n)}(\vec{r})\right] \simeq \\ &\simeq \frac{e}{2}\, \varrho\, (\vec{v}^{(p)}(\vec{r}) + (-)^\tau \vec{v}^{(n)}(\vec{r})) . \end{aligned} \tag{47}$$

Inserting this expression in eq. (46) we get:

$$\mathcal{M}_0(M1,\nu) = \frac{e}{2m}\sqrt{\frac{3}{4\pi}}\, I_\nu , \tag{48}$$

$$\mathcal{M}_1(M1,\nu) = \frac{e}{2m}\sqrt{\frac{3}{4\pi}}\, S_\nu , \tag{49}$$

where

$$\begin{aligned} I_\nu = \vec{I}\,\vec{e}_\nu &= \int d\vec{r}\, m\frac{\varrho}{2}\, \vec{r} \times \vec{v}^{(p)}\, \vec{e}_\nu + \int d\vec{r}\, m\frac{\varrho}{2}\, \vec{r} \times \vec{v}^{(n)}\, \vec{e}_\nu = \\ &= (\vec{I}^{(p)} + \vec{I}^{(n)})\, \vec{e}_\nu , \end{aligned} \tag{50}$$

$$S_\nu = \vec{S}\cdot\vec{e}_\nu = (\vec{I}^{(p)} - \vec{I}^{(n)})\, \vec{e}_\nu . \tag{51}$$

$\vec{e}_\nu$ being the spherical components of the unit vector in the intrinsic frame. Since all terms containing components of I annihilate the ground state the only nonvanishing reduced amplitudes are:

$$\langle n11 \| \mathfrak{M}_1(M1) \| 000 \rangle = -\sqrt{\frac{3}{4\pi}} \frac{e}{2m} \langle n1 | S_\xi | 00 \rangle . \tag{52}$$

For n = 0:

$$\langle 011 \| \mathfrak{M}_1(M1) \| 000 \rangle = -i \sqrt{\frac{3}{4\pi}} \frac{e}{2m} \int_0^{\frac{\pi}{2}} d\theta \; \Phi^{\star}_{02}(\theta) \frac{d}{d\theta} \Phi_{00}(\theta) =$$

$$= i \sqrt{\frac{3}{4\pi}} \frac{e}{2m} \frac{1}{\pi^{1/4}} \frac{\Gamma(\frac{5+\sqrt{5}}{2})}{\sqrt{\Gamma(\frac{2+\sqrt{5}}{2})}} \frac{1}{\theta_o} . \tag{53}$$

The transition probability to ψ_{01M1} is then:

$$B(M2) = \frac{1}{4} \left| \langle 011 \| \mathfrak{M}_1(M1) \| 000 \rangle \right|^2 \simeq 0.025 \; A^{3/2} \delta \left(\frac{e}{2m}\right)^2 , \tag{54}$$

or in Weisskopf units:

$$\frac{B(M1)}{B_W(M1)} \sim 0.014 \; A^{3/2} \delta . \tag{55}$$

For A = 180 and δ = 0.25 this ratio is $B/B_W \sim 9$, while for A = 24 and δ = 0.5 $B/B_W \sim 1$. It should be noted that only the orbital motion is taken into account in our model. Again the isoscalar contribution vanishes as well as the transition probability to I = K = 0 states.

5. - CONCLUSIONS.

The lowest lying excited states predicted by our model are:

i) a state with quantum numbers n = 1, I = 0, r_η = 1 (K = 0), which is not coupled to the ground state by e. m. radiation;

ii) a n=0 I=1 $r_\eta = -1$ (K=1) state which is strongly excited by isovector M1-radiation;

iii) a n=0 I=2 $r_\eta = -1$ (K=1) state weakly excited by isovector E2-radiation.

The semiclassical interpretation of these states is obscured to some extent by the dynamical variables we used to describe the motion of the system. A complete intrinsic oscillation for instance cannot be described by the variable θ alone because of its limited range of variation ($0 \leq \theta \leq \pi/2$). Such a description should require the extension of the range of θ to $-\pi/2 \leq \theta \leq \pi/2$. Had we done this since the one to one correspondence with the old variables $\alpha_p \beta_p$ $\alpha_n \beta_n$ must be preserved, we should have either restricted the range of the Euler angle γ to $0 \leq \gamma \leq \pi$ thereby blurring the description of the rotational motion or kept the full range for γ but imposed an appropriate constraint on the wavefunction.

A microscopic calculation based[6] on a schematic model predicts an isovector K = 1. The authors of this calculation however assume that the deformed nuclei are axially symmetric while our states, although characterized by a single value of K, do not have axially symmetric intrinsic deformation. This because the relative motion of protons and neutrons is simultaneously localized around the ζ and η axes. Were the motion localized only around a single axis the system would have resulted to be approximately axially symmetric due to the small value of θ_o. An effect of this symmetry breaking consists in the reduction of the intrinsic quadrupole moment in the ground state by a factor 1/2.

We conclude with some remarks on the assumptions of our model. The results of the present work hold strictly for light deformed nuclei only. For heavier nuclei it should be necessary to take into account the neutron excess. We do not expect, however, that our results should substantially change.

In describing the proton-neutron fluids as rigid rotors we neglected the superconducting properties of the nucleus. The relevant effect of neutron-neutron and proton-proton pairing correlations is to halve appro ximately the moment of inertia, which in turn would increase the excitation energies by a factor $\sqrt{2}$, would enhance the E2 transition probabilities by a factor 2 and would reduce the M1 transition probabilities by a factor 1/2. It is more difficult to predict the effect of neutron-proton pairing correlations. If such correlations are important in deformed nuclei, as it has been suggested[7], the collective modes described by the present model might even be inhibited.

REFERENCES AND FOOTNOTES.

(1) - M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948).

(2) - The possible occurrence of such an excitation mode was discussed by one of us (F. P.) at the Institute of Theoretical and Experimental Physics of Moscow in March 1975 and by R. Hilton in Dubna in June 1976. No quantitative treatment, however, has been done till now as far as we know.

(3) - See for instance: A. Bohr and B. R. Mottelson, Nuclear Structure (Benjamin, 1975), Vol. II, p. 47.

(4) - See ref. (3), p. 75.

(5) - I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series and Products (Academic Press, 1965).

(6) - T. Suzuki and D. J. Rowe, Nuclear Phys. A289, 461 (1977).

(7) - F. Palumbo, Lett. Nuovo Cimento 14, 572 (1975).

APPENDIX A

We present a list of commutation relations satisfied by the components of $\vec{I}$ and $\vec{S}$. Some of them are well known and are reported for sake of clarity. Some others are not known but easy to evaluate, so only a few of them are explicitly evaluated as illustrative examples. Let us recall the commutation relations satisfied by the components of the proton and neutron angular momenta $\vec{I}^{(p)}$ and $\vec{I}^{(n)}$ along the fixed axes $\vec{x}$, $\vec{y}$, $\vec{z}$:

$$\left[I_i^{(p)}, I_j^{(p)}\right] = i\varepsilon_{ijk} I_k^{(p)} \; ; \; \left[I_i^{(n)}, I_j^{(n)}\right] = i\varepsilon_{ijk} I_k^{(n)} \; ; \; \left[I_i^{(p)}, I_j^{(n)}\right] = 0 \; ;$$

$$\left[I_i^{(p)}, \xi_j^{(p)}\right] = i\varepsilon_{ijk}\xi_k^{(p)} \; ; \; \left[I_i^{(p)}, \eta_j^{(p)}\right] = i\varepsilon_{ijk}\eta_k^{(p)} \; ; \; \left[I_i^{(p)}, \zeta_j^{(p)}\right] = i\varepsilon_{ijk}\zeta_k^{(p)} \; ;$$

$$\left[I_i^{(n)}, \xi_j^{(n)}\right] = i\varepsilon_{ijk}\xi_k^{(n)} \; ; \; \left[I_i^{(n)}, \eta_j^{(n)}\right] = i\varepsilon_{ijk}\eta_k^{(n)} \; ; \; \left[I_i^{(n)}, \zeta_j^{(n)}\right] = i\varepsilon_{ijk}\zeta_k^{(n)} \; ;$$

$$\left[I_i^{(p)}, \xi_j^{(n)}\right] = \left[I_i^{(p)}, \eta_j^{(n)}\right] = \left[I_i^{(p)}, \zeta_j^{(n)}\right] = 0 \; ;$$

$$\left[I_i^{(n)}, \xi_j^{(p)}\right] = \left[I_i^{(n)}, \eta_j^{(p)}\right] = \left[I_i^{(n)}, \zeta_j^{(p)}\right] = 0 \; . \qquad \text{(A.1)}$$

ε_{ijk} being the completely antisymmetric tensor of third rank.

All commutation relations satisfied by the fixed components of $\vec{I}$, $\vec{S}$, $\vec{\xi}$, $\vec{\eta}$ and $\vec{\zeta}$ follow from definitions (2), (3) and relations (A.1). They are:

$$\left[I_i, I_j\right] = i\varepsilon_{ijk} I_k \; ; \; \left[I_i, S_j\right] = i\varepsilon_{ijk} S_k \; ; \; \left[S_i, S_j\right] = i\varepsilon_{ijk} I_k \; ;$$

$$\left[I_i, \xi_j\right] = i\varepsilon_{ijk}\xi_k \; ; \; \left[I_i, \eta_j\right] = i\varepsilon_{ijk}\eta_k \; ; \; \left[I_i, \zeta_j\right] = i\varepsilon_{ijk}\zeta_k \; ;$$

$$\left[S_i, \xi_j\right] = 2i(\delta_{ij} - \xi_i\xi_j)\,\mathrm{ctg}(2\theta) + i\eta_i\eta_j\,\mathrm{tg}\,\theta - i\zeta_i\zeta_j\,\mathrm{ctg}\,\theta \; ;$$

$$\left[S_i, \eta_j\right] = i(\varepsilon_{ijk}\zeta_k - \xi_i\eta_j)\,\mathrm{ctg}\,\theta \; ;$$

$$\left[S_i, \zeta_j\right] = i(\varepsilon_{ijk}\eta_k + \xi_i\zeta_j)\,\mathrm{tg}\,\theta \; . \qquad \text{(A.2)}$$

We have further :

$$\left[I_i, \cos\theta\right] = \left[I_i, \sin\theta\right] = 0 \; ;$$
$$\left[S_i, \sin\theta\right] = i\xi_i \cos\theta \; ; \quad \left[S_i, \cos\theta\right] = -i\xi_i \sin\theta \; . \tag{A.3}$$

Therefore for any $f(\theta)$:

$$\left[I_i, f(\theta)\right] = 0 \; ;$$
$$\left[S_i, f(\theta)\right] = i\xi_i \frac{d}{d\theta} f(\theta) \; . \tag{A.4}$$

It is to be noted that : $\left[S_i, \xi_i\right] \neq 0$, $\left[S_i, \eta_i\right] \neq 0$, $\left[S_i, \zeta_i\right] \neq 0$.

However :

$$\vec{S}\cdot\hat{\xi} - \hat{\xi}\cdot\vec{S} = 2i\,\mathrm{ctg}\,\theta \; ;$$
$$\vec{S}\cdot\hat{\eta} = \hat{\eta}\cdot\vec{S} \; ; \qquad \vec{S}\cdot\hat{\zeta} = \hat{\zeta}\cdot\vec{S} \; . \tag{A.5}$$

The component of $\vec{S}$ along $\hat{\xi}$ must be defined as :

$$S_\xi = \frac{1}{2}(\vec{S}\cdot\hat{\xi} + \hat{\xi}\cdot\vec{S}) \tag{A.6}$$

in order to be Hermitian.

The commutation relations of the components of $\vec{I}$ and $\vec{S}$ along $\hat{\xi}$, $\hat{\eta}$, $\hat{\zeta}$ follow from the previous ones. We have :

$$\left[I_\xi, I_\eta\right] = -iI_\zeta \qquad \text{and cyclic permutations} \; ;$$
$$\left[I_\xi, S_\xi\right] = 0 \; ; \quad \left[I_\xi, S_\eta\right] = -iI_\eta\,\mathrm{ctg}\,\theta \; ; \quad \left[I_\xi, S_\zeta\right] = iI_\zeta\,\mathrm{tg}\,\theta \; ;$$
$$\left[I_\eta, S_\xi\right] = 0 \; ; \quad \left[I_\eta, S_\eta\right] = iI_\xi\,\mathrm{ctg}\,\theta \; ; \quad \left[I_\eta, S_\zeta\right] = 0 \; ;$$
$$\left[I_\zeta, S_\xi\right] = 0 \; ; \quad \left[I_\zeta, S_\eta\right] = 0 \; ; \quad \left[I_\zeta, S_\zeta\right] = -I_\xi\,\mathrm{tg}\,\theta \; ; \tag{A.7}$$

$$\left[S_\xi, S_\eta\right] = i(I_\zeta - S_\eta \mathrm{ctg}\,\theta) \;;\; \left[S_\xi, S_\zeta\right] = -i(I_\zeta - S_\zeta \mathrm{tg}\,\theta) \;;\; \left[S_\eta, S_\zeta\right] = i I_\xi \quad . \tag{A.7}$$

Moreover all I- and S-components commute with $f(\theta)$ except for S_ξ which satisfies the relation:

$$\left[S_\xi, f(\theta)\right] = i \frac{d}{d\theta} f(\theta) \quad . \tag{A.8}$$

We now evaluate a few of the commutation relations presented for sake of illustration. Let us compute for instance the last of relations (A.3). From the definition (2) of $\hat{\xi}$, $\hat{\eta}$, $\hat{\zeta}$ we get :

$$\begin{aligned}\left[S_i, 2\cos\theta\right] &= \left[S_i, (\hat{\zeta}_p+\hat{\zeta}_n)_j \zeta_j\right] = \left[S_i, (\hat{\zeta}_p+\hat{\zeta}_n)_j\right]\zeta_j + (\hat{\zeta}_p+\hat{\zeta}_n)_j \left[S_i, \zeta_j\right] = \\ &= \left[S_i, (\hat{\zeta}_p+\hat{\zeta}_n)_j\right]\zeta_j + \zeta_j\left[S_i, 2\cos\theta\,\zeta_j\right] - \left[S_i, 2\cos\theta\right]\zeta_i\zeta_j\end{aligned} \tag{A.9}$$

where repeated indices denote summation. We get eventually :

$$\begin{aligned}\left[S_i, \cos\theta\right] &= \frac{1}{2}\zeta_j\left[S_i, (\hat{\zeta}_p+\hat{\zeta}_n)_j\right] = \frac{1}{2} i\,\varepsilon_{ijk}\zeta_j(\hat{\zeta}_p-\hat{\zeta}_n)_k = \\ &= i\,\varepsilon_{ijk}\zeta_j\eta_k \sin\theta = -i\,\xi_i \sin\theta \quad .\end{aligned}$$

Let us now evaluate the last of eqs. (A.2). We start with :

$$\left[S_i, (\hat{\zeta}_p+\hat{\zeta}_n)_j\right] = \left[S_i, 2\zeta_j\cos\theta\right] = \zeta_j\left[S_i, 2\cos\theta\right] + 2\left[S_i, \zeta_j\right]\cos\theta \quad .$$

It follows then :

$$\begin{aligned}\left[2\, S_i, \zeta_j\right]\cos\theta &= \left[S_i, (\hat{\zeta}_p+\hat{\zeta}_n)_j\right] - \zeta_j\left[S_i, 2\cos\theta\right] = \\ &= 2i\,\varepsilon_{ijk}\eta_k\sin\theta + 2i\,\xi_i\zeta_j\sin\theta \quad .\end{aligned}$$

The final results is :

$$\left[S_i, \zeta_j\right] = i(\varepsilon_{ijk}\eta_k + \xi_i\zeta_j)\,\mathrm{tg}\,\theta \quad .$$

APPENDIX B

In order to determine the restoring force constant C of the harmonic potential (20) we follow a procedure adopted by Goldhaber and Teller.[1] To this purpose it is sufficient to consider the range $0 \leq \theta \leq \pi/4$. In this range the potential is:

$$V(\theta) = \frac{1}{2} C \theta^2 . \qquad (B.1)$$

On the other hand as the angle θ gets larger than a critical value θ_c, $\frac{1}{2} \varrho \Delta v(\theta)$ neutron-proton pairs do not interact anymore causing an increase of the nuclear potential energy V_N

$$V_N(\theta) = \frac{1}{2} \varrho \Delta v(\theta) v_o . \qquad (B.2)$$

Here ϱ denotes the nuclear density, v_o the neutron-proton interaction potential and $\Delta v(\theta)$ the volume variation of the nucleus due to proton--neutron relative displacement.

At the critical value θ_c the two potentials must coincide:

$$\frac{1}{2} C \theta_c^2 = \frac{1}{2} \varrho \Delta v(\theta_c) v_o . \qquad (B.3)$$

This equation allows us to determine the restoring force constant C once θ_c and $\Delta v(\theta_c)$ are determined. The volume variation is evaluated as follows. The nuclear surface at the equilibrium (θ=0) is described by the equation:

$$\frac{\xi^2 + \eta^2}{R_1^2} + \frac{\zeta^2}{R_3^2} = 1 , \qquad (B.4)$$

or in spherical coordinates:

$$r(\Theta, \phi, \theta=0) = r(\Theta, \theta=0) = (R_1^{-2} \sin^2\Theta + R_3^{-2} \cos^2\Theta)^{-1/2} . \qquad (B.5)$$

As protons (neutrons) rotate through θ $(-\theta)$ about $\hat{\xi}$ the surface of each rotor is described by eq. (B.4) with $\xi\ \eta\ \zeta$ replaced by:

$$\xi' = \xi \quad ; \quad \eta' = \zeta \sin\theta + \eta \cos\theta \; ; \quad \zeta' = \zeta \cos\theta - \eta \sin\theta \; . \tag{B.6}$$

In spherical coordinates to first order in θ it becomes :

$$r(\Theta, \phi, \theta) \simeq r(\Theta, \theta=0)\,(1 - \theta\, f(\Theta, \phi)) \; , \tag{B.7}$$

where :

$$f(\Theta, \phi) = \frac{1}{2} \sin(2\Theta) \sin\phi\, (R_1^{-2} - R_3^{-2})\, r^2(\Theta, 0) \; . \tag{B.8}$$

The volume variation is then given by :

$$\Delta v(\theta) = \int_0^{2\pi} d\phi \int_0^{\pi} d\Theta \, \sin\Theta \left| \int_{r(\Theta,\phi,-\theta)}^{r(\Theta,\phi,\theta)} dr\, r^2 \right| =$$

$$= 8\theta \int_0^{\pi} d\phi \int_0^{\pi/2} d\Theta \, \sin\Theta \;\; r^3(\Theta, 0) \left| f(\Theta, \phi) \right|^2 \; , \tag{B.9}$$

having used eqs. (B.7) and retained the terms linear in θ. Inserting the explicit expressions (B.5) and (B.8) of $f(\Theta, \phi)$ and $r(\Theta, 0)$ in the previous eq. (B.9) we get :

$$\Delta v(\theta) \simeq 16\theta \left| R_1^{-2} - R_3^{-2} \right| \int_0^{\pi/2} d\Theta \, \sin^2\Theta \, \cos\Theta \, (R_1^{-2} \sin^2\Theta + R_3^{-2} \cos^2\Theta)^{-5/2} =$$

$$= 16\theta \left| R_1^{-2} - R_3^{-2} \right| \int_0^{1} dt\, t^2 \left[(R_1^{-2} - R_3^{-2})\, t^2 + R_3^{-2} \right]^{-5/2} \simeq \tag{B.10}$$

$$\simeq \frac{16}{3}\, \theta \, \frac{\left| R_3^2 - R_1^2 \right|}{R_1^2} \, R_3^3 \; .$$

It remains now to determine θ_c. To this purpose we observe that for $\theta > \theta_c$ the average distance between proton and neutron surfaces exceeds the range r_o of the proton-neutron interaction. We must therefore impose :

$$r_o = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} d\Theta \sin\Theta \left| r(\Theta, \phi, \theta_c) - r(\Theta, \phi, -\theta_c) \right| \simeq$$

$$\simeq \frac{4}{\pi} \theta_c \left| R_1^{-2} - R_3^{-2} \right| \int_0^{\pi/2} d\Theta \cos\Theta \sin^2\Theta \, (R_1^{-2} \sin^2\Theta + R_3^{-2} \cos^2\Theta)^{-3/2} \simeq$$

$$\simeq \frac{4}{3\pi} \theta_c \frac{\left| R_3^2 - R_1^2 \right|}{R_1^2} R_3 , \qquad \text{(B. 11)}$$

where we again used eqs. (B. 7), (B. 8) and (B. 5) and retained linear terms in θ only. θ_c is therefore given by:

$$\theta_c = \frac{3\pi}{4} \frac{R_1^2}{\left| R_3^2 - R_1^2 \right|} \frac{r_o}{R_3} . \qquad \text{(B. 12)}$$

The expression for c comes from (B. 3) using eqs. (B. 10) and (B. 12):

$$c = \varrho v_o \frac{\Delta v(\theta_c)}{\theta_c^2} \simeq \frac{64}{9\pi} \varrho \frac{v_o}{r_o} \left| R_3^2 - R_1^2 \right|^2 \left[\frac{R_3}{R_1} \right]^4 . \qquad \text{(B. 13)}$$

Putting:

$$\varrho = \varrho_o = \frac{A}{\frac{4}{3}\pi R^3} , \qquad R = 1.2 \, A^{1/3} ,$$

and

$$\frac{\left| R_3^2 - R_1^2 \right|}{3R_1^2} \sim \frac{\left| R_3^2 - R_1^2 \right|}{2R_1^2 + R_3^2} = \frac{2}{3} \delta ,$$

we obtain for C:

$$C \simeq \frac{128}{5\pi^2} \frac{v_o}{r_o} A^{4/3} \delta^2 . \qquad \text{(B. 14)}$$

Assuming the nuclear moment of inertia to be[4] :

$$2\mathcal{J}_o = \frac{2}{5} A M R^3 \left(1 + \frac{1}{3}\delta\right) ,$$

we get for the frequency :

$$\omega = \sqrt{\frac{C}{2\mathcal{J}_o}} \cong \frac{20}{3\pi}\left[\frac{v_o}{r_o}\right]^{1/2} \delta A^{-1/6} \simeq 60 A^{-1/6}\delta , \tag{B.15}$$

and for the oscillator parameter θ_o :

$$\theta_o^2 = \frac{1}{2\mathcal{J}_o\omega} \simeq \frac{5\pi}{1.2 \times 16}\left[\frac{r_o}{v_o M}\right]^{1/2} A^{-3/2} \delta^{-1} = 1.2 A^{-3/2} \delta^{-1} . \tag{B.16}$$

having put $v_o = 40$ MeV, $r_o = 2$ fm.

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