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A. Turrin: ALTERNATING-$\Delta\beta$ COUPLERS WITH PHASE-MATCHED SECTIONS.

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ABSTRACT : The switching characteristics of multiple-section $\Delta\beta$-reversal couplers having phase-matched sections are obtained. In particular, the switching diagram for a four-section coupler is given. It is found that the device length still remains a noncritical parameter even when the ratio of the total length of the phase-matched sections to the interaction length is about 1/5.

Switched alternating-phase mismatch couplers with two sections are now operating(1), and a clear discussion of these devices, and also of couplers with N sections, can be found in another paper by Kogelnik and Schmidt(2).

It has been shown in Ref. (2) (hereafter referred to as KS) that by letting the mismatch $\Delta\beta(z)$ between the propagation constants of the two guides alternate between suitable positive and negative values at suitable intervals along the axial coordinate z of the coupler, one can obtain an optical switch which has reasonable fabricational tolerances.

In switched directional couplers with a number N of alternating $\Delta\beta(z)$ sections the simplest $\Delta\beta(z)$ arrangement is that of KS: each section is L/N long and $\Delta\beta(z) = \pm\Delta\beta$ changes sign in going from the n'th to the (n+1)th section. Here, L is the coupler's interaction length.

Since it seems desirable to treat the effect of phase-matched (i.e. $\Delta\beta(z) = 0$) sections on the switching characteristics, in the present communication we shall consider a coupler having an even number N of phase-matched sections of length d/N each, interspersed with N sections of alternating $\Delta\beta$ of length D/N each, and disposed according to a periodic pattern. Thus, we can consider the coupler as composed of N/2 consecutive identical "unit cells", calling by this name the smallest set of contiguous sections which can be regarded as the periodic element of the coupler. The unit cell we are going to consider is represented in Fig. 1.

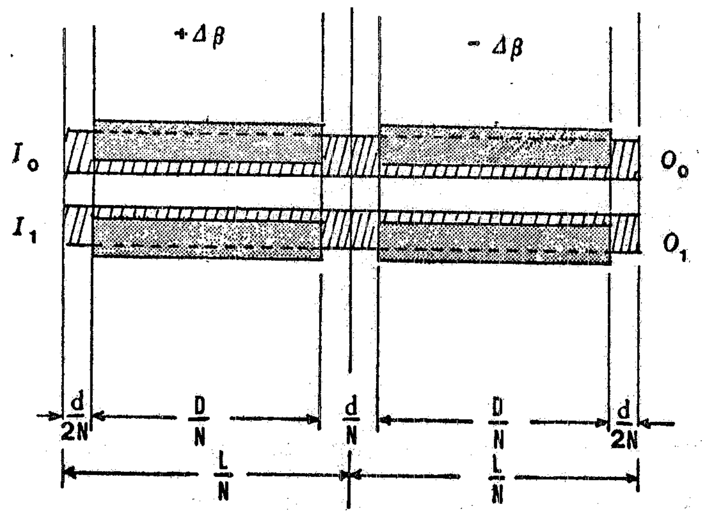

FIG. 1 - Unit cell of a N-section switch (N is an even number) composed of two sections of alternating $\Delta\beta$ having equal length D/N and equal spacing d/N each. L denotes the interaction length. I_o, I_1 ; O_o, O_1 are defined by Eq. (1) in the text.

As by KS, use of a constant coupling coefficient $\kappa(z) = \kappa$ will be made.

The linear transformation produced by the unit cell depicted in Fig. 1 may be written in matrix notation

$$\begin{vmatrix} O_o \\ O_1 \end{vmatrix} = B \begin{vmatrix} I_o \\ I_1 \end{vmatrix} , \tag{1}$$

where $I_{o,1}$ and $O_{o,1}$ are the nearly stationary modeamplitudes associated with the two guides, and the over-all transformation matrix B of this unit cell is given as the product of the matrices for the individual sections

$$B = \begin{vmatrix} c(1/2) & -is(1/2) \\ -is(1/2) & c(1/2) \end{vmatrix} A \begin{vmatrix} c(1/2) & -is(1/2) \\ -is(1/2) & c(1/2) \end{vmatrix} , \tag{1a}$$

where

$$A = \begin{vmatrix} C - i\frac{\delta}{r}S & -i\frac{\kappa}{r}S \\ -i\frac{\kappa}{r}S & C + i\frac{\delta}{r}S \end{vmatrix} \begin{vmatrix} c & -is \\ -is & c \end{vmatrix} \begin{vmatrix} C + i\frac{\delta}{r}S & -i\frac{\kappa}{r}S \\ -i\frac{\kappa}{r}S & C - i\frac{\delta}{r}S \end{vmatrix} . \tag{1b}$$

Here, and subsequently, we are using the following notations :

$$\begin{vmatrix} c(1/2) = \cos(\kappa d/(2N)) \\ s(1/2) = \sin(\kappa d/(2N)) \end{vmatrix} , \qquad \begin{vmatrix} c = \cos(\kappa d/N) \\ s = \sin(\kappa d/N) \end{vmatrix} , \tag{1c}$$

$$\delta = \Delta\beta/2 \quad , \qquad r = (\kappa^2 + \delta^2)^{1/2} \tag{1d}$$

and

$$C = \cos(rD/N) \quad , \qquad S = \sin(rD/N) . \tag{1e}$$

On carrying out the multiplication indicated in Eq. (1b), one obtains that the matrix A can be written in the form

$$A = \begin{vmatrix} a_1 & -ia_2 \\ -ia_2^* & a_1 \end{vmatrix} , \tag{2}$$

where a_1 is a real number given by the expression

$$a_1 = c(1 - 2S^2\kappa^2/r^2) - 2sSC\kappa/r , \tag{2a}$$

and the real part of a_2 is

$$\mathrm{Re}(a_2) = 2\,c\,S\,C\,\kappa/r + s(C^2 - S^2) \; . \tag{2b}$$

Similarly, using Eq. (2) in Eq. (1a) one finds that matrix B can be written in the form

$$B = \begin{vmatrix} b_1 & -ib_2 \\ -ib_2^* & b_1 \end{vmatrix} , \tag{3}$$

where b_1 is a real number given by the expression

$$b_1 = c\,a_1 - s\,\mathrm{Re}(a_2) \; , \tag{3a}$$

and b_2 has an expression which we need not specify.

It is apparent from Eqs. (1a), (1b), (1c), (1d) and (1e) that the determinants of the individual matrices are equal to unity; consequently, the determinant of the over-all transformation matrix B is equal to unity also.

Thus, the unit cell matrix B may be written in the form

$$B = \begin{vmatrix} \cos\theta & \gamma\sin\theta \\ -\gamma^*\sin\theta & \cos\theta \end{vmatrix} , \tag{4}$$

where

$$\cos\theta = b_1 \; , \qquad \gamma\sin\theta = -ib_2 \; , \tag{4a, b}$$

and the relationship

$$\gamma\gamma^* = 1 \tag{4c}$$

holds (as consequence of the power conservation law).

Combining Eqs. (4a), (3a), (2b) and (2a) we can write

$$\cos\theta = c^2(1 - 2S^2\kappa^2/r^2) - s\left[2\,c\,S(2)\,\kappa/r + s\,C(2)\right] \; , \tag{5}$$

where c, s, S and r are given by Eqs. (1c), (1d) and (1e) and

$$S(2) = \sin(2rD/N)\ , \qquad C(2) = \cos(2rD/N)\ . \tag{1f}$$

Finally, it follows by Eq. (4c) that the (N/2)-th power of the matrix B (see Eq. (4)) is:

$$B^{N/2} = \begin{vmatrix} \cos(N\theta/2) & \gamma\sin(N\theta/2) \\ -\gamma^{*}\sin(N\theta/2) & \cos(N\theta/2) \end{vmatrix}\ . \tag{6}$$

We can now derive the criteria which allow the determination of the parameters $\Delta\beta$, L and D for which the N-section switch permits light to propagate straight through (e. g. $O_oO_o^{*} = I_oI_o^{*} = 1$ and $O_1 = I_1 = 0$), or for which the light is switched from one guide to the other (e. g. $O_1O_1^{*} = I_oI_o^{*}$ and $O_o = I_1 = 0$).

The first condition is referred to[2] as a "bar-state" of the device and is represented[2] by the symbol ⊜; the second condition is called[2] a "cross-state" of the switch and is represented[2] by the symbol ⊗.

Inspection of Eq. (6) readily reveals that the bar-state condition is obtained when $\cos(N\theta/2)$ equals +1 or -1, and the cross-state condition occurs when $\cos(N\theta/2) = 0$.

Each of these three conditions determines values of $\cos\theta$ which may be inserted in Eq. (5) to plot the switching diagram.

As an example and for the sake of comparison with the KS results, we consider in detail the case of a four-section (i. e. two-unit cell) switched directional coupler (N = 4). To do this, we introduce, according to KS treatment, the two dimensionless variables L/ℓ and $\Delta\beta L/\pi$ (where $\ell = \pi/(2\kappa)$ is the coupling length) and the dimensionless parameter ε defined by the relationships

$$D = (1 - \varepsilon)L\ , \qquad d = \varepsilon L\ . \tag{7a, b}$$

The cross-state condition gives the values $\cos\theta = \pm 1/\sqrt{2}$, corresponding to the curves labeled by ⊗ in Fig. 2 (i. e. the curves (1, 7) and (3, 5), respectively).

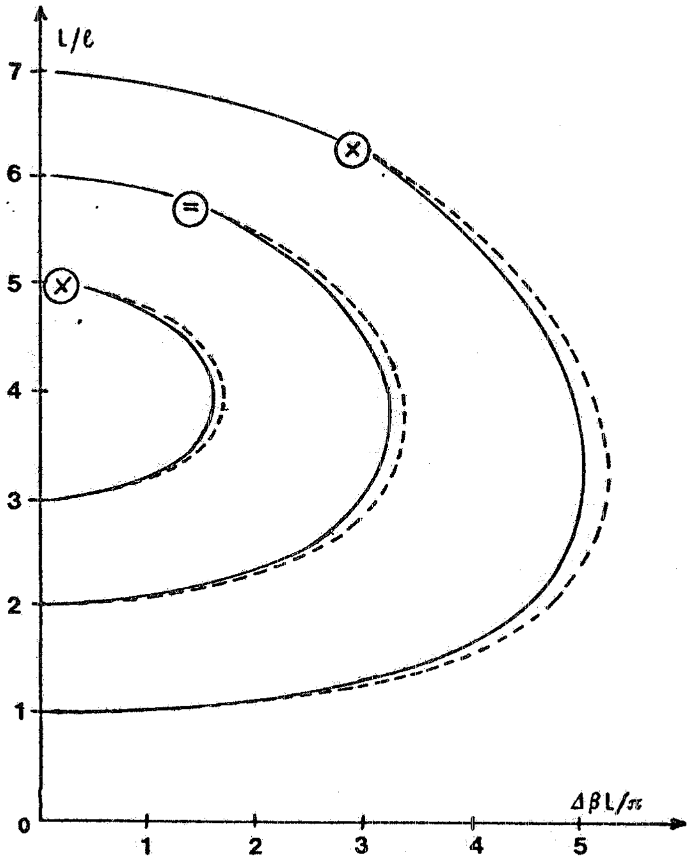

FIG. 2 - Switching diagram of a four-section (i. e. two-unit cell) alternating-$\Delta\beta$ coupler with phase-matched sections. The ⊗ and ⊜ symbols attached to the curves denote cross-state and bar-state conditions respectively. The solid curves are the KS curves (see Fig. 8 of KS paper), which correspond to the case where $\varepsilon = 0$. Dashed curves correspond to the case where $\varepsilon = 0.2$ ($\varepsilon = d/L$). Curves intersecting the L/ℓ axis at $L/\ell > 7$ are omitted for the sake of lithographic clearness.

The bar-state condition implies three conditions: $\cos\theta = 0$ and $\cos\theta = \pm 1$. Only the first condition is of interest, since only it gives the curves labeled by ⊜ in Fig. 2 (i. e. the curves (2, 6)). The solid curves in Fig. 2 correspond to the KS curves (represented in Fig. 8 of KS paper), which are redetermined by us by letting $\varepsilon = 0$. The dashed curves in Fig. 2 are calculated by letting $\varepsilon = 0.2$.

In conclusion, we have shown that insertion of phase-matched sections of moderate length in alternating $-\Delta\beta$ couplers do not implies more severe fabricational tolerance requirements than predicted by KS for couples in which phase-matched sections are absent.

REFERENCES.

(1) - R. V. Schmidt and H. Kogelnik, Appl. Phys. Lett. 28, 503 (1976).

(2) - H. Kogelnik and R. V. Schmidt, IEEE J. Quantum Electron. QE-12, 396 (1976).