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ABSTRACT.

The depolarization induced in a beam of spin 1/2 particles in rapidly jumping an intrinsic resonance in cyclic accelerators can be evaluated with the aid of the Froissart and Stora formula, provided that the Teng's condition is satisfied throughout the Q-jump pulse.

Using both an elementary and a rigorous procedure, we have derived a generalized version of the Froissart and Stora formula applicable to cases where Teng's condition is violated. It allows Teng's condition to be relaxed and replaced by a weaker requirement, thereby enabling one to estimate in any case the correct order of magnitude of the induced depolarization.

1. - INTRODUCTION.

At present, the feasibility of attaining polarized deuteron beams of energy ~400 GeV is predicated only[1] on a successful acceleration (without serious depolarization) of these particles through the large accelerators (SPS, FNAL, ISABELLE). The depolarization of these beams is ascribable to the traversal of few (although rather strong) intrinsic spin-resonances.

As it is well known, the remedial action which one must take in this case consists in changing rapidly[2] at proper times the vertical betatron wavenumber Q during the acceleration cycle. Under suitable conditions[3], the residue vertical polarization S_z of a selected oscillating particle can be evaluated by applying the Froissart and Stora[4] (FS) formula (Eq. (2) below). The condition for its validity has been derived by Teng[3], and is the following (Eq. (22) of Ref. (3)):

$$\Delta Q > \sim 4 (\dot{Q}/ \omega_o)^{1/2} . \qquad (1)$$

Here, ω_o is the angular velocity of the particle, $\dot{Q} \equiv dQ/dt$ is the (constant) rate of change of Q and ΔQ is the range of the Q-jump. Condition (1) is taken to assure that $\dot{Q}$ remains constant during the whole range of time where the resonance is effective, so that, to a good approximation, the FS formula

$$S_{z_{FS}} = 2 \exp\left[-(\pi\omega^2/2)/(\omega_o\dot{Q})\right] - 1 \simeq 1 - \pi\omega^2/(\omega_o\dot{Q}) \qquad (2)$$

holds. In Eq. (2), ω is the width of the resonance for a selected oscillating particle (vertical betatron oscillations), and the assumption $S_{z_{t \to -\infty}} = 1$ is implicit.

Since for multiGeV accelerators some difficulties may be encountered in satisfying completely requirement (1), we will generalize, in the present note, the FS treatment to include the effect of the magnitude of the Q-range, ΔQ, thereby allowing condition (1) to be relaxed and enabling one to estimate in any case the correct order of magnitude of the induced depolarization.

This note is divided into three paragraphs. In paragraph 2 we will derive the generalized FS formula (GFSF), i. e. Eq. (6) below, by a qualitative reasoning only and compare our results to the results given by the FS formula (Eq. (2) above) in two special cases (ZGS and CERN PS (protons)). Paragraph 3 will be devoted to a rigorous derivation of the GFSF on the basis of a suitable model. The last part of paragraph 3 will be spent to show that, in the limit where $(\Delta Q)^2 \gg (2\omega/\omega_o)^2$, the resulting Eq. (20) (which is valid for any magnitude of the Q-range, ΔQ) reduces to the (more useful) form (6).

In both paragraphs 2 and 3, the spin motion will be studied in the rotating perturbing-field approximation, and in a reference frame which is attached to the particle and which rotates about the main-field direction with angular velocity equal to the rotating-field angular velocity, $\omega_{res} = \omega_o G \gamma_{res}$. Here, γ is the particle's total energy measured in units of its rest energy and G is its gyromagnetic factor.

2. - AN APPROXIMATE APPROACH.

As soon as the spin-precession angular frequency, $\omega_p(t) = \omega_o G\gamma(t)$, is far off resonance, the vertical polarization $S_{z_{\gamma < \gamma_{res}}}$ of a selected particle is expressed by the equation

$$S_{z_{\gamma < \gamma_{res}}} = \dot{\chi}(t)/(\omega^2 + \dot{\chi}^2(t))^{1/2} , \tag{3}$$

where we assume $\omega \simeq$ constant and $S_{z_{t \to -\infty}} = 1$. In Eq. (3),

$$\dot{\chi}(t) = -(\omega_p(t) - \omega_{res}) = -\omega_o G(\gamma(t) - \gamma_{res}) \tag{4}$$

is the rate of advance of angular precession about the main field direction.

Expression (3) remains valid even when $\gamma(t)$ goes (from below) close[5] to γ_{res} during the acceleration cycle.

If the pulsed quadrupoles are now suddendly switched on (and properly timed), say, at $t = -t_o$, the residue polarization $S_{z_{\gamma > \gamma_{res}}}$ after the resonance tail is passed through is given[6,7] merely by the product of Eq. (3) by Eq. (2). Performing the product and writing for $\dot{\chi}(-t_o)$

$$\dot{\chi}(-t_o) = \delta \simeq \omega_o \Delta Q/2 , \tag{5}$$

we have for the induced depolarization

$$1 - S_{z_{t \to +\infty}} \simeq (\omega^2/\omega_o)\left[\pi/\dot{Q} + 2/(\omega_o(\Delta Q)^2)\right] , \tag{6}$$

i.e., the GFSF (the assumption has been made that $\delta^2 \gg \omega^2$).

As a simple numerical example, note that for the ZGS case[2] (protons) where Teng's condition (1) is violated only marginally, the contribution of the second term in Eq. (6) is $\approx$ 8%. On the contrary, for the CERN PS example[8] (protons) where Teng's condition is violated completely, such a contribution gets worse ($\approx$ 140%) (In the latter case, however, the resulting (little) depolarization is underestimated by a factor $\approx$ 2.4 only if use of Eq. (2) is made).

3. - AN ANALYTIC APPROACH.

As it is well known[4], the spin motion of a spin 1/2 particle is governed by the pair of coupled equations

$$2i\dot{f} = \omega g \exp(+i\chi), \qquad 2i\dot{g} = \omega f \exp(-i\chi) . \tag{7}$$

With the normalization condition $gg^* + ff^* = 1$, S_z is given by

$$S_z = 1 - 2gg^* . \tag{8}$$

Once an appropriate model for the Q-jump shape is adopted, the objects is to solve Eqs. (7).

As a simple model which exhibits sweeping of $\omega_p(t)$ through the resonance frequency ω_{res}, let

$$\omega = \text{constant} , \qquad \dot{\chi} = -\delta \tanh(\alpha t) , \tag{9}$$

where we assume α = constant and δ (= constant) to be given by Eq. (5).

Thus, in the following, the transition probability $gg^*_{t\to+\infty}$ in the long-time limit will be calculated for a particle initially in the state

$$S_{z_{t\to-\infty}} = \delta/(\omega^2 + \delta^2)^{1/2} \tag{3'}$$

(compare with Eq. (3)) by inserting Eqs. (9) in Eqs. (7) and solving these equations.

The first step consists of decoupling the spin-motion equations. We have, e.g., for g,

$$\ddot{g} - i\delta \tanh(\alpha t)\, \dot{g} + (\omega/2)^2 g = 0 . \tag{10}$$

Eq. (10) is easily transformed into the equation

$$x^2(1-x)g'' + x\Big[(1+ip) + (-1+ip)x\Big]g' + (q/2)^2(1-x)\, g = 0 \tag{11}$$

by the transformation

$$x = -\exp(2\alpha t) , \qquad (' \equiv d/dx ; \quad '' \equiv d^2/dx^2) . \tag{12}$$

The dimensionless constants p and q are given by

$$p = \delta/(2\alpha) \qquad \text{and} \qquad q = \omega/(2\alpha) . \tag{13}$$

On introduction of the new function y by the substitution[9]

$$g = x^{\ell} y \,, \tag{14}$$

Eq. (11) becomes[9]

$$x(1-x)y'' + \left[c - (a+b+1)x\right]y' - aby = 0 \,. \tag{15}$$

The constants ℓ, c, a and b are

$$2\ell_{\pm} = -ip \pm ir \;; \qquad c_{\pm} = 1 \pm ir \;; \qquad r = (p^2 + q^2)^{1/2} \;;$$
$$a_+ = -ip + ir \;; \qquad b_+ = -ip \;; \tag{16}$$
$$a_- = -ip \;; \qquad b_- = -ip - ir \,.$$

Eq. (15) is the well known hypergeometric differential equation[10]. Thus, in the vicinity of $x = -0$ (i.e. at $t \to -\infty$) the solution of Eq. (11) which satisfies the boundary condition (3') can be written[11] as follows:

$$g = D(-x)^{-ip/2-ir/2} F(-ip-ir, \, -ip; 1-ir; x) \,. \tag{17}$$

Here, D is the proper integration constant, namely,

$$DD^* = gg^*_{t\to-\infty} = (1 - S_z)/2_{t\to-\infty} = (1-p/r)/2 \tag{18}$$

and $F(-ip-ir, -ip; 1-ir; x)$ is a hypergeometric function ($F(a,b;c;x) \equiv F(b,a;c;x)$; $F(a,b;c;0) = 1$).

Note that solution (17) is consistent with the coupled nature of the original Eqs. (7). This can be easily verified by considering separately both Eqs. (7) at $x = -0$ (use the differentiation formula[12] for the F functions and remember that $ff^*_{t\to-\infty} = (1+p/r)/2$).

The asymptotic form for g as $t \to +\infty$ (i.e. $x \to -\infty$) can be found by making use of the appropriate linear transformation formula[13] for the $F(-ip-ir, -ip; 1-ir; x \to -\infty)$ function appearing in Eq. (17). Replacing in the resulting linearly-transformed[13] form of $g_{t\to+\infty}$ the $F(a,b;c;1/x)$ functions by unity and performing a fair amount of Γ-algebra, one finds

$$g_{t\to+\infty} = D(-x)^{ip/2-ir/2}\left[\Sigma - (-x)^{ir} \sinh(p\pi)/\sinh(r\pi)\right] , \tag{19}$$

where

$$\Sigma = r\Gamma^2(-ir)\left[(r-p)\,\Gamma(-ip-ir)\;\Gamma(ip-ir)\right]^{-1} . \tag{19'}$$

Here, the Γ's are gamma functions.

Finally, we develop the squared modulus $gg^*_{t\to+\infty}$ and use Eq. (18). We get for the desired transition probability $gg^*_{t\to+\infty}$.

$$2gg^*_{t\to+\infty} = 1 - S_{z_{t\to+\infty}} = (1+p/r)s_-s_+ + (1-p/r)\left[\sinh(p\pi)/\sinh(r\pi)\right]^2 , \tag{20}$$

where

$$s_\pm = \sinh\left[(r \pm p)\pi\right]/\sinh(r\pi) . \tag{20'}$$

Eq. (20) is the GFSF, written in a form which is valid for any magnitude of the Q-range, ΔQ.

In writing Eq. (20) we have dropped a rapidly oscillating term which comes in (its time-average value is zero) since we are interested only in the mean value of the residue polarization.

Specializing now the case of interest, where $\omega^2 \ll \delta^2$ (i.e., by Eqs. (13), the case $q^2 \ll p^2$) we have

$$r - p \simeq \omega^2/(4\alpha\delta) \tag{21}$$

(recall that r is given by Eq. (16)).

If we write

$$\alpha\delta\,(=|\ddot{\chi}(0)|\,) = \omega_0\dot{Q} \tag{22}$$

and use Eq. (5), Eq. (20) reduces, for $\omega^2 \ll \delta^2$, i.e. for

$$(\Delta Q)^2 \gg (2\,\omega\,/\,\omega_0)^2 , \tag{23}$$

to Eq. (6).

In conclusion, one may retain only condition (23) and calculate with great confidence the net resulting depolarization by making use of Eq. (6).

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