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The Microscopic Analysis of the ^{4}He-^{4}He Elastic Scattering at Intermediate Energies.

A. MAŁECKI and L. SATTA
Laboratori Nazionali di Frascati - Frascati

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In the last ten years there has been a considerable progress in studying nuclear structure by means of intermediate-energy protons [1,2]. The store of experimental data should now be supplemented by the measurements performed with beams of fast ions. It is expected that some effects of nuclear structure will be enhanced in nucleus-nucleus collisions due to mechanisms which are absent in particle-nucleus scattering; in particular, in nucleus-nucleus scattering are involved multi-nucleon collisions [5] which should be sensitive to the short-range nuclear correlations.

Among the nuclei studied with beams of energetic protons particular attention has been paid to the α-particle [2]. Recently the measurements of p-^{4}He elastic scattering has been extended to large momentum transfers by the use of accelerated alphas hitting a hydrogen target [3]. A natural continuation of this work is the experiment on the ^{4}He-^{4}He scattering, in phase of realization at Saclay [3].

The proton-nucleus scattering experiments at intermediate energies have successfully been interpreted in terms of the Glauber model [4] of multiple collisions. The model may straightforwardly be generalized to scattering of two nuclei [5,6].

In this letter we describe a new approximate and relatively simple version of the Glauber formula for nucleus-nucleus scattering. We give physical and numerical arguments for this approximation and discuss its relation to other works. Our approach is then applied to α-α elastic scattering in order to provide a theoretical framework to the forthcoming experimental results [3].

(*) On leave of absence from Instytut Fizyki Jadrowej, Kraków, Poland.
(1) J. SAUDINOS and C. WILKIN: *Ann. Rev. Nucl. Sci.*, **24**, 341 (1974).
(2) G. IGO: in *Sixth International Conference on High-Energy Physics and Nuclear Structure* (Santa Fe, N. Mex., 1975).
(3) J. BERGER *et al.* (SACLAY-CAEN-FRASCATI COLLABORATION): to be published in *Phys. Lett.*, B, and private communication.
(4) R. J. GLAUBER: in *Lectures in Theoretical Physics*, Vol. 1, edited by W. E. BRITTIN and L. G. DUNHAM (New York, N. Y., 1959), p. 315.
(5) W. CZYŻ and L. C. MAXIMON: *Ann. of Phys.*, **52**, 59 (1969).
(6) A. DAR and Z. KIRZON: *Phys. Lett.*, **37** B, 166 (1971); *Nucl. Phys.*, **237** A, 319 (1975).

The Glauber model is based on the two assumptions.

i) the eikonal approximation [4] for high-energy, small-angle scattering;

ii) the approximation of closure over the intermediate states [7] of colliding nuclei.

The Glauber amplitude for scattering of two nuclei A and B is written in the form of the impact parameter integral:

$$F_{AB} = \frac{ip}{2\pi}\int \mathrm{d}^2 b \exp[i\boldsymbol{q}\cdot\boldsymbol{b}]\Gamma_{AB}(b)\,, \tag{1}$$

p, q being the incident c.m. momentum and the momentum transfer, respectively. The nucleus-nucleus profile function Γ_{AB} for elastic scattering is [4,5]

$$\Gamma_{AB}(b) = 1 - \langle \psi_A \psi_B | \prod_{j=1}^{A} \prod_{k=1}^{B} [1 - \gamma_{jk}(\boldsymbol{b} - \boldsymbol{s}_j^A + \boldsymbol{s}_k^B)] | \psi_A \psi_B \rangle\,. \tag{2}$$

The elementary profile functions γ_{ik} depend on the transverse (*i.e.* in the plane perpendicular to the bisectrix of the c.m. scattering angle) co-ordinates $\boldsymbol{s}_j^A$, $\boldsymbol{s}_k^B$ of the constituent nucleons and are to be related to the nucleon-nucleon elastic-scattering amplitudes. ψ_A and ψ_B are the nuclear ground-state wave functions which depend both on the transverse and longitudinal intrinsic nucleon co-ordinates.

If one multiplies out the AB factors in eq. (2) one obtains terms, each of which is a product of a certain number n of the profiles γ; $1 \leqslant n \leqslant AB$. In particular there are many terms in which a given nucleon co-ordinate appears more than once. The terms with repetition of nucleon variables represent a peculiarity of nucleus-nucleus collisions; for particle-nucleus scattering the Glauber model does not contain such terms. A large variety of the repetition terms in scattering of two complex nuclei makes the evaluation of ground-state expectation value in eq. (2) a very difficult task, even for simple-model wave functions.

An enormous simplification of the Glauber formula may, however, be obtained by relaxing the closure approximation. Closure implies the possibility of excitations and de-excitations (connected between one another in such a way that the final result is elastic scattering) of the two nuclei during scattering. The effect of virtual nuclear excitations which we will call quasi-elastic shadowing [*] should not, however, be large. In fact, at high energies the inelastic transitions will prevalently lead to the break-up channels (for ^{4}He besides this is, because of the lack of discrete excitation, the unique possibility) and it is hardly probable that disintegrated products could recombine to the initial state. The neglect of quasi-elastic shadowing would reduce eq. (2) to the following simple form:

$$\begin{cases} \Gamma_{AB}(b) = 1 - (1 - S_{AB})^{AB}\,, \\ S_{AB} \quad = \langle \psi_A \psi_B | \gamma(\boldsymbol{b} - \boldsymbol{s}^A + \boldsymbol{s}^B) | \psi_A \psi_B \rangle\,. \end{cases} \tag{3}$$

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(7) L. L. FOLDY and J. D. WALECKA: *Ann. of Phys.*, **54**, 447 (1969).

(*) The analogous virtual excitations of the elementary projectile in particle-nucleus scattering are usually referred to as inelastic shadowing or the Gribov effect (8).

(8) J. PUMPLIN and M. ROSS: *Phys. Rev. Lett.*, **21**, 1778 (1968); V. N. GRIBOV: *Sov. Phys. JETP*, **29**, 483 (1969).

Equation (3) is obtained by inserting between the profile functions γ_{jk} in eq. (2) the operators $\sum_{n_A} |n_A\rangle\langle n_A|$, $\sum_{n_B} |n_B\rangle\langle n_B|$, $|n_A\rangle$, $|n_B\rangle$ being the complete sets of states for the two nuclei, and then by deleting the inelastic scates. For the sake of simplicity we have assumed in eq. (3) that the elementary profiles for the p-p and p-n interaction are identical.

An expression similar to eq. (3) has been obtained by CZYŻ and MAXIMON ([5]) in the discussion of the optical limit of the multiple-scattering amplitude. In contrast to their work (see also ref. ([6])) we do not, however, make neither any assumption on the nuclear ground-state wave functions, nor need any estimate of the repetition terms. Our eq. (3) is obtained only by ignoring quasi-elestic shadowing. The exponentiation of eq. (3) in the limit $AB \to \infty$ would give the well-known optical limit of the nucleus-nucleus profile ([4-6]). Thus in our approach the optical limit acquires a new physical interpretation as being essentially equivalent to the neglect of quasi-elastic shadowing.

For light nuclei the exponentiation would, of course, be unjustified. Let us also notice that the effect of translational invariance ([5,9]) which is very important in this case is included in eq. (3) since ψ_A and ψ_B are to be intrinsic wave functions. Therefore the correction connected with the c.m. constraint ([9]) will not lead, unlike as in ref. ([5]), to any inconsistency with the approximate form of the nucleus-nucleus profile.

The approximation of eq. (3) has been checked by us, on the example of the ^{4}He-^{4}He elastic scattering, by a term by term comparison with eq. (2) up to the fifth order in the nucleon-nucleon interaction—see fig. 1. To this end the independent particle model (IPM) has been used

$$|\psi_A|^2 = \prod_{j=1}^{A} \varrho_A(r_j^A)\,, \qquad |\psi_B|^2 = \prod_{k=1}^{B} \varrho_B(r_k^B)\,. \tag{4}$$

The single-particle density has been chosen as a double Gaussian:

$$\varrho(r) = \pi^{-\frac{3}{2}}[R_1^3 - \partial R_2^3]^{-1}\left[\exp\left[-\frac{r^2}{R_1^2}\right] - \delta \exp\left[-\frac{r^2}{R_2^2}\right]\right], \tag{5}$$

which gives a chance, in contrast to a single Gaussian, of fitting a diffraction structure of the elastic charge form factor of ^{4}He ([10]). The c.m. constraint on the nuclear model of eqs. (4) and (5) has been imposed using the Gartenhaus-Schwartz prescription ([9]).

The elementary profiles have been assumed in the form:

$$\gamma(b) = \frac{\sigma(1 - i\alpha)}{4\pi a} \exp\left[-\frac{b^2}{2a}\right], \tag{6}$$

which corresponds to a Gaussian q-dependence of the $\mathcal{N}$-$\mathcal{N}$ elastic-scattering amplitudes. The parameters σ (the total $\mathcal{N}$-$\mathcal{N}$ cross-section), α (the Re/Im ratio of the forward amplitude) and a (the slope) are, in general, energy dependent.

Looking at fig. 1 it may be stated that, although the difference between the two formulae for individual scattering orders may be large, the agreement is improved when one sums various contributions to the scattering amplitude.

([9]) S. GARTENHAUS and C. SCHWARTZ: *Phys. Rev.*, **108**, 482 (1957); A. MAŁECKI and P. PICCHI: *Lett. Nuovo Cimento*, **14**, 390 (1975).

([10]) R. F. FROSCH, J. S. MCCARTHY, R. E. RAND and M. R. YEARIAN: *Phys. Rev.*, **160**, 874 (1967).

A similar relation, as in fig. 1, between eqs. (2) and (3) has been found for the single Gaussian density ($\delta = 0$). In this case our eq. (3) has been also compared with the complete calculation of the Glauber multiple-scattering amplitude carried out in ref. ([5]); the agreement up to the third diffraction maximum is very good (*). Therefore we can

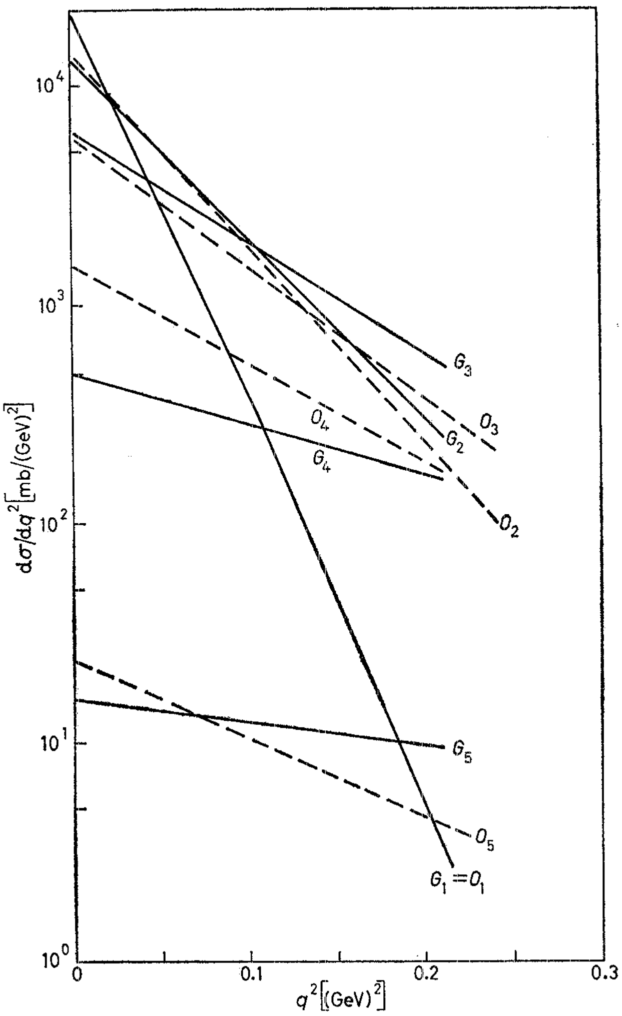

Fig. 1. – Comparison of the Glauber multiple-scattering amplitude for the α-α scattering with its approximated form, obtained by neglecting quasi-elastic shadowing. The curves correspond to contributions from the single, double, etc. scattering which result either from eq. (2) (full lines denoted G_n) or eq. (3) (broken lines denoted O_n). The double Gaussian nuclear density (eq. (5)) with the parameters $R_1 = 1.25$ fm, $R_2 = 0.77$ fm, $\delta = 1.0$, providing a good fit to the ^{4}He charge form factor ([10]), has been used. The $\mathcal{N}$-$\mathcal{N}$ parameters are appropriate for scattering at $P_{\text{lab}} = 1.25$ GeV/c per nucleon: $\sigma = 40$ mb, $\alpha = -0.2$, $a = 3$ (GeV)$^{-2}$.

(*) On the contrary, in ref. ([5]) a considerable difference between the multiple scattering and the optical limit results in the case of 4×4 scattering has been found. It should, however, be stressed again that our eq. (3) differs from the optical limit of ref. ([5]) by the way of introducing the c.m. correction and by the lack of exponentiation.

conclude that the approximated Glauber formula with no shadowing should work up to $q^2 = (0.4 \div 0.5)\ (\text{GeV})^2$. Then it is expected to understimate the cross-section.

The physical essence of the « no shadowing » multiple-scattering formula implies that nuclear structure enters here only through the elastic form factors $\Phi_A(q)$, $\Phi_B(q)$ of the two nuclei. In fact one has

$$S_{AB}(b) = \frac{1}{2\pi i p}\int \mathrm{d}^2 q \exp[-i\boldsymbol{q}\cdot\boldsymbol{b}] f_{\mathcal{N}\mathcal{N}}(q)\, \Phi_A(q)\, \Phi_B(q)\,, \tag{7}$$

$f_{\mathcal{N}\mathcal{N}}$ being the nucleon-nucleon elastic-scattering amplitude.

For a comparison of eq. (3) with experiment one may apply the phenomenological nuclear form factors, established in an analysis of the electron-nucleus scattering. Any disagreement with experiment, if one believes in the Glauber model, might then be interpreted as quasi-elastic shadowing in nucleus-nucleus scattering. The size of shadowing could be a measure of usefulness of nucleus-nucleus scattering as a tool for obtaining new (*i.e.* those not available in electron scattering) informations on nuclear structure. It follows from our discussion on the validity of eq. (3) that in the α-α experiment the interesting domain begins near the third diffraction maximum.

Another aspect of nuclear structure regards the sensitivity of nucleus-nucleus scattering to details of nuclear wave functions. It is well known ([11]) that nuclear form factor is strongly affected by the short-range correlations between nucleons. To check this effect in nucleus-nucleus scattering we have calculated eq. (3) using the model of correlated pairs ([11]):

$$|\psi|^2 = \prod_{j=1}^{A} \varrho(r_j)\left\{1 + \sum_{l=1}^{A/2} (2^l \cdot l!)^{-1} \sum_{j_1\neq k_1\neq \ldots \neq j_l \neq k_l}^{A} [\Delta(j_1, k_1) \ldots \Delta(j_l, k_l)]\right\}, \tag{8}$$
$$\Delta(j, k) = G^2(\boldsymbol{r}_j - \boldsymbol{r}_k) - 1\,.$$

The successive terms of (8) correspond to an expansion in numbers of correlated pairs: independent particles, one correlated pair, etc. The two inputs of the model (**) are the single-particle density $\varrho(r)$ (taken by us as a single Gaussian with the size parameter R) and the correlation operator $G(r_{jk})$ assumed in the form ([11])

$$G^2 = \frac{g^2 + (M-1)g}{M}\,, \qquad g(r_{jk}) = 1 - \exp\left[-\lambda \frac{r_{jk}^2}{R^2}\right], \tag{9}$$

the coefficient $M = -1 + (1+2\lambda)^{\frac{3}{2}}(1+4\lambda)^{-\frac{3}{4}}$ being determined by normalization.

The result of the correlated model for the ^{4}He-^{4}He elastic scattering is presented in fig. 2. The parameters R, λ of the model provide a good fit to the ^{4}He elastic-charge form factor in a wide range of momentum transfer ([10,11]). For comparison we show also the result of IPM with a single Gaussian. Its parameter R_1 has been fixed by the value of the root-mean-square radius of the α-particle which in both cases is the same:

(11) A. MAŁECKI and P. PICCHI: *Riv. Nuovo Cimento*, **2**, 119 (1970).

(**) To avoid misunderstandings one should stress that we consider here the model correlations and not the « self-consistent » correlations for which $\int \mathrm{d}^3 r_2 G^2(r_{12})\varrho(r_2) = 1$. Such correlations would affect the elastic form factor very little (through the c.m. term only); for their realization one needs, however, a sophisticated form of $\varrho(r)$, the Gaussian form being clearly insufficient.

1.64 fm in agreement with a low-q experiment (12). In fig. 2 the Coulomb interaction of the α-particles has been included (13) assuming charges extended over the volume of the nuclei.

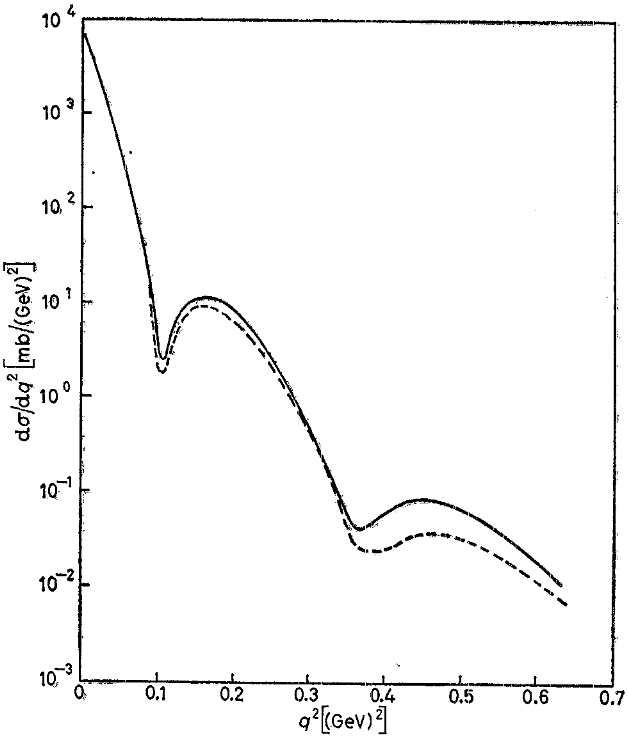

Fig. 2. – Comparison of the ^{4}He-^{4}He elastic cross-section in two nuclear models: – – – IPM with the single Gaussian density ($R_1 = 1.36$ fm); —— the model of correlated pairs ($R = 1.265$, $\lambda = 0.652$) The $\mathcal{N}$-$\mathcal{N}$ parameters are given in the caption of fig. 1.

The effect of the correlations in nucleus-nucleus scattering seems to be enhanced as compared with proton-nucleus (14) scattering. However it is still much smaller than in electron-nucleus scattering. The short-range correlations are most effective in the diffraction maxima. It should be pointed out that these observations are based on the « no shadowing » multiple-scattering formula which probably will fail for $q^2 > 0.5$ (GeV)^2. On the other hand the role of the short-range correlations should increase at large momentum transfer. Therefore our discussion cannot be conclusive and poses challenging problem to experimenters and theoreticians: i) to measure small cross-sections downward the third maximum, and ii) to calculate the complete Glauber profile of the nucleus-nucleus scattering with correlated forms of nuclear densities.

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(12) M. FRANK, D. HAAS and H. PRANGE: *Phys. Lett.*, **19**, 391, 719 (1965).
(13) W. CZYŻ and L. C. MAXIMON: *Ann. of Phys.*, **60**, 484 (1970).
(14) R. DYMARZ and A. MAŁECKI: Frascati report LNF-76/23 (1976).