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G. Dattoli[(*)], R. Mignani[(+)], D. Prosperi[(x)]: DIPOLE MAGNETIC MOMENTS OF CHARMED BARYONS IN A NEW U(3) SCHEME.

In the last years, many works have been devoted to studying the electromagnetic properties of charmed hadrons, in particular the dipole magnetic moments of charmed baryons[(1-3)]. This problem has been approached both in the hypothesis of an exact SU(4) symmetry[(1)] and of a partially broken one[(2, 3)]. In the first case, the magnetic moments of charmed hadrons have been expressed in terms of those of the uncharmed ones, by assuming invariance under U(4) and U(8) symmetries. In the latter approach, the magnetic moments μ_i of the quarks have been supposed to be proportional to their charge-to-mass ratios ($\mu_i \propto q_i/m_i$), so introducing in a natural way symmetry breaking effects[(2, 3)]. Indeed, the results of refs. (2, 3) come out to be identical to those of ref. (1) when the symmetry breaking is switched off.

Recently, it has been pointed out that the theoretical investigations of charmed hadron properties could take advantage of the introduction of a new SU(3) scheme, built up by including u-, d- and c-quarks[(4, 5)]. In particular, this approach has been applied in ref. (5) to derive formulas relating the magnetic moments of charmed baryons to those of the nucleons. But all results so obtained in ref. (5) do not agree with the previous

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ones of refs. (1-3), so that the validity of the proposed approach has been not definitively proved.

The aim of the present paper is showing that the basic idea of refs. (4, 5) is correct. We shall prove that quarks u, d and c can be really considered as belonging to the fundamental representation of a suitable U(3) scheme. Moreover, using the group properties of this "new" U(3), we shall obtain formulae for the magnetic moments of charmed baryons which agree with those of refs. (1-3).

Let us first introduce a "planar representation" of the SU(4) weight diagrams by plotting $\tilde{Y} = B + S + C$ versus $I_3^{(3)}$. The fundamental representation of SU(4) we obtain in this way is shown in Fig. 1. Indeed,

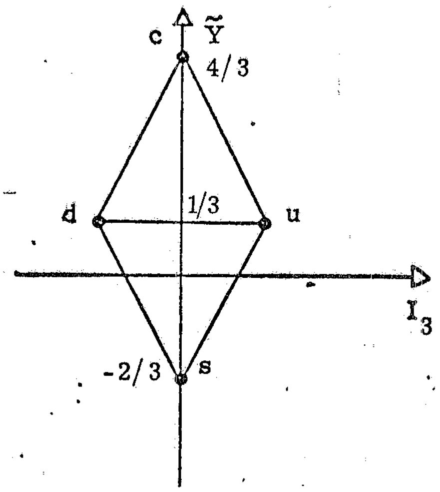

FIG. 1 - Planar weight diagram of the SU(4) fundamental representation in the Y, I_3-plane.

if we neglect the s-quark and suitably shift the vertical axis ($\tilde{Y} \longrightarrow Y' = \tilde{Y} - 2/3$), we arrive to the u-, d- and c-quark representation shown in Fig. 2: at sight, it coincides with the [3] representation of a new SU(3) or U(3) group.

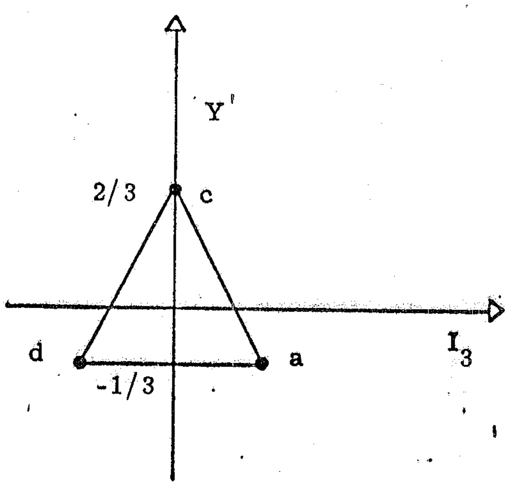

FIG. 2 - Fundamental representation of the "new" U(3) group built up with the u-, d- and c-quarks.

This last ambiguity can be eliminated by observing that, in terms of Y', the Gell-Mann-Nakano-Nishijima formula reads:

$$q = I_3 + \frac{1}{2} Y' + \frac{1}{3}. \qquad (1)$$

A simple inspection of eq. (1) allows us to conclude that the algebra of the new group we are interested in is U(3). More precisely, in addition to the eight Gell-Mann and Neeman matrices $\lambda_i (i = 1, \ldots, 8)$, we need now to introduce a new generator proportional to the unit matrix 1, that is $\lambda_0 = \sqrt{2/3}\ 1$. Therefore, the charge operator for the $[3]$ representation can be written in terms of the diagonal generators of the U(3) algebra as follows:

$$Q = \frac{1}{2} \lambda_3 - \frac{1}{2\sqrt{3}} \lambda_8 + \frac{1}{\sqrt{6}} \lambda_0, \qquad (2)$$

This expression, which can be obtained from eq. (1) by replacing I_3 and Y' with $\lambda_3/2$ and $-\lambda_8/\sqrt{3}$, respectively, coincides with eq. (1) of ref. (5).

Let us now recall that, in the standard SU(3) approach to hadron electromagnetic properties, it is often useful to work in the U-spin formalism. In the U-spin representation, the commuting operators are[6]

$$Q \ , \ u^2 \ , \ u_3 \ , \qquad (3)$$

and the third U-spin component is defined as $u_3 \equiv (2Y-Q)/2$.

We can easily develop a similar approach for the new U(3) scheme. We define an "U'-spin representation" whose commuting operators are

$$Q' \ , \ u'^2 \ , \ u'_3 \ . \qquad (4)$$

The "new charge operator" Q' must be connected to Q by the relation

$$Q' = Q - \frac{1}{\sqrt{6}} \lambda_0 \qquad (5)$$

Eq. (5) ensures us that Q' has the same transformation properties of Q in the usual SU(3) scheme. Further, by analogy, we define u'_3 as:

$$u'_3 = \frac{1}{2} (2Y' - Q') = u_3 + \frac{1}{2} (2C - 1). \qquad (6)$$

The $\overline{3}$- representation in the (Q', u'_3) plane is displaied in Fig. 3. Similar diagrams for the $[8]$ and $[\overline{10}]$ baryon representations are also shown in Figs. 4 and 5, respectively; the corresponding Q'-, u'- and u'_3-values are reported in Tables 1a) and 1b).

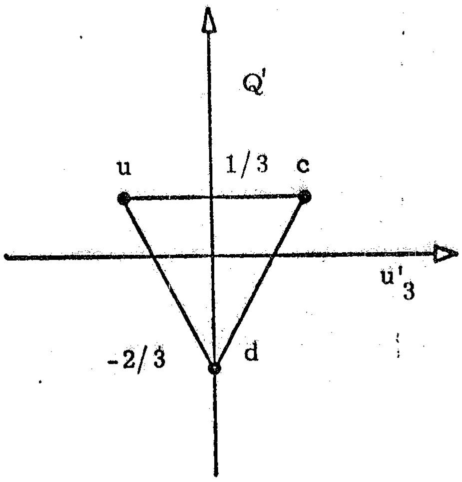

FIG. 3 - Fundamental representation of the "new" U(3) group in the Q', u'_3--plane.

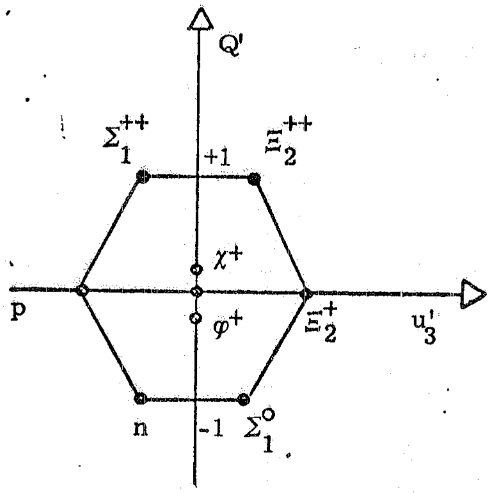

FIG. 4 - Baryon octet ($J^{\pi}=1/2^+$) in the "U'-spin" representation.

$$|\chi^+\rangle = \frac{1}{2}\left(-|\Sigma_1^+\rangle + \sqrt{3}\,|\Lambda_1^+\rangle\right) \quad u'=1$$

$$|\varphi^+\rangle = \frac{1}{2}\left(\sqrt{3}\,|\Sigma_1^+\rangle + |\Lambda_1^+\rangle\right) \quad u'=0$$

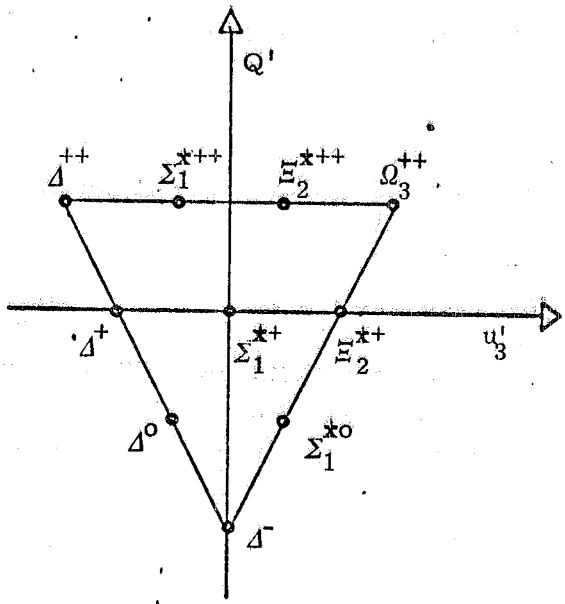

FIG. 5 - Baryon decuplet ($J^{\pi}=3/2^+$) in the "U'-spin" representation.

TABLE 1a) - Values of Q', u' and u'_3 for the baryons belonging to the $[\bar{8}]$ representation.

State	Q'	u'	u'_3
Ξ_2^{++}	+1	1/2	+1/2
Σ_1^{++}	+1	1/2	-1/2
Ξ_2^{+}	0	1	+1
χ^{+}	0	1	0
φ^{+}	0	0	0
Σ_1^{o}	-1	1/2	+1/2
n	-1	1/2	-1/2
p	0	1	-1

TABLE 1b) - Values of Q', u' and u'_3 for the baryons belonging to the $[\overline{10}]$ representation.

State	Q'	u'	u'_3
Δ^{++}	+1	3/2	-3/2
Σ_1^{*++}	+1	3/2	-1/2
Ξ_2^{*++}	+1	3/2	+1/2
Ω_3^{++}	+1	3/2	+3/2
Δ^{+}	0	1	-1
Σ_1^{*+}	0	1	0
Ξ_2^{*+}	0	1	+1
Δ^{o}	-1	1/2	-1/2
Σ_1^{*o}	-1	1/2	+1/2
Δ^{-}	-2	0	0

Let us now introduce a "fictitious magnetic dipole vector" $\vec{\mu}'$, given by

$$\vec{\mu}'(h) = 2\mu_o \langle h \mid \sum_i Q'_i \vec{s}_i \mid h \rangle , \tag{7}$$

where h is the hadron state of interest, Q'_i and $\vec{s}_i$ are the "new charge" and spin of the i-th quark, respectively; lastly, μ_o is the same scale factor appearing in the standard definition of the physical magnetic dipole vector $\vec{\mu}$. The transformation properties of $\vec{\mu}'$ in U(3) are clearly the same of $\vec{\mu}$ in the standard SU(3). We can also define a "fictitious magnetic moment" μ' by the expression

$$\mu'(h) \equiv \left[\mu'_3(h)\right]_{J_3} = J, \tag{8}$$

where $\vec{J} = \sum_i \vec{s}_i$ is the total hadron angular momentum (we confine ourselves to consider only $\vec{L}=0$ states). By taking into account eq. (6), it is straightforward to relate μ' to the physical magnetic momentum μ; we have:

$$\mu'(h) = \mu(h) - \frac{2}{3}\mu_o J. \tag{9}$$

Let us emphasize that the difference $(\mu' - \mu)$ does not depend on the details of the SU(3) wave functions: it is equal to $-\mu_o/3$ for the baryon $[\bar{8}]$ representation ($J=1/2$) and to $-\mu_o$ for the baryon $[\overline{10}]$ representation ($J=3/2$).

At this point, we are able to calculate μ' in the U'-spin representation by using the standard formulas valid for μ in the U-spin representation. Thus, the general expression for μ' takes the form[8]:

$$\mu'/\mu_o = a\,Q' + b\left\{u'(u'+1) - \frac{1}{4}(Q')^2 - \frac{1}{6}C_2^{(3)}\right\}, \qquad (10)$$

where $C_2^{(3)}$ is the quadratic Casimir operator defined in ref. (8) (its iegenvalues are 6 and 12 for the 8- and 10-dimensional representation, respectively).

For the baryon octet $[\overline{8}]$, it is a=b=2/3[7], and we have:

$$\mu'(\overline{8})/\mu_o = \frac{2}{3}\left\{(-\frac{1}{2}Q'+1)^2 - u'(u'+1)\right\}. \qquad (11)$$

In the case of the baryon decuplet $[\overline{10}]$, the method of ref. (7) gives a=1, b=0, and therefore eq. (10) becomes

$$\mu'(\overline{10})/\mu_o = Q'. \qquad (12)$$

Eventually, by eq. (11), one easily finds $\mu_o = \mu_p$, and μ/μ_p is given by the following relations:

a) <u>Baryons octet</u> ($J^\pi = 1/2^+$) (see Fig. 4 and Table 1a):

$$\mu/\mu_p = -\frac{2}{3}\left\{(Q-3)^2\,\frac{1}{4} - u'(u'+1) - \frac{1}{2}\right\}; \qquad (13)$$

b) Baryon decuplet ($J^\pi = 3/2^+$) (see Fig. 5 and Table 1b):

$$\mu/\mu_p = Q' + 1 = Q \qquad (14)$$

Explicit expressions for the magnetic moments of the various baryon states can be immediately derived from eqs. (13), (14). For instance, one has, for the $1/2^+$-baryons of the 8-dimensional representation:

$$\begin{cases} \mu_{\Xi_2^{++}} = \mu_{\Sigma_1^{++}} = \frac{2}{3}\mu_p, \\ \mu_{\Xi_2^{+}} = \mu_p, \quad \mu_{\Sigma_1^{o}} = \mu_n = -\frac{2}{3}\mu_p, \\ \mu_{\Lambda_1^{+}} + \mu_{\Sigma_1^{+}} = \frac{2}{3}\mu_p. \end{cases} \qquad (15)$$

It is easily seen that the above relations coincide with those of refs. (1-3) in the limit of an exact symmetry, and thus the correctness of our procedure is confirmed. By the way, such an approach could be usefully applied as well to calculate other electromagnetic properties of charmed hadrons, e.g. magnetic form factors or mass shifts.

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