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A. Turrin: USE OF THE BLOCH-VECTOR MODEL IN TAPERED COUPLING PROBLEMS.

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ABSTRACT.

The formal similarity of the first-order coupled-mode equations used in two-mode coupled theory and the Schrödinger equation of a perturbed two-level atom in the rotating-wave approximation is recognized. The use of the Bloch equations in a formal description of the exchange of power between the two modes is proposed.

Potentially the most attractive method of interconnecting different components in an integrated optical circuit is distributed coupling.

Analytic calculations[1-3] of the coupler efficiency suggest that the method is feasible; however, more detailed calculations will be required to permit more precise estimates for accurate design of tapered couplers.

In their recent derivation of the coupler efficiency, Matsuhara et al.[1], Arnaud[2] and Smith[3] restricted the analysis to the case where the coupling coefficient c(z) is constant and the mismatch $\Delta(z) = \beta_1(z) - \beta_2(z)$ between the propagation coefficients is a linear function of the axial distance z along the coupler. Their choice of c(z) and $\Delta(z)$ was limited, of course, to functions that render the coupled-wave equations reducible to a known type of differential equation. The corresponding differential equations for the nearly stationary modeamplitudes $A_{1,2}(z)$ were solved in terms of special functions and were expanded asymptotically in z, leading to a very simple expression for the asymptotic coupler efficiency $\eta(+\infty)$.

Analytic techniques applied to more generalized analogous cases[4, 5, 6] are unlikely to succeed if actual strong nonlinearities are to be introduced into the coupled-mode equations.

In this note, we focus our attention on the close analogy between population inversion induced in a two-level atom by chirping a laser field through the resonance frequency and power transfer in a two-mode coupler. Then, following the quantum-mechanical analogy, we can define a Bloch vector for the coupler. Within the framework of the vector model of the coupler, the efficiency $\eta(z)$ can be described quite simply, and in this representation tapered coupler problems can more readily be solved in terms of three **real** unknown functions.

The first-order coupled-mode equations for the nearly stationary modeamplitudes $A_{1,2}$ of the electromagnetic field in a coupler are

$$-idA_1/dz=(\Delta/2)A_1+cA_2 \quad , \quad -idA_2/dz=-(\Delta/2)A_2+cA_1 \qquad (1)$$

where we take both $c=c(z)$ and $\Delta = \Delta(z)$ to be real.

Now the quantum-mechanical equations for the two-level atomic dynamics[7] in the rotating-wave approximation and Eqs. (1) are in fact of exactly the same form.

Just as in the two-level atom case, it is convenient to define the three-component real vector[7] $\bar{r}= X\bar{i} + Y\bar{j} + Z\bar{k}$

$$X=A_1A_2^*+A_2A_1^*, \quad Y=i(A_1A_2^*-A_2A_1^*), \quad Z=A_1A_1^*-A_2A_2^*. \qquad (2)$$

Note that

$$r^2=X^2+Y^2+Z^2=A_1A_1^*+A_2A_2^* \qquad (3)$$

is just the sum of the power in the two modes ($P_i=A_iA_i^*$; $i=1,2$). Thus the magnitude r of $\bar{r}$ is constant.

From Eqs. (2) and (1) we get

$$dX/dz=\Delta Y, \quad dY/dz=-\Delta X+2cZ, \quad dZ/dz=-2cY, \qquad (4)$$

i. e. we obtain the "Bloch equations" for the tapered coupler, and is perhaps worth drawing attention to the fact that Eqs. (4) can be transformed back[8] into Eqs. (1) by use of suitable transformations.

The similarity of Eqs. (4) and the optical Bloch equations is clear if we notice that in Eqs. (4) the time is replaced by z. Δ finds its analogy in the "detuning" and 2c in "$p\,\mathcal{E}/\hbar$" (p is the dipole transition matrix element between the two states and $\mathcal{E}(t)$ is the field envelope of the laser pulse).

With this is mind, the power exchange between the two modes is described by the evolution of a Bloch vector $\bar{r}$ "precessing" around the vector

$$\bar{\omega} = -2c\,\bar{i} - \Delta\,\bar{k} \qquad (5)$$

in a fictitious space $\bar{i}, \bar{j}, \bar{k}$.

With the normalization condition $r^2 = P_{tot} = P_1 + P_2 = 1$, the precession "frequency" of the Bloch vector is the magnitude ω of $\bar{\omega}$, and we have

$$Z = P_1 - P_2 = 2P_1 - 1 = 1 - 2P_2, \qquad (6)$$

so that the initial conditions $P_1(-\infty)=0$, $P_2(-\infty)=1$ give $Z(-\infty)=-1$, and the coupler efficiency $\eta(z)$ becomes

$$\eta(z) = P_1(z) = (1/2)(1+Z). \qquad (7)$$

An intuitive grasp of the Bloch-vector motion is possible, as Froissart and Stora[9] and Treacy[10] have shown. Suppose the Bloch vector $\bar{r}$ and $\bar{\omega}$ to be initially ($z=-\infty$) along $-\bar{k}$ (i. e. $X=0, Y==0, Z=-1$; $c(-\infty)=0$, $\Delta(-\infty) > 0$).

If the variations of $c(z)$ and $\Delta(z)$ remain sufficiently slow so that $\bar{\omega}$ can change direction only a negligible amount in one precession "period," then $\bar{r}$ precessing around $\bar{\omega}$ remains along $\bar{\omega}$ all the "time" z. Moreover, in the case where there exists a cross-over point (i. e. in the case where $\Delta(z)$ is made to change "slowly" from a positive value to a negative value) we can flip the Bloch vector ($Z(+\infty) \simeq 1$), provided that $c(+\infty)=0$. Physically, this means an almost complete exchange of power between the two modes. Obviously in the other extreme case where $\Delta(z)$ changes abruptly at the cross-over point, the Bloch vector is not able to follow $\bar{\omega}$ and flip cannot occur.

A complete and systematic discussion of the Bloch equations in the adiabatic following limit (we are mainly interested in) can be found in Section II of a 1975 paper by Lehmberg and Reintjes[11]. Figures 3 and 7 of the quoted Reference[11] show solutions for X, Y, Z for the case analogous to the one in which a linear variation of $\Delta(z)$ and a gaussian change in c(z) is considered.

In summary, we have shown that the efficiency of a two-mode coupler can be evaluated by solving the "Bloch equations" (Eqs. (4)), where the three components of the "Bloch vector" (Eqs. (2)) are real functions of the complex slowly-varying modeamplitudes. The efficiency $\eta(z)$ of the tapered coupler is closely related to the Z component of the Bloch vector (Eqs. (7)). This alternative set of equations, which gives a geometrical representation of the complex first-order coupled-mode equations, may be used as a convenient and useful tool for design of tapered couplers.

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