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N. Lo Iudice and F. Palumbo: NEW ISOVECTOR COLLECTIVE MODES IN DEFORMED NUCLEI.

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N. Lo Iudice[x] and F. Palumbo: NEW ISOVECTOR COLLECTIVE MODES IN DEFORMED NUCLEI.

ABSTRACT. - We study a model of deformed nucleus in which the proton and neutron fluids are described as rigid rotators with axial symmetry. The nucleus as a whole is no longer axially symmetric. Relative rotational oscillations modes of the proton-neutron fluids are predicted.

The giant dipole resonance is a collective excitation existing in all nuclei. It has a semiclassical interpretation as an oscillation of protons against neutrons[1]. This two-fluid picture suggests the existence of additional modes of excitation in deformed nuclei. For instance the neutron and proton deformed fluids might perform rotational oscillations in opposition of phase around a common axis[(x)].

In this paper we study the properties of a deformed nucleus in which protons and neutrons are described as identical rigid rotators (Z = N) with axial symmetry.

Let us denote by $\vec{\xi}_p, \vec{\eta}_p, \vec{\zeta}_p, \vec{\xi}_n, \vec{\eta}_n, \vec{\zeta}_n$ the versors of the principal axes of these rotators, by $\vec{I}_p$ and $\vec{I}_n$ their angular momenta, and by $\mathcal{J}_o$ their common moment of inertia with resp. to the axes $\vec{\xi}_p, \vec{\eta}_p, \vec{\xi}_n, \vec{\eta}_n$. The kinetic energy in the nuclear centre of mass system is

$$T = \frac{1}{2\mathcal{J}_o}\left(I^2_{\xi_p} + I^2_{\eta_p} + I^2_{\xi_n} + I^2_{\eta_n}\right) . \qquad (1)$$

We assume the potential to be a function of the angle between the symmetry axes $\vec{\xi}_p$ and $\vec{\xi}_n$, which we denote by 2θ. It is then convenient to express T in a form which exhibits its θ-dependence. To this end we

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define the principal axes for the whole nucleus

$$\vec{\xi} = \frac{1}{\sin(2\theta)} \vec{\zeta}_p \times \vec{\zeta}_n ,$$

$$\vec{\eta} = \frac{1}{2\sin\theta} (\vec{\zeta}_p - \vec{\zeta}_n) , \qquad (2)$$

$$\vec{\zeta} = \frac{1}{2\cos\theta} (\vec{\zeta}_p + \vec{\zeta}_n) ,$$

and the 0(4) generators

$$\vec{I} = \vec{I}_p + \vec{I}_n , \qquad \vec{S} = \vec{I}_p - \vec{I}_n . \qquad (3)$$

The components of $\vec{S}$ along the principal axes are

$$S_\xi = i \frac{\partial}{\partial\theta} , \qquad S_\eta = \mathrm{ctg}\,\theta \, I_\zeta , \qquad S_\zeta = \mathrm{tg}\,\theta \, I_\eta . \qquad (4)$$

The above realization requires lengthy calculations on the algebra of angular momenta that will be reported in a more detailed presentation of this work.

In this way we have replaced the four dynamical variables $\vec{\zeta}_p$ and $\vec{\zeta}_n$ by the three Euler angles and θ. The correspondence is one-to-one if we allow the Euler angles to vary over their full range and θ between zero and $\frac{\pi}{2}$.

The Hamiltonian expressed in the new variables has the following form

$$H = \frac{1}{4\mathcal{J}_o} \left[I^2 - \frac{d^2}{d\theta^2} + \mathrm{ctg}^2\theta \, I_\zeta^2 + \mathrm{tg}^2\theta \, I_\eta^2 \right] + V(\theta) , \qquad (5)$$

where the potential must have the property

$$V(\theta) = V(\frac{\pi}{2} - \theta) , \qquad (6)$$

in order to be consistent with the symmetry properties of the system of two rigid rotators.

Eq. (5) clearly shows that the system we are dealing with does not have axial symmetry. Therefore its eigenfunctions have the form

$$\Psi^{(\sigma)}_{IMn} = \sqrt{\frac{2I+1}{8\pi^2}} \sum_K D^I_{MK} \Phi^{(\sigma)}_{Kn}(\theta) . \tag{7}$$

Being the system composed of two axially symmetric rigid rotators, the Ψ^{σ}_{IMn}'s must have the following symmetries

$$P \Psi^{(\sigma)}_{IMn} = \Psi^{(\sigma)}_{IMn} , \tag{8.1}$$

$$P_p \Psi^{(\sigma)}_{IMn} = P_n \Psi^{(\sigma)}_{IMn} = \Psi^{(\sigma)}_{IMn} , \tag{8.2}$$

where P is the parity operator, and P_p, P_n are the parity operators for protons and neutron respectively.

We will consider only eigenfunctions with $I = 0, 1, 2$. They have the following form

$$\begin{aligned}
\Psi_{00n} &= \frac{1}{\sqrt{8\pi^2}} \Phi_{0n} \\
\Psi_{1Mn} &= \sqrt{\frac{3}{8\pi^2}} (D^1_{M,1} - D^1_{M,-1}) \Phi_{1n} \\
\Psi^{(1)}_{2Mn} &= \sqrt{\frac{5}{8\pi^2}} (D^2_{M,1} + D^2_{M,-1}) \Phi_{1n} \\
\Psi^{(2)}_{2Mn} &= \sqrt{\frac{5}{8\pi^2}} \left[(D^2_{M,2} + D^2_{M,-2}) \Phi^{(2)}_{2n} + D^2_{20} \Phi^{(2)}_{0n} \right]
\end{aligned} \tag{9}$$

with the symmetry constraint

$$\Phi^{(\sigma)}_{Kn}(\theta) = (-1)^K \Phi^{(\sigma)}_{Kn}(\frac{\pi}{2} - \theta) \tag{10}$$

as a consequence of eqs. (8.2).

The intrinsic wave-functions $\Phi^{(2)}_{2n}$ and $\Phi^{(2)}_{20}$ satisfy coupled differential equations, while Φ_{0n} and Φ_{1n} satisfy single differential equations. We will confine ourselves to the latter. In analogy with the Goldhaber and

Teller model[1] we assume the potential to be

$$V(\theta) = \begin{cases} \frac{1}{2} C \theta^2 , & \theta \gtrsim 0 \\ \frac{1}{2} C(\frac{\pi}{2} - \theta)^2 & \theta \lesssim \frac{\pi}{2} \ . \end{cases} \tag{11}$$

An estimate of the constant C, made following a procedure similar to the one used by Goldhaber and Teller[1] gives

$$C = \frac{32}{9\pi^2} A \frac{R}{r_o} v_o \delta^2 , \tag{12}$$

where R is the nuclear radius, r_o the range of the proton-neutron interaction, v_o the proton-neutron interaction energy and δ the Nilsson deformation parameter[2].

The eigenvalue equation for Φ_{0n} and Φ_{1n} in the region $\theta \gtrsim 0$ is

$$\left\{ - \frac{d^2}{d\theta^2} + \frac{K^2}{\theta^2} + (\frac{16}{15} K^2 + 2 \mathcal{J}_o C) \theta^2 - \left[4 \mathcal{J}_o E_{IKn} - I(I+1) + \frac{2}{3} K^2 \right] \right\} \varphi_{Kn} = 0 . \tag{13}$$

The same equation holds in the region $\theta \lesssim \frac{\pi}{2}$ when θ is replaced by $\frac{\pi}{2} - \theta$. The motion is localized in these two regions within a range of the order

$$\theta_K = \frac{1}{\sqrt{2 \mathcal{J}_o \omega_K}} \tag{14}$$

where

$$\omega_K = \sqrt{(C + \frac{8K^2}{15 \mathcal{J}_o}) \frac{1}{2 \mathcal{J}_o}} \sim 24 \delta A^{-1/6} . \tag{15}$$

Assuming[3]

$$\mathcal{J}_o = \frac{1}{2} \mathcal{J}_{rig} (1 + \frac{1}{3}\delta) = \frac{1}{5} m R^2 A (1 + \frac{1}{3}\delta) , \tag{16}$$

typical values of $\theta_K \sim \theta_o$ result to be of the order of a few percent.

This allows us to solve eq. (11) separately in the two regions $\theta \gtrsim 0$ and $\theta \lesssim \frac{\pi}{2}$, and to extend each region from zero to infinity.

The solutions which make the Hamiltonian (5) hermitean must satisfy the condition

$$\lim_{\theta \to (0, \frac{\pi}{2})} \left[\Phi^{(\sigma)}_{Kn} \frac{d}{d\theta} \Phi_{K'n'} - \Phi_{K'n'} \frac{d}{d\theta} \Phi_{Kn} \right] = 0 \quad . \tag{17}$$

For K = 0 the normalized solutions of eq. (13) satisfying the above conditions are

$$\varphi_{0n} = \left[2^{2n-1} (2n!) \sqrt{\pi}\, \theta_o \right]^{-1/2} H_{2n} \left(\frac{\theta}{\theta_o} \right) e^{-\frac{\theta^2}{2\theta_o^2}} , \tag{18}$$

with eigenvalues

$$E_{I0n} = (2n + \frac{1}{2})\, \omega_o + \frac{I(I+1)}{4\mathcal{J}_o} \quad . \tag{19}$$

It should be noted that only even Hermite polynomials appear as a consequence of eq. (17).

For $K \neq 0$ we have

$$\varphi_{Kn} = \left[\frac{2n!}{\Gamma(n+\varrho+1)} \; \frac{1}{\theta_o} \right]^{1/2} \left(\frac{\theta}{\theta_o} \right)^{\frac{1}{2}+\varrho} L_n^{\varrho} \left(\frac{\theta^2}{\theta_o^2} \right) e^{-\frac{\theta^2}{2\theta_o^2}} , \tag{20}$$

where L_n^{ϱ} are Laguerre polynomials and

$$\varrho = \frac{1}{2} \sqrt{1 + 4K^2} \tag{21}$$

with eigenvalues

$$E_{IKn} = (2n + \varrho + 1)\,\omega_o + \frac{I(I+1) - \frac{2}{3}K^2}{4\mathscr{I}_o} \qquad (22)$$

The approximate eigenfunctions in the full range $0 \leq \theta \leq \frac{\pi}{2}$ are

$$\Phi_{Kn} = \frac{1}{\sqrt{2}} \left[\varphi_{Kn}(\theta) + (-1)^K \varphi_{Kn}(\tfrac{\pi}{2} - \theta) \right] , \qquad (23)$$

showing how axial symmetry breaks down. The first excited states, having quantum numbers $K = 0$, $n = 1$ and $K = 1$, $n = 0$, fall at typical energies of the order of 5 MeV.

The $K = 0$, $n = 1$ excited states describe the rotational oscillations referred to at the beginning, while the $K = 1$, $n = 0$ excited states are associated with the other classical picture where the $\vec{\xi}_p$ and $\vec{\xi}_n$ axes form a fixed angle (no oscillation) in the intrinsic frame of reference, the angle being determined by the equilibrium between the centrifugal force and the neutron-proton attraction.

A microscopic calculation[5] based on the vibrating potential model predicts a state which could be interpreted as a rotational oscillation. In such a calculation, however, axial symmetry is assumed from the beginning. Had we also assumed axial symmetry, the agreement would be complete. But this assumption is inconsistent with the symmetry condition (10). Our model is indeed incompatible with a picture of the nucleus as a two-fluid system with axial symmetry.

REFERENCES AND FOOTNOTES.

(1) - M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948).

(x) - The possible occurrence of such an excitation mode was discussed by one of us (F. P.) at the Institute of Theoretical and Experimental Physics of Moscow in March 1975 and by R. Hilton in Dubna in June 1976. No quantitative treatment, however, has been done till now as far as we know.

(2) - See for instance: A. Bohr and B. R. Mottelson, Nuclear Structure (Benjamin, 1975), Vol. II, p. 47.

(3) - See ref. (2), p. 75.

(4) - I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series and Products (Academic Press, 1965).