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UNIFIED FIELD THEORIES WITH U(N) INTERNAL SYMMETRIES: GAUGING THE SUPERCONFORMAL GROUP

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Gauge theories for extended SU(N) conformal supergravity are constructed which are invariant under local scale, chiral, proper conformal, supersymmetry and internal SU(N) transformations. The relation between intrinsic parity and symmetry properties of their generators of the internal vector mesons is established. These theories contain no cosmological constants, but technical problems inherent to higher derivative actions are pointed out.

1. Introduction

Supergravity theory [1] in ordinary four-dimensional space-time, by which we mean the gauge theory of the graded Poincaré or de Sitter group, has the great advantage over super-Riemannian [2] theory in that it is formulated in terms of relati-

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vely few fields. Matter, however, is not unified with the spin-2 and spin-$\frac{3}{2}$ gauge fields of supergravity from the start but must be added, in a locally supersymmetric way.

Investigations into the renormalizability of such matter-supergravity systems have revealed that they consist of two classes: (i) models which are at least one- [3] and two-loop [4] finite and contain a global internal O(N) symmetry (for example, the O(2) model [5]), and (ii) non-renormalizable models such as the Maxwell-Einstein system [6]. The O(N) internal symmetry can be gauged provided a cosmological term and an apparent mass term for the spin-$\frac{3}{2}$ gauge field are suitably added to the Lagrangian [7]. However, in this case the corresponding field theory does not have a direct particle interpretation in Minkowski space IO(3,1) but rather in O(3,2) de Sitter space [8]. From a pure geometrical point of view, these O(N) theories can be regarded as gauge theories of the simple graded Lie algebra OSp(4, N) [9] and in fact two of them (N = 1,2) have been re-obtained using an entirely geometrical procedure [10.11]. After a group contraction (in the limit of infinite de Sitter radius!) O(3,2) limits to IO(3,1) and O(N) to $\frac{1}{2}N(N-1)$ central U(1) charges. In this way, the usual supergravity theories (with the non-semisimple graded Poincaré group as the gauged group) can be re-derived.

The change in the internal symmetry of the gauged group can be understood simply by noting that the dimensionless O(N) gauge coupling g, given by the ratio of the Newton constant and of the Sitter radius, vanishes after group contraction. For $N > 8$ the O(N) models contain particles with helicities exceeding 2 and it is not clear whether such theories are viable. For $n \leqslant 8$, on the other hand, the groups O(N) do not contain SU(3) $\otimes$ SU(2) $\otimes$ U(1) as a subgroup which is commonly thought to be the minimal gauge group of the non-gravitational interactions. Consequently it becomes a serious problem for supergravity to be regarded as a truly unified theory of nature. Another problem is connected with the presence of a cosmological term which does not allow a standard quantization procedure for the class of theories with a gauged O(N) symmetry.

In this paper we present an investigation which might be a step towards solving these problems. It is known that the graded conformal group allows U(N) internal symmetries rather than O(N) [12]. Gauging these SU(2,2|N) graded groups [9] for $N \geqslant 5$ one would obtain supersymmetric actions with an internal SU(3) $\otimes$ SU(2) $\otimes$ U(1) gauged symmetry. Moreover, in these theories no cosmological term is present and quantization in asymptotically flat space is possible.

These models look particularly interesting because a complete fusion is achieved among internal and space-time symmetries. In fact the U(N) dimensionless coupling constant coincides with the dimensionless gravitational coupling of the Weyl action, which describes the pure spin-2 sector of these gauged superconformal theories. This is to be contrasted with gauged OSp(4, N) graded Lie groups in which two unrelated couplings are present: the Newton constant and the de Sitter radius.

Recently three of us constructed a gauge theory for U(1) superconformal group [13]. The resulting complete action contains the Weyl action for the spin-2 part, together with terms describing one spin-$\frac{3}{2}$, one axial vector boson and a spin-$\frac{1}{2}$ fermion.

The occurrence of the higher derivative Weyl action is expected, as it is the only locally scale invariant gravitational action without dimensional constants. Although it is manifestly renormalizable [14], it contains ghosts and, from the known relations between non-unitarity and non-renormalizability in gauge theories, we realize that such theories are equivalent to non-renormalizable theories. Nevertheless, if the ghost problem could be dealt with in the future, these theories might become realistic.

We present a gauge theory for the graded conformal group with U(N) internal symmetry, following the approach of MacDowell and Mansouri [10]. This implies that we gauge the conformal and superconformal generators K and S, as well as the generators of chiral transformations A, dilatations D and the graded Poincaré group M, P, Q. In this sense our approach differs from that of Freund et al. [15] who not gauge K and S. Another difference is that we work in ordinary space, as a result of which we do not obtain the large number of fields due to expanding the scale parameters in fermionic co-ordinates. However, as in ref. [13], we find that the gauge fields of K and S (and of course of M) are non-propagating and can be eliminated explicitly from the theory. The gauge theory below thus follows straightforwardly from the graded Lie algebra of the group SU(2,2|N), the only non-trivial aspect being the choice of constraints and their solution such that they are compatible with the equations of motion.

Our conventions are $\eta_{ab} = (+++-)$, $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$, where γ^a are real and the charge conjugation matrix $C = \gamma^0$. $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ with $\gamma_4 = i\gamma_0$, hence γ_5 is imaginary, antisymmetric, hermitian, and $\gamma_5^2 = +1$. We emphasise that this article draws heavily upon ref. [10] and is a continuation of ref. [13]. Many of the details used below are explained there.

2. Graded Lie algebra of SU(2,2|N)

The generators of SU(2,2|N) consist of the 15 generators P^a, K^a, M^{ab} and D of the ordinary conformal group C, together with the generators A of U(1) and G^i of SU(N), and finally, the $8N$ spinorial charges Q^i_α and S^i_α with $i = 1, N$ and $\alpha = 1,4$. An explicit $(4+N) \times (4+N)$ matrix representation is given by matrices whose diagonals consist of (4×4) and $(N \times N)$ blocks. For the bosonic generators [16]

$$C = \left(\begin{array}{c|c} P, K, M, D & 0 \\ \hline 0 & 0 \end{array}\right), \qquad A = -\tfrac{1}{4}i \left[\begin{array}{c|c} 1 & \\ \hline & \frac{4}{N}1 \end{array}\right], \qquad G^i = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \tau^i \end{array}\right), \tag{1}$$

with

$$P^a = -\tfrac{1}{2}\gamma^a(1-\gamma_5), \qquad K^a = \tfrac{1}{2}\gamma^a(1+\gamma^5), \qquad M^{ab} = \tfrac{1}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a) \equiv \sigma^{ab},$$

$$D = -\tfrac{1}{2}\gamma_5 .$$

The $N \times N$ matrices τ^i are generalizations of the usual SU(2) and SU(3) representations. The $(N-1)N$ off-diagonal matrices have only two non-zero entries while the normalization of the N diagonal matrices is explained after eq. (6). They are of the form:

$$\tau^i = \begin{bmatrix} & -\frac{1}{2}i \\ -\frac{1}{2}i & \end{bmatrix}, \quad \begin{bmatrix} & -\frac{1}{2} \\ +\frac{1}{2} & \end{bmatrix}, \quad \frac{1}{\sqrt{2}}\begin{bmatrix} i/N & & & 0 \\ & \ddots & i(1-N)/N & \\ & & & \ddots \\ 0 & & & i/N \end{bmatrix} \tag{2}$$

except for the case $N = 2$, where the traceless diagonal matrix has entries $(-\frac{1}{2}i, +\frac{1}{2}i)$. The matrices Q^i_a, S^i_a have non-vanishing entries only in the $(4+i)^{\text{th}}$ row and column where they are given by

$$Q^i_\alpha = \left[\begin{array}{c|c} & -[\frac{1}{2}(1+\gamma_5)C]_{.\alpha} \\ \hline \frac{1}{2}(\gamma_5 - 1)_{\alpha.} & \end{array}\right], \qquad (S^i_\alpha) = -(Q^i_\alpha)^* . \tag{3}$$

The dot takes the value n in the nth column and row. All generators have zero graded trace [9]. For $N = 4$, A becomes a central charge, appearing in anticommunication relations for $\{Q, S\}$. This result is known [12].

From the (anti) commutation relations we abstract the graded Lie algebra for SU(2,2$|N$). For the bosonic sector one finds

$$\begin{aligned}
&[M^{ab}, P^c] = \eta^{bc}P^a - \eta^{ac}P^b , \qquad \text{idem for } K , \\
&[M^{ab}, M^{cd}] = \eta^{bc}M^{ad} + 3 \text{ more terms} , \\
&[P^a, D] = P^a , \qquad [K^a, D] = -K^a , \\
&[P^a, K^b] = 2(\eta^{ab}D - M^{ab}) .
\end{aligned} \tag{4}$$

The boson and fermion generators satisfy the relations

$$\begin{aligned}
&[S^m_b, P^a] = (\gamma^a)^c_b Q^m_c , && [S^m_a, A] = \tfrac{1}{4}i\left(\frac{4}{N} - 1\right)(\gamma^5)^b_a S^m_b , \\
&[Q^m_b, K^a] = (\gamma^a)^c_b S^m_c , && [Q^m, A] = \tfrac{1}{4}i\left(\frac{4}{N} - 1\right)(\gamma^5)^b_a Q^m_b , \\
&[S^m_a, D] = -\tfrac{1}{2}S^m_a , && [Q^m_a, M^{bc}] = (\sigma^{bc})^d_a Q^m_d , \\
&[Q^m_a, D] = \tfrac{1}{2}Q^m_a , && [S^m_a, M^{bc}] = (\sigma^{bc})^d_a S^m_d , \\
&[Q^m_a, G^i] = (\gamma_5)^b_a (\tau^i_S)^{mn} Q^n_b + (\tau^i_A)^{mn} Q^n_a , \\
&[S^m_a, G^i] = -(\gamma_5)^b_a (\tau^i_S)^{mn} S^n_b + (\tau^i_A)^{mn} S^n_a ,
\end{aligned} \tag{5}$$

while for the anticommutators one finds

$$\{Q^m_c, Q^n_d\} = -\tfrac{1}{2}(\gamma^a C)_{cd}\, \delta^{mn} P_a ,$$

$$\{S^m_c, S^n_d\} = \tfrac{1}{2}(\gamma^a C)_{cd}\delta^{mn}K_a ,$$

$$\{Q^m_c, S^n_d\} = \delta^{mn}[-\tfrac{1}{2}C_{cd}D + (\sigma^{ab}C)_{cd}M_{ab}] + (i\gamma_5 C)_{cd}\delta^{mn}A$$
$$+ (i\gamma_5 C)_{cd}(2i\tau^i_S)^{mn}G^i_S + C_{cd}(-2\tau^i_A)^{mn}\, G^i_A . \qquad (6)$$

Summation over pairs (ab) is for $a > b$ only. The only N dependence is in the $[Q, A]$ and $[S, A]$ commutators since the N dependence in $\{Q, S\}$ is eliminated by our choice of normalization of A and τ^i. The indices S and A in the last commutator indicate that only (anti) symmetric matrices τ^i and generators G^i are to be taken. Clearly, $\frac{1}{2}N(N-1)$ internal symmetry generators have even intrinsic parity; they correspond to the residual O(N) symmetry of the graded subalgebra OSp(4, N) which is contained in SU(2,2|N) [17]. The remaining $\frac{1}{2}N(N+1)$ generators with symmetric structure constants in (m, n) have odd intrinsic parity.

More manifestly covariant formulae are obtained by considering the chiral projections of Q and S

$$Q^m_{a,\mathrm{L}} = \tfrac{1}{2}[1 + \gamma_5]_{ab}\, Q^m_b , \quad Q^m_{a,\mathrm{R}} = \tfrac{1}{2}[1 - \gamma_5]_{ab}\, Q^m_b , \qquad \text{idem } S_\mathrm{L}, S_\mathrm{R} . \qquad (7)$$

In terms of these, one finds that Q_L and S_R (and Q_R and S_L) commute while

$$[Q^m_{a,\mathrm{L}}, G^i] = (\tau^i)^{mn}\, Q^n_{a,\mathrm{L}} , \qquad [S^m_{a,\mathrm{L}}, G^i] = -(\tau^i)^{nm}\, S^n_{a,\mathrm{L}} ,$$

$$[Q^m_{a,\mathrm{R}}, G^i] = -(\tau^i)^{nm}\, Q^m_{a,\mathrm{R}} , \qquad [S^m_{a,\mathrm{R}}, G^i] = (\tau^i)^{nm}\, S^n_{a,\mathrm{R}} ,$$

$$\{Q^m_{a,\mathrm{L}}, S^n_{b,\mathrm{L}}\} = \tfrac{1}{2}(1 + \gamma_5)[\delta^{mn}(-\tfrac{1}{2}CD + \sigma^{cd}CM_{cd} + CiA) - 2(\tau^i)^{mn}\, CG^i]_{ab} ,$$

$$\{Q^m_{a,\mathrm{R}}, S^n_{b,\mathrm{R}}\} = \tfrac{1}{2}(1 - \gamma_5)[\delta^{mn}(-\tfrac{1}{2}CD + \sigma^{cd}CM_{cd} - CiA) + 2(\tau^i)^{nm}CG^i]_{ab} . \qquad (8)$$

Thus Q^m_L and S^m_R transform according to the vector representation of SU(N), and Q^m_R and S^m_L transform according to the complex conjugated vector representation. For these complex charges we define a barred symbol as usual: $\bar{Q}^m_\mathrm{L} = (Q^m_\mathrm{L})^\dagger\gamma^4$. For Majorana charges Q^m one then has the relation

$$C\bar{Q}^{m\,\mathrm{T}}_\mathrm{R} = iQ^m_\mathrm{L} , \qquad C\bar{Q}^{m\,\mathrm{T}}_\mathrm{L} = iQ^m_\mathrm{R} , \qquad \text{idem } S^m . \qquad (9)$$

3. Curvatures

From the structure constants we form the curvatures,

$$R^A_{\mu\nu} = -\partial_\mu h^A_\nu + \partial_\nu h^A_\mu + f^A_{BC}\, h^B_\nu h^C_\mu ,$$

$$[O_A, O_B\} = f_{BA}{}^C O_C ,$$

$$h^A_\mu = e^a_\mu, f^a_\mu, \omega^{ab}_\mu, B_\mu, \bar{\psi}^{a,m}_\mu, \bar{\varphi}^{a,m}_\mu, A_\mu, \Lambda^i_\mu ,$$

$$O_A = P^a, K^a, M^{ab}, D, Q^m_a, S^m_a, A, G^i . \tag{10}$$

(see table 1). The action we take is the most general geometrical, parity-conserving action without dimensional constant, to which we add the Maxwell actions for the internal symmetry gauge fields.

$$I = \int d^4x\, \epsilon^{\mu\nu\rho\sigma} [\alpha(R_{\mu\nu}(M)^{ab} R_{\rho\sigma}(M)^{cd})\, \epsilon_{abcd} + \beta(R_{\mu\nu,\mathrm{L}}(S)^{ma} R_{\rho\sigma,\mathrm{L}}(Q)^{mb}$$
$$+ R_{\mu\nu,\mathrm{R}}(S)^{ma} R_{\rho\sigma,\mathrm{R}}(Q)^{mb})\, (\gamma_5 C)_{ab} + \gamma R_{\mu\nu}(A)\, R_{\rho\sigma}(D)]$$
$$+ \int d^4x \sqrt{-g}\, g^{\mu\rho} g^{\nu\sigma} [\delta R_{\mu\nu}(A)\, R_{\rho\sigma}(A) + \eta R_{\mu\nu}(G^i)\, R_{\rho\sigma}(G^i)] . \tag{11}$$

This is the same action as in the U(1) case [13] except for the Maxwell terms for the SU(N) generators.

The curvatures transform under gauge transformations according to the adjoint

Table 1
Table of curvatures for SU(2,2|N)

$$R^P_{\mu\nu} = -2(\partial_\mu e^a_\nu - \omega^{ad}_\mu e_{d\nu}) + \tfrac{1}{2}\bar{\psi}^n_\mu \gamma^a \psi^n_\nu + 2e^a_\mu B_\nu ,$$

$$R^K_{\mu\nu} = -2(\partial_\mu f^a_\nu - \omega^{ad}_\mu f_{d\nu}) - \tfrac{1}{2}\bar{\phi}^n_\mu \gamma^a \phi^n_\nu - 2f^a_\mu B_\nu ,$$

$$R^{L\,ab}_{\mu\nu} = -2(\partial_\mu \omega^{ab}_\nu + \omega^{ac}_\nu \omega^{\ b}_{\mu c}) - 8e^a_\mu f^b_\nu - 2\bar{\psi}^n_\mu \sigma^{ab} \phi^n_\nu ,$$

$$R^D_{\mu\nu} = -\partial_\mu B_\nu + 4e^a_\mu f_{a\nu} + \bar{\psi}^n_\mu \phi^n_\nu ,$$

$$R^A_{\mu\nu} = -\partial_\mu A_\nu + \partial_\nu A_\mu - 2i\bar{\psi}^n_\mu \gamma_5 \phi^n_\nu ,$$

$$R_{\mu\nu,\mathrm{L}}(Q)^n = \Big(D_\nu \bar{\psi}^n_\mu - D_\nu \bar{\psi}^n_\nu + \bar{\phi}^n_\mu \gamma_\nu - \bar{\phi}^n_\nu \gamma_\mu + B_\nu \bar{\psi}^n_\mu$$
$$- \tfrac{1}{2} i \left(\frac{4}{N} - 1\right) A_\nu \bar{\psi}^n_\mu + 2\Lambda^i_\nu \bar{\psi}^m_\mu (\tau^i)^{mn}\Big) \tfrac{1}{2}(1 + \gamma_5) ,$$

$$R_{\mu\nu,\mathrm{L}}(S)^n = \Big(D_\nu \bar{\phi}^n_\mu - D_\mu \bar{\phi}^n_\nu - \bar{\psi}^n_\mu \gamma_a f^a_\nu + \bar{\psi}^n_\nu \gamma_a f^a_\mu - B_\nu \bar{\phi}^n_\mu$$
$$+ \tfrac{1}{2} i \left(\frac{4}{N} - 1\right) A_\nu \bar{\phi}^n_\mu - 2\Lambda^i_\nu \bar{\phi}^m_\mu (\tau^i)^{nm}\Big) \tfrac{1}{2}(1 + \gamma_5)$$

$$R^{\Lambda^i}_{\mu\nu} = -\partial_\mu \Lambda^i_\nu + \partial_\nu \Lambda^i_\mu + f^i_{kj} \Lambda^j_\nu \Lambda^k_\mu - \bar{\psi}^m_\nu (1 + \gamma_5)\, \phi^n_\mu (\tau^i)^{mn} + \bar{\psi}^m_\nu (1 - \gamma_5)\, \phi^n_\mu (\tau^i)^{nm} .$$

All expressions are to be antisymmetric in μ, ν and $[\tau_i, \tau_j] = f^k_{ij} \tau_k$. Note that f, B and ω are non-propagating fields.

representation

$$\delta h_\mu^A = D_\mu \epsilon^A \equiv \partial_\mu \epsilon^A + f_{BC}^A h_\mu^B \epsilon^C \,,$$
$$\delta R_{\mu\nu}^A = f_{BC}^A R_{\mu\nu}^B \epsilon^C \,. \tag{12}$$

It follows at once that the action is invariant under M, D, A and G^i transformations. It is not invariant under P but it is invariant under general co-ordinate transformations by construction, whose relation to P transformations with parameter $\epsilon^P \equiv \xi^a$ is given by

$$\delta_{\text{gen}}(\xi)_{\text{coord}} h_\mu^A = D_\mu(h_\nu^A \xi^\nu) + R_{\mu\nu}^A \xi^\nu \,. \tag{13}$$

As in the U(1) case, one can impose the following constraints:

$$R_{\mu\nu}^P = 0\,, \qquad R_{\mu\nu}^Q + \tfrac{1}{2}\gamma_5 \widetilde{R}_{\mu\nu}^Q = 0\,, \tag{14}$$

which expresses ω in terms of e, ψ and B, and ϕ_μ^n in terms of ψ_μ^n, $\omega(e, \psi, B)$, e, B, A and a new spin-$\frac{1}{2}$ field χ^n. The transformation law of the no longer independent fields ω and ϕ_μ^n under M, A, D, K, S and G^i transformations is the same as the original gauge transformation law in (12), provided one fixes the transformation laws for χ^n appropriately. It follows that one can use the transformation laws of curvatures in (12) to investigate the invariance of (11) under M, A, D, K, S and G^i gauge transformations. Invariance under M, A, D, K and G^i is manifest, while in the variation δI under S transformations the M, Q curvatures cancel if $\beta = -8\alpha$ and the Q, D curvatures cancel if $\beta = 2i\gamma$. The A, Q terms obtained from the β, γ and δ terms cancel upon using (14) if $\delta = -8\alpha/N$. The $R(\Lambda^i)\,R(Q)$ terms cancel if $\beta = 2\eta$.

Thus the action in (11) is M, A, D, K, S and G^i gauge invariant as well as generally co-ordinate invariant if

$$\beta = -8\alpha = 2i\gamma = \delta N = 2\eta\,. \tag{15}$$

The field of special conformal gauge transformations can be eliminated from the action upon using its field equation. In the purely bosonic conformal group the resulting action was the Weyl action $R_{\mu\nu}^2 - \frac{1}{3}R^2$ [13]. Here the f^2 and $efR(A)$ terms in $R(L)R(L)$ and $R(A)R(D)$ in the action contribute also to the kinetic terms of the axial boson. After eliminating the spin-$\frac{1}{2}$ fields χ^n from the action by fixing the S gauges appropriately [13], the kinetic terms for the A, Λ^i, e and ψ fields are found to be

$$\mathcal{L}^{\text{kin}} = 2\alpha\left(1 - \frac{4}{N}\right) F_{\mu\nu}^2(A) - 4\alpha\, F_{\mu\nu}^2(\Lambda^i)$$
$$+ 8\alpha(R_{\mu\nu}^2 - \tfrac{1}{3}R^2) - 4\alpha\bar{\psi}_\mu(\partial\!\!\!/ \Box g_{\mu\nu} - \partial\!\!\!/ \partial_\mu\partial_\nu + \epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\Box)\,\psi_\sigma\,. \tag{16}$$

For $N < 4$ the U(1) field has the same sign of kinetic term as the SU(N) fields. For

$N = 4$ there is no kinetic term for A, and ψ and ϕ are U(1) singlets. Therefore, there is no residual U(1) symmetry for $N = 4$, since only terms of the form $F_{\mu\nu}(A)\,\bar{\psi}\phi$ could remain.

4. Conclusions

We have obtained the gauge theories of extended U(N) conformal supergravity. This allows the application of supergravity to real nature, since the internal symmetry groups can describe weak, electromagnetic and strong unified field theories, but first it should be understood how to do quantum field theory with higher derivative actions. These theories are invariant under local dilatations, local proper conformal boosts, local chiral rotations, local U(N) internal transformations and local S supersymmetry transformations, and of course under local Lorentz and general co-ordinate transformations. The algebra of the corresponding generators K, D, M, S, A and G^i closes. No non-geometrical terms were needed, only the two constraints $R(P) = R(Q) + \frac{1}{2}\gamma_5\widetilde{R}(Q) = 0$. Hence in these theories matter is truly unified with gauge fields, unlike in the O(N) models in de Sitter space, where nongeometrical terms in the transformation law for ψ are needed [11].

The gauge fields ω, f, ϕ and B are non-propagating and were eliminated explicitly from the action. This was expected, since moments of local currents yield in general non-propagating fields, and M, K and D are related to the energy momentum Noether tensor and S to the Q supersymmetry current through moments of x^μ [18, 19]. Explicit spin decomposition [20] of the spin-2 (Weyl) and (higher derivative) spin-$\frac{3}{2}$ kinetic terms reveals that they contain only spin-2 and -$\frac{3}{2}$ and not lower spin gauge components. Again this is to be expected, since globally conformal actions have the property of containing only the highest possible spin [21].

We do not have (yet?) Q supersymmetry invariance. By adding curvatures to the local P transformations, one obtains general co-ordinate transformations and it might likewise be possible to obtain local Q invariance by adding curvatures to the local Q transformation laws. In superspace, P and Q play the same role, but here this is not so sure. There is only one spin-$\frac{3}{2}$ field in the action, but one can have local invariances without accompanying gauge field (K, M, D, for example, in our case). Hence the situation with Q is not clear at the moment. In SO(2) supergravity one needs no non-geometrical spin-$\frac{1}{2}$ terms in the action, so that we expect our action to be unique at least for U(2). For U(N) with $N > 2$ full Q invariance presumably implies that non-gauge particle fields must be suitably added to our starting Lagrangian given by eq. (11).

For $N = 4$ the explicit matrix representation given in the text shows that the U(1) field A yields a central charge: it is present in the $\{Q, S\}$ anticommutator. However, one can omit central charges from a (graded) Lie algebra without violating the Jacobi identities. Hence the possibility for $N = 4$ to have SU(4) or U(4) can also be understood from this point of view. But our models can only realize SU(4).

A most interesting aspect is the counting of states. In the U(N) theories one has one spin-2 and N spin-$\frac{3}{2}$ higher derivative fields, one axial boson A from the chiral transformation, N spin-$\frac{1}{2}\chi^n$ as residue from ϕ_μ^n after solving the Q constraints, and $(N^2 - 1)$ vector bosons whose parity is even or odd depending on whether their generators are antisymmetric or symmetric. One should not attempt to put these fields in spin $(J, J + \frac{1}{2})$ multiplets since one deals with higher derivative theories. For example, the scalar multiplet with fields A, B, ψ, F and G becomes, upon adding a gradient to all fields, a higher derivative theory in which the scalars F and G become propagating and whose particle content is a higher derivative spin $(\frac{1}{2}, 0, 0)$ multiplet and two propagating spin-0 bosons only. In fact, one might turn the argument around and speculate that the axial vector boson A corresponds to a non-propagating field in ordinary supergravity. Signals that there is such an axial vector auxiliary field are known [22].

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