To be submitted to
Lett. Nuovo Cimento

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-77/58(P)
22 Dicembre 1977

G. Dattoli and R. Mignani: FORMULATION OF ELECTRO-MAGNETISM IN A SIX DIMESNIONAL SPACE-TIME.

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-77/58(P)
22 Dicembre 1977

G. Dattoli[(x)] and R. Mignani[(o)] : FORMULATION OF ELECTROMAGNETISM IN A SIX DIMENSIONAL SPACE-TIME.

ABSTRACT

We give a preliminary formulation of electromagnetism in a six dimensional space-time with three time-coordinates. It is shown that such a formalism provides a further support to the hypothesis by Mignani and Recami that charged tachyons in an e.m. field behave as magnetic monopoles.

It has been recently suggested by Recami and one of the present authors (R. M.)[(1)] that the introduction of two extra time-coordinates can help to interpret the imaginary quantities entering superluminal Lorentz transformations (SLT)[(2)]. In such framework, time is assumed to be a vector in an Euclidean 3-dimensional space T^3, so that an event P is represented in the Euclidean six-dimensional space $M^6 = R^3 \times i\, T^3$ [(1, 3,4, 5)]:

$$P \equiv (x, y, z, it_x, it_y, it_z) \qquad (1)$$

(x) Address after Dec. 31, 1977: CNEN, Divisione Nuove Attività, Centro di Frascati.

(o) Istituto di Fisica dell'Università di L'Aquila, and INFN - Sezione di Roma.

or, alternatively, as a vector in a complex 3-dimensional space C^3(1, 3, 4):

$$P \equiv (x+it_x,\ y+it_y,\ z+it_z) . \qquad (2)$$

The lenght of P is given by the quadratic form(1, 3, 4):

$$P^2 = x^2 + y^2 + z^2 - (t_x^2 + t_y^2 + t_z^2) . \qquad (3)$$

As to interpreting physically the extra coordinates, one assumes that, for bradyons, only the modulus of the time vector is observable(3), whereas, for tachyons, the components of the position vector are assumed having no meaning individually(1).

Actually, it has been shown in ref. (1) that a SLT in M^6 is expected to effect transition from the six-vector $P = (x, y, z, it_x, it_y, it_z)$, to the corresponding six-vector $P' \equiv (t'_x, t'_y, t'_z, ix', iy', iz')$(6). In other words, a full symmetry is achieved between space and time(3) only considering SLT's, besides the usual Lorentz transformations (LT).

The problem of subluminal and superluminal LT's in six dimensional has been very recently discussed by some authors(7,8), both from a formal and a physical viewpoint. Although some questions are left open, it seems that the six dimensional approach can give insight into a fully understanding the space-time structure of physical theories(1 - 4, 7, 8) It seems therefore worthwhile generalizing the six-dimensional formalism to other physical quantities.

In the present paper, we want to do a first step in this direction by giving a preliminary formulation of electromagnetism in M^6. In particular, we shall give another formal proof that electrically charged tachyons are expected to behave as (space-like) magnetic monopoles(2, 9, 1).

First of all, let us assume that, if v_α ($\alpha = 1, \ldots, 6$) is any vector in M^6, subluminal (superluminal) observers are able to physically distinguish only between the first (second) three components, in the sense that only the modulus of the others is observable.

Then let us define the differential vector operator ∇_α (α = i, . . , 6) in M^6 as follows:

$$\nabla_\alpha \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, -i\frac{\partial}{\partial t_x}, -i\frac{\partial}{\partial t_y}, -i\frac{\partial}{\partial t_t} \right) \equiv (\nabla_r, -i\nabla_t) , \tag{4}$$

with self-clear meaning of the notations. By ∇_α, we can express all the operators of the vector analysis. For instance, the D'Alembert operator now reads(10, 11):

$$\diamond \equiv \nabla_\alpha \nabla_\alpha = \nabla_r^2 - \nabla_t^2 . \tag{5}$$

Obviously, $\diamond$ reduces to $\Box$ when we replace ∇_t^2 with $\frac{\partial^2}{\partial t^2}$.

Then, we define a six vector electromagnetic potential A_α as follows:

$$A_\alpha \equiv (A_x, A_y, A_z, i\varphi_x, i\varphi_y, i\varphi_z) . \tag{6}$$

Let us first do some considerations at a qualitative stage. By applying to A_α a SLT, we obtain(12)

$$A'_\alpha \equiv (\varphi_x, \varphi_y, \varphi_z, -iA_x, -iA_y, -iA_z) . \tag{7}$$

According to the above-made assumptions only $\varphi = (\varphi_x^2 + \varphi_y^2 + \varphi_z^2)^{1/2}$ and $A = (A_x^2 + A_y^2 + A_z^2)^{1/2}$ are measurable quantities for bradyons and tachyons, respectively. By taking into account that a SLT does'not induce a change in the dimensions of physical quantities and that ∇_r, $\frac{\partial}{\partial t}$ go into ∇_t, $\frac{\partial}{\partial \tau}$, respectively, we will eventually obtain:

$$\underline{E}' = \nabla_t \times \varphi , \qquad \underline{H}' = \frac{\partial \varphi}{\partial r} + \nabla_t A . \tag{8}$$

Moreover, in absence of "charges" and "currents", the propagation equations for $\underline{E}'$ and $\underline{H}'$ will read:

$$\tilde{\Box} \underline{E}' = 0 , \qquad \tilde{\Box} \underline{H}' = 0 , \tag{9}$$

where $\tilde{\Box}$ is defined as:

$$\tilde{\Box} \equiv \underset{\sim}{\nabla}_t^2 - \frac{\partial^2}{\partial \tau^2} . \qquad (9')$$

Let us now approach the problem in a more formal way, and define in six dimensional space-time the action S of a charged particle, interacting with an electromagnetic field A_α:

$$S = -m \int_1^2 d\tau - e \int_1^2 A_\alpha dx_\alpha , \qquad (10)$$

where τ is the proper time[13] and $dx_\alpha = (d\underset{\sim}{\tau}, i\, d\underset{\sim}{t})$, so that eq. (10) can be rewritten in the form:

$$S \equiv -m \int_1^2 d\tau - e \int_1^2 (\underset{\sim}{A} \cdot d\tau - \underset{\sim}{\varphi} \cdot dt) . \qquad (11)$$

Let us for simplicity neglect the first term. For a subluminal observer, S coincides with the usual definitions of action:

$$S = -e \int_1^2 (A \cdot d\tau - \varphi\, dt) . \qquad (12)$$

By applying a SLT (e.g., the trascendet one[2]), it is possible to derive the action S' for a superluminal observer (i.e., according to the duality principle[2] the action describing a "charged" tachyon in an electromagnetic field):

$$S' = -e \int_1^2 (\underset{\sim}{\varphi} \cdot d\underset{\sim}{t} - A\, d\tau) . \qquad (13)$$

From eq. (13) it is not a difficult task to derive the following Lagrangian:

$$\mathcal{L} = eA - e\underset{\sim}{\varphi} \cdot \underset{\sim}{\sigma} , \qquad (14)$$

where $\sigma = dt / d\tau$. By applying to L the time gradient $\underset{\sim}{\nabla}_t$ we obtain:

$$\nabla_t \mathcal{L} = e \nabla_t A - e \nabla_t (\varphi \cdot \sigma) = e \nabla_t A - e(\varphi \cdot \nabla_t)\sigma + e \sigma \times (\nabla_t \times \varphi) , \tag{15}$$

and therefore, by a standard procedure, one can write the Lorentz force on a tachyon as:

$$F = e \nabla_t A + e \frac{\partial \varphi}{\partial \tau} + e \sigma \times (\nabla_t \times \varphi) . \tag{16}$$

From equation (16) we derive the previous eqs. (8) for E' and H', in terms of which F now reads:

$$F = e(H' + \sigma \times E') . \tag{17}$$

It can be immediately deduced from eq. (17) that an infinite speed tachyon behaves as a purely magnetic charge a rest (in the sense that it only "feels" the magnetic field)[14].

Now we can straightforwardly derive from eqs. (8) the first group of the "Maxwell" equations:

$$\nabla_t \cdot E' = 0 , \qquad \nabla_t \times H' = \frac{\partial E'}{\partial \tau} , \tag{18}$$

in a similar way, one can argue that the second group of the "Maxwell" equations has the form:

$$\nabla_t \times E' = - \frac{\partial H'}{\partial \tau} + \varrho , \qquad \nabla_t \cdot H' = - J , \tag{19}$$

where we introduced the current four vector $\varrho = (\varrho_x, \varrho_y, \varrho_z, iJ)$. In absence of "charges" and "currents", we have:

$$\nabla_t \times E' = - \frac{\partial H'}{\partial \tau} , \qquad \nabla_t \cdot H' = 0 , \tag{20}$$

from which one easily derives the propagation equations (eqs. (9)).

As a final remark, let us remind that the interpretation of extra timelike coordinates adopted by us suffers from some difficulties (e. g. non linearity of the transformation equations between four vector components)[7].

To overcome such a problem, a very interesting proposal has been recently formulated by Cole[7], i.e. an averaging about the time directions (in the bradyonic case). The changes in the six-dimensional formulation of electromagnetism, needed by the time-average approach, will be discussed by the present authors in a forthcoming paper.

Acknowledgments.

After the completetion of the present work, we become acquainted with a paper by Višyn[15], where very similar results are obtained. We are very grateful to the Author for having brought to our attention his paper, which has confirmed to us that our ideas were not wrong at all.

Thanks are also due to prof. E. Recami for the advice of the book by Kalitzin.

REFERENCES AND NOTES. -

(1) - R. Mignani and E. Recami, Lett. Nuovo Cimento 16, 449 (1976).

(2) - See E. Recami and R. Mignani, Riv. Nuovo Cimento 4, 209 (1974), and references therein.

(3) - P. Demers, Can. J. Phys. 53, 1687 (1975).

(4) - See also N. Kalitzin, Multitemporal Theory of Relativity, R.G. Zaikov and D.G. Fakirov, Eds. (Bulgarian Academy of Sciences, Sofia, 1975).

(5) - Natural units $c = \hbar = 1$ are used throughout.

(6) - Here, the primed components are the ones one would have got by the dual subluminal transformation of the SLT considered. For the meaning of duality in extended relativity, see ref. (2).

(7) - E.A.B. Cole, Nuovo Cimento 40A, 171 (1977); Nuovo Cimento, to appear.

(8) - P.T. Pappas, Int. J. Theor. Phys., to appear; G. Ziino, Nuovo Cimento, to appear.

(9) - R. Mignani and E. Recami, Lett. Nuovo Cimento 9, 367 (1974); 11, 417 (1974); Nuovo Cimento 30A, 533 (1976); E. Recami and R. Mignani, Lett. Nuovo Cimento 9, 479 (1974); Phys. Letters 62B, 41 (1976); in 'The Uncertainty Principle and Foundations of Quantum Mechanics', Price and Chissick, Eds. (Wiley, 1977).

(10) - J. Kaluza, Zum Unitatsproblem der Physik (Sitz. Preuss. Akad. Wiss., 1921), p. 966.

(11) - The "exagonal" notation is a generalization of the Kaluza-Einstein symbol ⬡ for the five-dimensional D'Alembert operator. See ref. (10).

(12) - Modulo a Lorentz transformation.

(13) - Let us remind that only the proper time, and not ds, is invariant under SLT's. See ref. (2).

(14) - By the way, this accords with the duality principle and its interpretation for a charged world as "electromagnetic" duality. See refs. (2, 9).

(15) - V. Vyšin, Submitted to Nuovo Cimento.