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E. Etima and M. Greco :

A NEW TREATMENT OF RADIATIVE DECAYS OF MESONS

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A New Treatment of Radiative Decays of Mesons (*).

ETIM-ETIM (**)
CERN - Geneva

M. GRECO
Laboratori Nazionali dell'INFN - Frascati

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Summary. — Motivated by the remarkably good agreement in the descriptions of deep inelastic processes and some radiative meson decays by the quark model and the method of infinite vector-meson saturation, we argue in general that these two approaches are equivalent. We explicit the nature of this equivalence by endowing vector-meson–dominated vertices with the asymptotics implied by quark current algebra. With this leverage we obtain satisfactory predictions for SU_3 meson decays, including those for which the quark model by itself fails, excepting the $\rho \to \pi\gamma$ decay. Applied to the radiative decays of the new mesons, this scheme avoids the difficulties of nonrelativistic calculations and predicts considerably smaller widths.

1. – Introduction.

The study of the radiative decays of the newly discovered states is presently an industry. Unfortunately the nonrelativistic bound-state picture which describes fairly well their spectroscopy gives rather large decay widths (1).

(*) To speed up publication, the authors of this paper have agreed to not receive the proofs for correction.
(**) On leave of absence from Laboratori Nazionali dell'INFN, Frascati.
(1) T. APPELQUIST and H. D. POLITZER: *Phys. Rev. Lett.*, **34**, 43 (1975); A. DE RUJULA and S. L. GLASHOW: *Phys. Rev. Lett.*, **34**, 46 (1975); M. K. GAILLARD, B. W. LEE and J. L. ROSNER: *Rev. Mod. Phys.*, **47**, 2 (1975); H. HARARI: rapporteur talk at *Lepton-Photon Symposium* (Stanford, Cal., 1975); F. J. GILMAN: invited talk at *Coral Gables, Florida, 1975*, SLAC-PUB-1720 (February 1976).

Since, in these calculations, one must take care of several corrections as, for instance, those associated with the existence of many decay channels ([2]), there promises to be no simple recipe to reduce these large widths.

In this paper we study these radiative decays in a relativistic model in which the basic currents of SU_3 and SU_4 ($SU_3 \times SU_3$ and $SU_4 \times SU_4$) are dominated by vector-mesons, with appropriate quantum numbers, and exhibit a quark structure asymptotically. The model derives its motivation from the remarkable agreement between the quark parton model and the method of infinite vector-meson saturation ([3]) in the description of deep inelastic processes and of some radiative meson decays ([4]). We provide a basis for a more general phenomenological equivalence between these two approaches by imposing on vector-meson–dominated vertices the asymptotics of quark current algebra. The model is not completely defined by this requirement, however, because the purely hadronic part of the amplitude involves a large degree of unknown. Guided by analyticity, we abstract from the dual resonance model a prescription for implementing the saturation with vector-meson poles and deduce therefrom the strong-interaction couplings. This is done in sect. **2**, where we define the vertices $V \to P\gamma$, $P \to V\gamma$, $P \to \gamma\gamma$ and fix our notation. In sect. **3** the model is tested on SU_3 meson decays. The predictions are very good, including those for which the quark model by itself fails, excepting the $\rho \to \pi\gamma$ decay. The appropriate corrections to the simple VMD are also obtained. In sect. **4** we examine the decays of the new mesons in SU_4 and discuss briefly the mixing of the three pseudoscalars η, η' and η_c, eventually responsible for the decays $\psi(\psi') \to \eta\gamma$ and $\psi(\psi') \to \eta'\gamma$. Section **5** contains our conclusions. A short appendix presents the calculation of an important parameter used in the paper.

2. – Vertex function and current algebra constraint.

In a vector-meson pole dominance model the radiative decays of pseudoscalar (P) and vector (V) mesons $P \to \gamma\gamma$, $P \to V\gamma$ and $V \to P\gamma$, with widths

$$\Gamma(P \to \gamma\gamma) = \frac{\alpha^2\pi}{4} g^2_{P\gamma\gamma} m_P^3 , \tag{2.1a}$$

$$\Gamma(P \to V\gamma) = \frac{\alpha}{8} g^2_{VP\gamma} \frac{(m_P^2 - m_V^2)^3}{m_P^3} , \tag{2.1b}$$

$$\Gamma(V \to P\gamma) = \frac{\alpha}{24} g^2_{VP\gamma} \frac{(m_V^2 - m_P^2)^3}{m_V^3} , \tag{2.1c}$$

([2]) E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. Lane and T. M. Yan: *Phys. Rev. Lett.*, **34**, 369 (1975); E. Eichten, K. Gottfried, T. Kinoshita, K. Lane and T. M. Yan: *Phys. Rev. Lett.*, **36**, 500 (1976).

([3]) A. Bramón, E. Etim and M. Greco: *Phys. Lett.*, **41** B, 609 (1972).

([4]) P. G. O. Freund and S. Nandi: *Phys. Rev. Lett.*, **32**, 1811 (1974).

can be deduced from the amplitude for one of them. For instance, from the $P \to \gamma\gamma$ vertex defined in terms of the hadronic electromagnetic current by

$$\varepsilon_{\mu\nu\lambda\tau} q_1^\lambda q_2^\tau F(q_1^2, q_2^2; p^2) = i\int d^4x \exp[iq_1 x] \langle 0|T(j_\mu(x) j_\nu(0))|0\rangle , \tag{2.2}$$

one gets

$$\begin{cases} g_{P\gamma\gamma} = F(0, 0; p^2 = m_P^2) = F_P(0, 0) , \\ g_{VP\gamma} \dfrac{m_V^2}{f_V} = \lim\limits_{q^2 \to m_V^2} (m_V^2 - q^2) F_P(0, q^2) , \end{cases} \tag{2.3}$$

where m_V^2/f_V is the coupling of the vector meson of mass m_V to the photon.

Consider the case $P \equiv \pi^0$ to begin with. $F_\pi(q_1^2, q_2^2)$ has a double series of poles in q_2^2 and q_1^2 corresponding to the $I = 1$ and $I = 0$ vector mesons which can couple to the photon. Simple VMD ([5]) assumes that only the ρ and ω contribute (ideal mixing: mixing angle $\theta_V = \theta_{id} = \arcsin(1/\sqrt{3})$) so that

$$F_\pi(0, 0) = 2g_{\rho\omega\pi}/f_\rho f_\omega . \tag{2.4}$$

In the quark model, on the other hand, the $F_P(0, 0)$ are related to PCAC triangle anomalies ([6]), *e.g.*

$$F_P(0, 0; p^2 = 0) = -S_P/2\pi^2 f_P , \tag{2.5}$$

where f_P is defined by

$$\langle 0|j_{5\mu}^{(P)}(0)|p\rangle = if_P p_\mu . \tag{2.6}$$

In the fractionally charged colour quark model $|S_\pi| = \frac{1}{2}$ and extrapolating (2.5) to $p^2 = m_\pi^2$, one obtains a good prediction for $\Gamma(\pi^0 \to \gamma\gamma)$. Numerically eqs. (2.4) and (2.5) agree, but this is not enough to make the two approaches equivalent. In fact they are not. For instance, according to the Bjorken-Johnson-Low theorem ([7]), the asymptotic limit of the time-ordered product in eq. (2.2) for $q_{10}, q_{20} \to \infty$ with $|\boldsymbol{q}_1|, |\boldsymbol{q}_2|$ finite is determined by the commutator of the currents

$$q_{10} q_{2i} \varepsilon_{0ijk} F_P(q_1^2, q_2^2) \xrightarrow[\text{BJL}]{} -\frac{1}{q_{10}} \int d^3x \exp[-i\boldsymbol{q}_1 \boldsymbol{x}] \langle 0|[j_j(0, \boldsymbol{x}), j_k(0)]|p\rangle . \tag{2.7}$$

([5]) M. GELL-MANN, D. SHARP and W. G. WAGNER: *Phys. Rev. Lett.*, **8**, 216 (1962).
([6]) S. L. ADLER: *Phys. Rev.*, **177**, 2426 (1969); J. S. BELL and R. JACKIW: *Nuovo Cimento*, **60** A, 47 (1969); S. L. ADLER: *Lectures at the Brandeis Summer School*, edited by S. DESER, M. GRISARU and H. PENDLETON (1970).
([7]) J. D. BJORKEN: *Phys. Rev.*, **148**, 1467 (1966); K. JOHNSON and F. E. LOW: *Prog. Theor. Phys. Suppl.*, **37-38**, 74 (1966).

The commutator is easily evaluated once the form of the current is specified. In the fractionally charged quark model

$$(2.8)\qquad [j_\mu(0,\boldsymbol{x}), j_\nu(0)]_{\text{antisym}} = -\frac{2i}{3}\varepsilon_{0\mu\nu\lambda}\left[j^{(3)}_{5\lambda}(0) + \frac{1}{\sqrt{3}}j^{(8)}_{5\lambda}(0) + \frac{2\sqrt{2}}{\sqrt{3}}j^{(0)}_{5\lambda}(0)\right]\delta^{(3)}(\boldsymbol{x}),$$

where we have considered only the (Lorentz) antisymmetric part. $j^{(a)}_\mu(j^{(a)}_{s\mu})$ are the SU_3 vector and axial vector currents and $j^{(0)}_{s\mu} = \bar{\psi}\gamma_\mu\gamma_5\psi/\sqrt{6}$. From eqs. (2.6), (2.7) and (2.8), one finds (8)

$$(2.9)\qquad F_\mathrm{P}(q_1^2, q_2^2) \xrightarrow[q_1^2\to\infty]{} \frac{2}{q^2}N_\mathrm{P} f_\mathrm{P},$$

where N_P are the appropriate numerical factors in eq. (2.8). A factor of 3 has been included to account for colour.

In the usual field-current identity formulation of the VMD (9), on the other hand, the commutator in (2.7) vanishes identically because the space components of the vector-meson fields commute; the expansion of the right-hand side of (2.2) in powers of $1/q_0$ gives, in leading order, a $1/q^4$ asymptotic behaviour (8), in disagreement with (2.9).

Motivated by our previous successful descriptions of deep inelastic processes in the extended VMD (3), we impose on $F_\mathrm{P}(q_1^2, q_2^2)$ to have the asymptotics of eq. (2.9) when saturated with infinite series of vector-meson poles. Furthermore, we require it to have the analyticity in q_1^2 and q_2^2 of a strong-interaction vertex. To ensure this, we follow the procedure of Sugawara and Ademollo and Del Giudice (10) and make the following ansatz:

$$(2.10)\qquad F_\mathrm{P}(q_1^2, q_2^2) = k\int_0^1\!\!\int_0^1 \mathrm{d}x\,\mathrm{d}y\, X^{-\alpha(q_1^2)}y^{-\alpha(q_2^2)}(1-x)^{\gamma-1}(1-y)^{\gamma-1}(1-xy)^{\beta-2\gamma} =$$
$$= k\frac{\Gamma(1-\alpha(q_1^2))\Gamma(1-\alpha(q_2^2))[\Gamma(\gamma)]^2}{\Gamma(1+\gamma-\alpha(q_1^2))\Gamma(1+\gamma-\alpha(q_2^2))}\cdot$$
$$\cdot{}_3F_2(1-\alpha(q_1^2), 1-\alpha(q_2^2), 2\gamma-\beta; 1+\gamma-\alpha(q_1^2), 1+\gamma-\alpha(q_2^2); 1),$$

where $\alpha(q^2)$ are the vector-meson Regge trajectories $(\alpha(0)=\frac{1}{2}, \alpha'=1/2m_\rho^2)$, β and γ are fixed parameters corresponding to fictitious quark-quark and quark–pseudoscalar-meson trajectories. k is a normalization constant and ${}_3F_2(a, b, c; d, e; 1)$ the generalized hypergeometric function. A more general

(8) E. ETIM and P. PICCHI: *Lett. Nuovo Cimento*, **2**, 887 (1969).
(9) N. M. KROLL, T. D. LEE and B. ZUMINO: *Phys. Rev.*, **157**, 1376 (1967).
(10) H. SUGAWARA: Tokyo University preprint, unpublished (1969); M. ADEMOLLO and E. DEL GIUDICE: *Nuovo Cimento*, **63** A, 639 (1969); D. AMATI, S. D. ELLIS and J. H. WEIS: *Nucl. Phys.*, **84** B, 141 (1975).

representation for $F(q_1^2, q_2^2)$ than (2.10) could be used, provided it has the poles and the analyticity we have required. Because we have a highly restrictive scaling condition to satisfy from quark current algebra

$$F_P(q_1^2, q_2^2) \xrightarrow[q_1^2/q_2^2 \text{fixed}]{q^2\to\infty} (q_1^2)^{-\sigma} f(q_1^2/q_2^2) \tag{2.9'}$$

with $\sigma = 1$ and $f(x) = \text{constant}$ from (2.9), such generalization would only slightly increase the labour of extracting the coupling constants. In the case in which the generalization consists in adding satellites, we have another argument. Because of the scaling law in (2.9) and (2.9′), eq. (2.10) gives scaling structure functions in the deep inelastic region which our model respects [3]. Satellites contribute nonleading terms. For $P \equiv \pi^0$ the ρ and ω mass shell conditions give

$$k = \tfrac{1}{4} g_{\rho\omega\pi}/f_\rho f_\omega . \tag{2.11}$$

The parameter β is fixed to be one from eq. (2.9):

$$F_\pi(q_1^2, q_2^2) \xrightarrow[q_2^2/q_1^2\to 1]{q_1^2\to\infty} \begin{cases} k\Gamma(\gamma)\Gamma(\beta-\gamma)(-\alpha' q_1^2)^{-\beta} , & \beta > \gamma , \\ k\dfrac{\Gamma(\gamma)\Gamma(\beta)\Gamma(\gamma-\beta)}{\Gamma(2\gamma-\beta)}(-\alpha' q_1^2)^{-\beta} , & \beta < \gamma . \end{cases} \tag{2.12}$$

From the value $\gamma = \frac{3}{2}$, determined as discussed below, $F_\pi(q_1^2, q_2^2)$ becomes

$$F_\pi(q_1^2, q_2^2) \xrightarrow[q_1^2\to\infty]{} \frac{\pi}{4}\frac{g_{\rho\omega\pi}}{f_\rho f_\omega}\left(-\frac{2m_\rho^2}{q_1^2}\right), \tag{2.13}$$

whence from (2.8) and (2.9)

$$\frac{g_{\rho\omega\pi}}{f_\rho f_\omega} = -\frac{4f_\omega}{\pi m_\rho^2} . \tag{2.14}$$

In principle γ can be determined in many ways. It is related to the form factor for the transition $\gamma(q^2) \to \pi\omega$, *i.e.*

$$\lim_{q_2^2\to m_\omega^2 \equiv m_\rho^2} F_\pi(q_1^2, q_2^2) = k\frac{2m_\omega^2}{m_\omega^2 - q_2^2} B(\gamma, 1-\alpha(q_1^2)) = \frac{m_\omega^2}{f_\omega}\frac{g_{\omega\pi\gamma}(q_1^2)}{m_\omega^2 - q_2^2} , \tag{2.15}$$

where $B(x, y)$ is the beta-function.

At $q_1^2 = 0$

$$g_{\omega\pi\gamma}(q_1^2 = 0) = \frac{g_{\rho\omega\pi}}{2f_\rho}\frac{\Gamma(\frac{1}{2})\Gamma(\gamma)}{\Gamma(\frac{1}{2}+\gamma)} \tag{2.16}$$

and for large q_1^2

$$g_{\omega\pi\gamma}(q_1^2) \xrightarrow[q_1^2\to\infty]{} \frac{g_{\rho\omega\pi}}{2f_\rho}\Gamma(\gamma)(-\alpha' q_1^2)^{-\gamma} . \tag{2.17}$$

In the absence of any experimental information on the q^2 fall-off of the $\pi\omega$ transition form factor, we determine γ from the decay $\pi^0 \to \gamma\gamma$, for which one gets

$$F_\pi(0,0) = \frac{g_{\rho\omega\pi}}{2f_\rho f_\omega}[B(\tfrac{1}{2},\gamma)]^2 {}_3F_2(\tfrac{1}{2},\tfrac{1}{2},2\gamma-1;\tfrac{1}{2}+\gamma,\tfrac{1}{2}+\gamma;1)\,. \tag{2.18}$$

First extrapolate $F(0,0;p^2=m_\pi^2)$ to $F(0,0;0)$ and neglect terms of order $\alpha' m_\pi^2 = m_\pi^2/2m_\rho^2$ (*i.e.* assign a zero intercept to the pion trajectory $\alpha_\pi(0) = \alpha_\rho(0)-\frac{1}{2}$). Next make use of the KSFR relation [11]

$$2f_\pi^2 = m_\rho^2/f_\rho^2\,, \tag{2.19}$$

which has been shown elsewhere [12] to be satisfied in our scheme, and of the Crewther relation [13]

$$|S_\pi| = R_n/4: \tag{2.20}$$

for our value, $R_n = 8\pi^2/f_\rho^2$, of the ratio $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^+ \to \mu^+\mu^-)$ for normal hadrons, to rewrite S_π with the aid of (2.14) as

$$|S_\pi| = 4\pi^2 f_\pi/m_\rho^2 = \pi^3 g_{\rho\omega\pi}/f_\rho f_\omega\,. \tag{2.21}$$

Now substitute (2.21) into (2.5) and compare with (2.18) to recover $\gamma = \frac{3}{2}$ and

$$F_\pi(0,0) = \frac{\pi}{2}\frac{g_{\rho\omega\pi}}{f_\rho f_\omega}\,. \tag{2.22}$$

With this value of γ eq. (2.16) becomes

$$g_{\omega\pi\gamma}(0) = \frac{\pi}{4}\frac{g_{\rho\omega\pi}}{f_\rho}\,, \tag{2.23}$$

implying a correction of about 20% to the simple VMD.

With all the parameters fixed we are in a position to calculate the SU_3 meson decays. We do this in the next section.

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[11] K. KARAWABAYASHI and M. SUZUKI: *Phys. Rev. Lett.*, **16**, 255 (1966); FAYYAZUDDIN and RIAZUDDIN: *Phys. Rev.*, **147**, 1071 (1966).
[12] E. ETIM, M. GRECO and Y. SRIVASTAVA: *Lett. Nuovo Cimento*, **16**, 65 (1976).
[13] R. J. CREWTHER: *Phys. Rev. Lett.*, **28**, 1421 (1971).

3. – Radiative decays of SU_3 mesons.

To begin with let us consider the decays $\omega \to 3\pi$, $\omega \to \pi^0\gamma$ and $\pi^0 \to \gamma\gamma$. From eqs. (2.14), (2.19) and $f_\omega^2 = 9f_\rho^2$, one finds

$$g^2_{\rho\omega\pi} = \frac{72}{\pi^2 m_\rho^2} f_\rho^2 \simeq (330 \pm 40)\ (\text{GeV})^{-2}\,, \tag{3.1}$$

where the error comes from the experimental value of $\Gamma(\rho \to e^+e^-) = (6.45 \pm \pm 0.75)$ keV ([14]). Recall that in the $\omega \to 3\pi$ decay, one cannot use the simple-pole model in the three channels (s, t, u); double counting is involved. A correct procedure is to use finite-energy dispersion ([15]) relations or else make an explicit model for the amplitude ([16]), *e.g.* a Veneziano amplitude. Either way there is a 20% reduction relative to that of the simple-pole model. With this correction and $g^2_{\rho\pi\pi}/4\pi \simeq 2.65$ (*i.e.* $\Gamma_\rho \simeq 150$ MeV), the $\omega \to 3\pi$ width becomes

$$\Gamma(\omega \to \pi^+\pi^-\pi^0) \simeq \frac{g^2_{\rho\omega\pi} g^2_{\rho\pi\pi}}{4\pi} \left(10.5\cdot 10^{-6}\ (\text{GeV})^3\right) \simeq 9.8\ \text{MeV} \tag{3.2}$$

with a 20% estimated error. Similarly using (2.14), (2.23) and (2.22) in (2.1*a*) and (2.1*b*), one finds

$$\Gamma(\pi^0 \to \gamma\gamma) \simeq 10.5\ \text{keV}\,, \tag{3.3a}$$

$$\Gamma(\omega \to \pi^0\gamma) \simeq 1.0\ \text{MeV} \tag{3.3b}$$

with the same error.

For the $\rho \to \pi\gamma$ decay we have $g_{\rho\pi\gamma} = \frac{1}{3} g_{\omega\pi\gamma}$ and

$$\Gamma(\rho \to \pi\gamma) \simeq 105\ \text{keV}\,, \tag{3.4}$$

in disagreement with the recent experimental value $\Gamma(\rho \to \pi\gamma) = (35 \pm \pm 10)$ keV ([17]) obtained with the Primakoff effect. In view of the well-known subtleties of this type of measurement and of our results for the decays $\varphi \to \eta\gamma$ and $K^{0*} \to K^0\gamma$, discussed below, which agree with the new data ([18]), it seems highly desirable to have a new measurement of the $\rho \to \pi\gamma$ rate.

([14]) PARTICLE DATA GROUP: *Phys. Lett.*, **50** B, 1 (1974).
([15]) A. BRAMÓN and M. GRECO: *Nuovo Cimento*, **14** A, 322 (1973).
([16]) H. GOLDBERG and Y. SRIVASTAVA: *Phys. Rev. Lett.*, **22**, 749 (1969).
([17]) B. GOBBI, J. L. ROSEN, H. A. SCOTT, S. L. SHAPIRO, L. STRAWCZYNSKI and C. M. MELTZER: *Phys. Rev. Lett.*, **33**, 1450 (1974).
([18]) W. C. CARITHERS, P. MÜHLEMANN, D. UNDERWOOD and D. G. RYAN: *Phys. Rev. Lett.*, **35**, 349 (1975); C. BEMPORAD *et al.*: *Lepton-Photon Symposium* (Stanford, Cal., 1975).

Consider the $\varphi \to 3\pi$ and $\varphi \to \pi^0\gamma$ decays. These are usually assumed to occur because the φ-ω mixing angle θ_V is different from the ideal one (*i.e.* $g_{\varphi\rho\pi} = g_{\rho\omega\pi}\,\mathrm{tg}\,(\theta_V - \theta_{id})$). In the $\varphi \to 3\pi$ decay there is little correction to the $\varphi \to \rho\pi$ pole model [15], so that

$$\Gamma(\varphi \to \pi^+\pi^-\pi^0) = \frac{g^2_{\varphi\rho\pi} g^2_{\rho\pi\pi}}{4\pi} (38 \cdot 10^{-5}\ (\mathrm{GeV})^3)$$

and, given the ratio $\Gamma(\varphi \to 3\pi)/\Gamma(\omega \to 3\pi)$ [14], $\theta_V - \theta_{id} \simeq 2.6°$. For $\varphi \to \pi^0\gamma$ $g_{\varphi\pi\gamma} = g_{\omega\pi\gamma}\,\mathrm{tg}\,(\theta_V - \theta_{id})$ and [19]

$$\Gamma(\varphi \to \pi^0\gamma) \simeq 4.7\ \mathrm{keV}\,. \tag{3.5}$$

Another consequence of the small deviation of θ_V from θ_{id} is a small contribution to $\pi^0 \to \gamma\gamma$ from vector mesons of the φ-family coupling to the photon. The correction to $\Gamma(\pi^0 \to \gamma\gamma)$ is about 5% and can be computed from

$$F_\pi(0, 0) = \frac{\pi}{2} \frac{g_{\rho\omega\pi}}{f_\rho f_\omega} \left(1 - \frac{1}{2} \frac{\Gamma(0.88)}{\Gamma(1.88)}\,\mathrm{tg}\,(\theta_V - \theta_{id})\right). \tag{3.6}$$

Now let us consider the radiative decays with the η and η'. With quadratic η and η' mixing the quark model predictions for these decays [19] agree rather well with the data and with the predictions of low-energy theorems for $\eta(\eta') \to \gamma\gamma$ and $\eta(\eta') \to \pi^+\pi^-\gamma$ [20]. The exceptions are the recent measurements of $\varphi \to \eta\gamma$ and $K^{0*} \to K^0\gamma$. These are about a factor three smaller than the theoretical values. The predictions of our model agree with these new measurements.

We proceed as in sect. **2** and define

$$F_8(q_1^2(I=1), q_2^2(I=1)) = \frac{1}{4} \frac{g_{\eta_8\rho\rho}}{f_\rho^2} B\left[\frac{3}{2}, 1 - \alpha(q_1^2)\right] B\left[\frac{3}{2}, 1 - \alpha(q_2^2)\right] \cdot \\ \cdot {}_3F_2\left(1 - \alpha(q_1^2), 1 - \alpha(q_2^2), 2; \frac{5}{2} - \alpha(q_1^2), \frac{5}{2} - \alpha(q_2^2); 1\right) \tag{3.7}$$

with the BJL limit

$$F_8(q_1^2(I=1), q_2^2(I=1)) \xrightarrow[\mathrm{BJL}]{} \frac{\pi}{8} \frac{g_{\eta_8\rho\rho}}{f_\rho^2} \left(-\frac{2m_\rho^2}{q_1^2}\right). \tag{3.8}$$

If we combine with (2.8) and (2.9), we deduce

$$g_{\eta_8\rho\rho}/f_\rho^2 = -4\sqrt{3}\, f_8/\pi\,. \tag{3.9}$$

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[19] A. Bramón and M. Greco: *Phys. Lett.*, **48** B, 137 (1974).

[20] M. S. Chanowitz, Min-Shih Chen and Ling-Fong Li: *Phys. Rev. D*, **7**, 3104 (1973); M. S. Chanowitz: *Phys. Rev. Lett.*, **35**, 977 (1975).

Doing the same thing for the isoscalar part and for the singlet η_1 we obtain

$$(3.10)\qquad \begin{cases} g_{\eta\gamma\gamma} \simeq \dfrac{g_{\pi^0\gamma\gamma}}{\sqrt{3}} (\cos\theta_P - 2\sqrt{2}\sin\theta_P)\,, \\ g_{\eta'\gamma\gamma} \simeq -\dfrac{g_{\pi^0\gamma\gamma}}{\sqrt{3}} (\sin\theta_P + 2\sqrt{2}\cos\theta_P)\,, \end{cases}$$

provided $f_\pi = f_8 = f_1$. In eq. (3.10) terms of about a few percent have been neglected. These come from the difference between the vertex functions dominated by the ρ and the φ families. The origin of this difference will turn out, however, to be important for the decay $\varphi \to \eta\gamma$.

Our predictions for $\rho \to \eta\gamma$ ($\rho \to \eta'\gamma$) and $\omega \to \eta\gamma$ ($\omega \to \eta'\gamma$) agree with those of the quark model ($g_{\rho\eta\gamma} = -g_{\rho\eta'\gamma} = 3g_{\omega\eta\gamma} = -3g_{\omega\eta'\gamma} = g_{\omega\pi^0\gamma}/\Gamma 2$) [19,21]. For $\eta \to \pi^+\pi^-\gamma$ there is a large correction to the simple-pole model due to the fact that the physical dipion mass squared is far away from m_ρ^2. This correction is evaluated as in the case of $\omega \to \pi^+\pi^-\pi^0$ [15,22]. The rate obtained in this way agrees with the prediction of low-energy theorems [20]. The above correction does not apply to $\eta' \to \pi^+\pi^-\gamma$ because $m_{\pi\pi}^2 \simeq m_\rho^2$.

In contrast to the above, our prediction for the decay $\varphi \to \eta\gamma$ differs significantly from that of the quark model. This comes about from the pure φ-family dominance of the corresponding vertex function. The coupling constant is

$$(3.11)\qquad g_{\varphi\eta\gamma} = \frac{2\sqrt{2}}{3\sqrt{3}} (\sqrt{2}\cos\theta_P + \sin\theta_P) \frac{f_\varphi}{f_\omega} g_{\omega\pi\gamma} \left(\frac{2}{\pi} B\left(\frac{3}{2}, 1 - \alpha_\varphi(0)\right)\right).$$

The factor in the last brackets is reponsible for the modification of the quark model result and is about a factor two smaller. If $\alpha_\varphi(q^2) \equiv \alpha_\rho(q^2)$, then this factor is, of course, unity. The same consideration applies to the decay $K^{0*} \to K^0\gamma$ for which we get

$$(3.12)\qquad g_{K^{0*}K^0\gamma} = 2g_{K^{+*}K^+\gamma} = \sqrt{2}\, g_{\varphi\eta\gamma}\,.$$

All our predictions are displayed in the table together with the experimental data and the quark model results. With the exception of the $\rho \to \pi\gamma$ decay, there is on the whole a better agreement between theory and experiment. Our theoretical predictions are affected by a $(10 \div 20)\%$ error and have been scaled down by an overall factor of 0.9 to correct for the uncertainties introduced by the approximations involved in the determination of our coupling constants.

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(21) R. H. DALITZ: *Lectures at the Les Houches Summer School*, edited by C. DEWITT and M. JACOB (1965); G. MORPURGO: *Lectures at Erice Summer School*, edited by A. ZICHICHI (1971).

(22) A. BRAMÓN and M. GRECO: *Lett. Nuovo Cimento*, **2**, 583 (1974).

TABLE I.

Decay widths	Theory	Experimental data [14]	Quark model [19]
$\Gamma(\omega \to 3\pi)$ (MeV)	8.8	9.00 ± 0.06	input
$\Gamma(\omega \to \pi^0\gamma)$ (MeV)	0.9	0.87 ± 0.05	input
$\Gamma(\omega \to \eta\gamma)$ (keV)	7.3	8.9 ± 40	7.3 ± 0.5
$\Gamma(\rho \to \pi\gamma)$ (keV)	95	35 ± 10 [17]	93 ± 6
$\Gamma(\rho \to \eta\gamma)$ (keV)	55	< 160	55 ± 4
$\Gamma(\varphi \to 3\pi)$ (MeV)	input (θ_V)	0.66 ± 0.06	input (θ_V)
$\Gamma(\varphi \to \pi^0\gamma)$ (keV)	4.2	5.9 ± 2.1	4.2 ± 0.3
$\Gamma(\varphi \to \eta\gamma)$ (keV)	45	63 ± 15 [18]	175 ± 13
$\Gamma(\pi^0 \to \gamma\gamma)$ (eV)	9.0	7.8 ± 0.9	input
$\Gamma(\eta \to \gamma\gamma)$ (keV)	0.43	0.324 ± 0.046	0.375 ± 0.042
$\Gamma(\eta' \to \gamma\gamma)$ (keV)	7.3	BR$= 1.9 \pm 0.3$	6.35 ± 0.73
$\Gamma(\eta \to \pi^+\pi^-\gamma)$ (eV)	41	42 ± 7	41 ± 16
$\Gamma(\eta' \to \pi^+\pi^-\gamma)$ (keV)	120	BR$=27.4 \pm 2.2$	118 ± 9
$\Gamma(\eta' \to \omega\gamma)$ (keV)	11	—	11 ± 1
$\Gamma(K^{0*} \to K^0\gamma)$ (keV)	56	75 ± 35 [18]	217 ± 16
$\Gamma(K^{+*} \to K^+\gamma)$ (keV)	14	< 80	52 ± 4

4. – Radiative decay of the new mesons.

Nonrelativistic calculations [1,2,23] of the radiative decays of the new mesons give rather high transition rates, even after unitarity corrections [2]. We think that the situation here may be similar to the failure of the quark model in the decays $\varphi \to \eta\gamma$ and $K^{0*} \to K^0\gamma$. We, therefore, propose to apply our treatment of the SU_3 meson decays to the present problem, in particular to the ψ, $\psi' \to \eta_c(\eta, \eta')\gamma$ and $\eta_c \to \gamma\gamma$ decays.

We assume that the ψ-particles belong to a new family of states lying on Regge trajectories with a slope α'_c. The implications of this assumption for the vector states ($\psi_0 \equiv J/\psi(3.1)$, $\psi_1 \equiv \psi'(3.7)$, $\psi_2 \equiv (4.15) \ldots$) in e^+e^- annihilation have been discussed elsewhere [12]. We get a contribution to R given by $R_c = \frac{2}{3} R_n = 16\pi^2/3f_\rho^2 \simeq 1.7$ instead of the canonical value of $\frac{4}{3}$. We define the electromagnetic current in SU_4 to be

$$j_\mu(x) = j_\mu^{(3)}(x) + \frac{1}{\sqrt{3}} j_\mu^{(8)}(x) + \frac{2\sqrt{2}}{3} j_\mu^{(c)}(x) , \tag{4.1}$$

[23] R. BARBIERI, R. GATTO, R. KÖGERLER and Z. KUNZST: *Phys. Lett.*, **57** B, 455 (1975); **60** B, 183 (1976).

where

$$j_\mu^{(c)}(x) = \tfrac{1}{2}\left(j_\mu^{(0)}(x) - \sqrt{3}\, j_\mu^{(15)}(x)\right). \tag{4.2}$$

Our normalization of the $SU_4\,\lambda_i$ $(i = 0, 1, \ldots, 15)$ matrices is

$$\mathrm{Tr}\,(\lambda_i \lambda_j) = \tfrac{1}{2}\delta_{ij}\,,$$

$$\mathrm{Tr}\,(\lambda_k(\lambda_i \lambda_j)) = d_{ijk}\,,$$

$$\lambda_0 = \frac{1}{2\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}, \qquad \lambda_{15} = \frac{1}{2\sqrt{6}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix}. \tag{4.3}$$

The fourth quark has the quantum number assignments of the GIM model [24]. From the commutator

$$\begin{aligned}[j_\mu(0, \boldsymbol{x}), j_\nu(0)]_{\text{antisym}} = -\frac{2i}{3}\varepsilon_{0\mu\nu\lambda}\Bigg[& j_{5\lambda}^{(3)}(0) + \frac{1}{\sqrt{3}} j_{5\lambda}^{(8)}(0) + \\ &+ \frac{2\sqrt{2}}{\sqrt{3}}\left(\frac{\sqrt{3}\,j_{5\lambda}^{(0)}(0) + j_{5\lambda}^{(15)}(0)}{2}\right) + \frac{4\sqrt{2}}{3}\left(\frac{j_{5\lambda}^{(0)}(0) - \sqrt{3}\,j_{5\lambda}^{(15)}(0)}{2}\right)\Bigg]\delta^{(3)}(\boldsymbol{x})\,,\end{aligned} \tag{4.4}$$

we isolate the SU_3 singlets $(j_{5\lambda}^{(0)} - \sqrt{3}\,j_{5\lambda}^{(15)})/2$ and $(\sqrt{3}\,j_{5\lambda}^{(0)} + j_{5\lambda}^{(15)})/2$ of which the first is pure $\bar{c}c$ and the second corresponds to the old SU_3 singlet in (2.8), associated with η_1. We associate $(j_{5\lambda}^{(0)} - \sqrt{3}\,j_{5\lambda}^{(15)})/2$ with η_c and define the corresponding f_{η_c} (cf. eq. (2.6)) by

$$\langle 0|\frac{j_{5\lambda}^{(0)}(0) - \sqrt{3}\,j_{5\lambda}^{(15)}(0)}{2}|P \equiv \eta_c\rangle = i f_{\eta_c} p_\lambda\,. \tag{4.5}$$

By the same token we dominate the current in (4.2) with vector mesons of the ψ-family. The vertex function for the decays $\psi_n \to \eta_c\gamma$ and $\eta_c \to \gamma\gamma$ can be obtained from the function

$$\begin{aligned}F_{\eta_c}(q_1^2, q_2^2) = k' B(1 - \alpha(q_1^2), \gamma)\, B(1 - \alpha(q_2^2), \gamma)\,\cdot \\ \cdot {}_3F_2(1 - \alpha(q_1^2), 1 - \alpha(q_2^2), 2\gamma - \beta; 1 + \gamma - \alpha(q_1^2), 1 + \gamma - \alpha(q_2^2); 1)\,,\end{aligned} \tag{4.6}$$

where $\alpha(q^2)$ is the ψ trajectory with

$$1/\alpha'_c = m_{\psi_1}^2 - m_{\psi_0}^2 \simeq 4\ (\text{GeV})^2\,, \qquad \alpha_c(0) \simeq -\tfrac{3}{2}\,.$$

([24]) S. L. GLASHOW, J. ILIOPOULOS and L. MAIANI: *Phys. Rev. D*, **2**, 1285 (1970).

The ψ mass spectrum is $m_n^2 = m_0^2(1 + an)$, $a = (\alpha_c' m_0^2)^{-1} \simeq 0.4$. Compare this with the ρ-family, where $a = 2$. It is interesting to observe that, if η_c is the first member of the trajectory with intercept half-unity below $\alpha_\psi(s)$, exactly as in the case of π and ρ, one gets $m_{\eta_c}^2 \simeq 8\ (\text{GeV})^2$, in agreement with experiment [25].

The parameters β and γ in eq. (4.6) are the same as before. The normalization constant k' is related to the on-shell coupling $\psi\psi\eta_c$, *i.e.*

$$k' = \frac{1}{a^2}\frac{g_{\psi\psi\eta_c}}{f_\psi^2} \tag{4.7}$$

with m_ψ^2/f_ψ the ψ-γ coupling. In the BJL limit

$$F_{\eta_c}(q_1^2, q_2^2) \xrightarrow[\text{BJL}]{} \frac{8\sqrt{2}}{3}\frac{f_{\eta_c}}{q_1^2}. \tag{4.8}$$

From eq. (4.6), the left-hand side is

$$F_{\eta_c}(q_1^2, q_2^2) \xrightarrow[\text{BJL}]{} \frac{\pi}{2a^2}\frac{g_{\psi\psi\eta_c}}{f_\psi^2}(-\alpha_c' q_1^2)^{-1} \tag{4.9}$$

and, therefore,

$$\frac{g_{\psi\psi\eta_c}}{f_\psi^2} = -\frac{16\sqrt{2}}{3\pi}\frac{af_{\eta_c}}{m_\psi^2}. \tag{4.10}$$

The $\psi\eta_c\gamma$ coupling constant is obtained from the residue of $F_{\eta_c}(q_1^2, q_2^2 = 0)$ at $q_1^2 = m_\psi^2$. It is given by

$$g_{\psi\eta_c\gamma} = \frac{\pi}{40}\frac{g_{\psi\psi\eta_c}}{a^2 f_\psi} = -\frac{2\sqrt{2}}{15a}\frac{f_\psi f_{\eta_c}}{m_\psi^2}, \tag{4.11}$$

while the $\eta_c \to \gamma\gamma$ coupling is

$$F_{\eta_c}(0, 0) = \frac{1}{a^2}\frac{g_{\psi\psi\eta_c}}{f_\psi^2}\left[B\left(\frac{5}{2}, \frac{3}{2}\right)\right]^2 {}_3F_2\left(\frac{5}{2}, \frac{5}{2}, 2; 4, 4; 1\right). \tag{4.12}$$

The hypergeometric function can be reduced to $64(9\pi - 28)/\pi$. With this (4.12) becomes

$$F_{\eta_c}(0, 0) = \frac{\pi g_{\psi\psi\eta_c}}{a^2 f_\psi^2}\frac{9\pi - 28}{4} = -\frac{16\sqrt{2} f_{\eta_c}}{3am_\psi^2}\frac{9\pi - 28}{4}. \tag{4.13}$$

The factor $(9\pi - 28)/4 \simeq 0.07$ represents the reduction of the physical $\eta_c \to \gamma\gamma$

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(25) H. B. WIIK: *Lepton-Photon Symposium* (Stanford, Cal., 1975).

amplitude relative to the amplitude for a soft η_c ($m_{\eta_c} \simeq 0$). In fact, in the exact SU_4 limit with $\alpha_\rho = \alpha_\psi$ and $\alpha_\pi = \alpha_{\eta_c}$, one gets

$$F(0, 0; p^2_{\eta_c} = 0) = -\frac{16\sqrt{2} f_{\eta_c}}{3am^2_\psi}. \tag{4.14}$$

This agrees with the ratio of the anomalies $S_{\eta_c}/S_\pi = 4\sqrt{2}/3$ from eq. (4.4). Therefore, compared to naive calculations, there is a large reduction factor in this model. It may also be thought of as arising from the extrapolation of the two-photon legs from $q^2 \simeq m^2_\psi$ to $q^2 \simeq 0$. The reduction mechanism is thus the same as that operating in the decays $\varphi \to \eta\gamma$ and $K^{0*} \to K^0\gamma$.

So far we have expressed the decay constants in terms of f_{η_c}. To determine it theoretically, we follow the method of ref. [11]. The details of the calculation are given in the appendix. The result is the analogue of the KSFR relation

$$2f^2_\psi f^2_{\eta_c} = m^2_\psi \tag{4.15}$$

with $f_{\eta_c}/f_\pi \simeq 1.7$ and $f_{\eta_c} \simeq 160$ MeV. Substituting (4.15) in (4.11) and (4.13) gives

$$g_{\psi\eta_c\gamma} = -\frac{2}{15}\alpha'_c m_\psi, \tag{4.16}$$

$$F_{\eta_c}(0, 0) = -\frac{16}{3}\frac{9\pi - 28}{4}\alpha'_c\frac{m_\psi}{f_\psi} \tag{4.17}$$

and, with $m_{\eta_c} = 2.8$ GeV,

$$\begin{cases} \Gamma(\psi \to \eta_c\gamma) \simeq 0.60 \text{ keV}, \\ \Gamma(\eta_c \to \gamma\gamma) \simeq 0.51 \text{ keV}. \end{cases} \tag{4.18}$$

For the ψ' decay we find

$$g_{\psi'\eta_c\gamma} = -\frac{f_{\psi'}\alpha'_c m_\psi}{14 f_\psi} \tag{4.19}$$

and

$$\Gamma(\psi' \to \eta_c\gamma) \simeq 4.3 \text{ keV}. \tag{4.20}$$

Compared with the predictions of nonrelativistic calculations [1,2,23], ours are considerably smaller. They agree roughly with the results of the dual unitarization scheme [26]. Our suppression factors have been very effective.

We now consider the decays ψ, $\psi' \to \eta(\eta')\gamma$ and introduce a small mixing

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([26]) CHAN HONG-MO, J. KWIECINSKI and R. G. ROBERTS: *Phys. Lett.*, **60** B, 367, 469 (1976).

among η, η' and η_c. Call the physical mesons $\bar{\eta}$, $\bar{\eta}'$ and $\bar{\eta}_c$. For small mixing angles (of the order of a percent) the rotation matrix is particularly simple:

$$(4.21)\qquad \begin{cases} \bar{\eta} \simeq \eta\sqrt{1-\alpha^2} + \alpha\eta_c\,, \\ \bar{\eta}' \simeq \eta'\sqrt{1-\beta^2} + \beta\eta_c\,, \\ \bar{\eta}_c \simeq -\alpha\eta - \beta\eta' + \sqrt{1\,\alpha^2 - \beta^2}\,\eta_c\,. \end{cases}$$

The quantities α and β measure the $\bar{c}c$ component present in the physical $\bar{\eta}$ and $\bar{\eta}'$, respectively. There is a serious problem in the extrapolation of $F_{\eta_c}(q_1^2, q_2^2)$ from $p^2 \simeq m^2_{\eta_c}$ to $p^2 \simeq m^2_\eta$, $m^2_{\eta'}$ for fixed values of q_1^2 and q_2^2. We have no knowledge of this p^2-dependence. Consequently determining α and β from data is not unambiguous. In principle the problem admits a solution. One associates a trajectory $\alpha_P(p^2)$ with the p^2 leg, the same as for currents, and studies the amplitude

$$(4.22)\qquad H(q_1^2, q_2^2; p^2) \sim \int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\, x^{-\alpha(q_1^2)} y^{-\alpha(q_2^2)} z^{\delta-1}(1-x)^{\zeta-1}(1-y)^{\zeta-1}\cdot$$
$$\cdot(1-z)^{-\alpha_P(p^2)-1}[(1-xz)(1-yz)]^{\gamma-\zeta+\alpha_P(p^2)}(1-xyz)^{-2\gamma+\beta-\alpha_P(p^2)},$$

where δ and ζ are new parameters. The residue of $H(q_1^2, q_2^2; p^2)$ at $\alpha_P(p^2) = 0$ gives the vertex function. Unfortunately (4.22) introduces extra parameters on which we have not enough constraints. Only $H(0, 0; 0)$ is fixed by the anomaly. We are therefore unable to satisfactorily account for the p^2-dependence of the vertex function. Consequently we will incorporate in the quantities α and β both the effect of mixing and of extrapolation in p^2.

A straightforward calculation using (4.16) and (4.21) gives

$$(4.23)\qquad \begin{cases} \Gamma(\psi \to \bar{\eta}\gamma) \simeq 90\alpha^2\,\text{keV}\,, \\ \Gamma(\psi \to \bar{\eta}'\gamma) \simeq 72\beta^2\,\text{keV}\,. \end{cases}$$

Using the experimental values $\Gamma(\psi \to \eta\gamma) = (95 \pm 29)$ eV ([25]) and $\Gamma(\psi \to \eta'\gamma) = (0.38 \pm 0.24)$ keV ([27]), one gets $\alpha^2 = 1.05\cdot10^{-3}$ and $\beta^2 = 5.3\cdot10^{-3}$. For the decays $\psi' \to \eta(\eta')\gamma$, and by neglecting a possible variation of α and β with q_1^2,

$$(4.24)\qquad \begin{cases} \Gamma(\psi' \to \bar{\eta}\gamma) \simeq 52\alpha^2\,\text{keV} \simeq 0.055\,\text{keV}\,, \\ \Gamma(\psi' \to \bar{\eta}'\gamma) \simeq 45\beta^2\,\text{keV} \simeq 0.23\,\text{keV}\,. \end{cases}$$

From eqs. (4.6), (4.10) and (4.21) one finds

$$(4.25)\qquad \Gamma(\psi' \to \psi\bar{\eta}) \simeq 400\alpha^2\,\text{keV}$$

and, therefore, $\alpha^2 \simeq 2.5 \cdot 10^{-2}$ from the observed value of about 10 keV [27]. Comparing this with the value obtained from (4.23), one sees that there is a considerable variation (of about a factor five in the amplitude) of the p^2 extrapolation with the photon mass. However, the smallness of the $c\bar{c}$ component ($\alpha^2, \beta^2 \leqslant 10^{-2}$), required to account for the above decays, makes this simple mixing pattern very plausible.

5. – Conclusions.

Hadronic electromagnetic-current amplitudes manifest vector-meson pole dominance at low momentum transfers and exhibit pointlike quark structure at high energies. Both in total e^+e^- annihilation into hadrons and in deep inelastic scattering we have exhibited a model with infinite vector mesons which has these two properties. By explicitly requiring the quark model asymptotics to hold for general current amplitudes saturated with infinite series of vector mesons, we have shown in this paper that such a scheme describes satisfactorily radiative decays of mesons, including those cases in which the quark model by itself fails. We also obtain the right corrections to simple VMD and interesting relations such as that of Crewther. Applied to the radiative decays of the new mesons, we have shown that a suppression mechanism operative also in the decays $\varphi \to \eta\gamma$ and $K^{0*} \to K^0\gamma$ is of the right strength to produce reasonable decay widths, usually about a factor two smaller than predicted by nonrelativistic calculations. The decay $\rho \to \pi\gamma$ is not well understood and constitutes the exception in an otherwise satisfactory set of predictions.

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APPENDIX

We show here briefly how we obtained our value for the constant f_{η_c} defined by

$$\langle 0 | \frac{j_{5\mu}^{(0)}(0) - \sqrt{3}\, j_{5\mu}^{(15)}(0)}{2} | \eta_c(p) \rangle = i f_{\eta_c} p_\mu \qquad \text{(A.1)}$$

in eq. (4.5). We follow a procedure [12] already used with chiral SU_3 currents

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[27] DESY-HEIDELBERG COLLABORATION: J. Heintze's report at the *Lepton-Photon Symposium* (Stanford, Cal., 1975); H. L. LYNCH: *Lectures at the Cargèse Summer School* (1975), SLAC-PUB-1643 (August 1975).

to derive

$$2f_\rho^2 f_\pi^2 = m_\rho^2\,, \qquad f_K/f_\pi \simeq 1.25\,.$$

If we now invoke asymptotic chiral SU_4 and assume, as in the case of A_1, that the chiral partner A_c of $\psi_0 \equiv \psi(3.1)$ lies half-way between ψ and ψ', *i.e.* $m_{A_c}^2 - m_\psi^2 \simeq \frac{1}{2}\alpha_c'$, we get the analogous relation

$$2f_\psi^2 f_{\eta_c}^2 \simeq m_\psi^2\,, \tag{A.2}$$

where m_ψ^2/f_ψ is the coupling of the ψ to the photon. We start by considering the Green's functions

$$\Delta_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2)\Delta(q^2) = \int d^4x\, d^4y \exp[iqx]\langle 0|T(\theta(y) j_\mu(x) j_\nu(0))|0\rangle\,, \tag{A.3a}$$

$$\Delta^A_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2)\Delta^c(q^2) + g_{\mu\nu}\Delta^{nc}(q^2) = \\ = \int d^4x\, d^4y \exp[iqx]\langle 0|T(\theta(y) A_\mu(x) A_\nu(0))|0\rangle\,, \tag{A.3b}$$

$$\Pi_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2)\Pi(q^2) = i\int d^4x \exp[iqx]\langle 0|T(j_\mu(x) j_\nu(0))|0\rangle\,, \tag{A.3c}$$

$$\Pi^A_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2)\Pi^c(q^2) + q_\mu q_\nu \Pi^{nc}(q^2) = \\ = i\int d^4x \exp[iqx]\langle 0|T(A_\mu(x) A_\nu(0))|0\rangle\,, \tag{A.3d}$$

where $\theta(x)$ is the trace of the energy-momentum tensor, $j_\mu^{(x)}$ the electromagnetic current and A_μ a SU_4 chiral partner of a vector current in j_μ. $\Delta(q^2)$ and $\Delta^c(q^2)^\mu$ satisfy the trace identities

$$\Delta(q^2) = -2q^2 \frac{\partial \Pi(q^2)}{\partial q^2} - \frac{R}{6\pi^2}\,, \tag{A.4a}$$

$$\Delta^c(q^2) = -2q^2 \frac{\partial}{\partial q^2}(\Pi^c(q^2) + \Pi^{nc}(q^2)) - \frac{3R}{40\pi^2} \tag{A.4b}$$

with R the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. Saturating $j_\mu^{(x)}$ with the ψ-family and taking $\theta(x)$ as bilinear in these vector fields, we obtain, with as usual

$$m_n^2 = m_\psi^2(1 + an)\,, \qquad am_\psi^2 = 1/\alpha_c'\,, \qquad f_{\psi_n}^2/m_{\psi_n}^2 = f_\psi^2/m_\psi^2\,,$$

$$\Delta_\psi(q^2) = -\frac{R_\psi}{6\pi^2} - \frac{2q^2}{a^2 m_\psi^2 f_\psi^2}\zeta\left(\frac{1}{a} - \frac{q^2}{am_\psi^2}\right), \tag{A.5}$$

where $\zeta(z)$ is the generalized Riemann zeta-function and

$$R_\psi = \frac{12\pi^2}{af_\psi^2} \tag{A.6}$$

is the ψ-family contribution to R. In the limit $q^2 \to \infty$

$$\Delta_\psi(q^2) \to \frac{1}{q^2}\frac{m_\psi^2}{f_\psi^2}(2\alpha_c' m_\psi^2 - 1) + O(1/q^4)\,. \tag{A.7}$$

For $\Delta^c_\psi(q^2)$ we saturate in the simplest way consistent with asymptotic chiral symmetry and PCAC, *i.e.*

$$\text{(A.8)} \qquad \operatorname{Im} \Pi^c(s) + \operatorname{Im} \Pi^{nc}(s) = \pi \frac{m^2_{A_c}}{f^2_{A_c}} \sum_n \delta(s - m^2_{A_{cn}}) + \pi f_{\eta_c} \delta(s - m^2_{\eta_c}) ,$$

$$\text{(A.9)} \qquad \Delta^2_\psi(q^2) = \frac{-R_\psi}{6\pi^2} - \frac{2m^2_{A_c}}{a^2 m^2_\psi f^2_{A_c}} q^2 \zeta\left(\frac{m^2_{A_c}}{am^2_\psi} - \frac{q^2}{am^2_\psi}\right) - 2f^2_{\eta_c} \frac{q^2}{(m^2_{\eta_c} - q^2)^2} .$$

In the limit $q^2 \to \infty$ we get

$$\text{(A.10)} \qquad m^2_\psi / f^2_\psi = m^2_{A_c} / f^2_{A_c} ,$$

$$\text{(A.11)} \qquad \Delta^c_\psi(q^2) \to \frac{1}{q^2} \frac{m^2_\psi}{f^2_\psi} \left[(2\alpha'_c m^2_{A_c} - 1) - \frac{2f^2_{\eta_c} f^2_\psi}{m^2_\psi} \right] + O(1/q^4) .$$

Exactly as in the case of chiral SU_3, equality of the coeffcients of $1/q^2$ in (A.7) and (A.11) yields (A.2), with our assumption about the position of A_c.

● RIASSUNTO

Alla luce dell'ottimo accordo tra il modello a quark ed il modello di saturazione con infiniti mesoni vettoriali nella descrizione dei processi altamente anelastici e dei decadimenti radiativi dei mesoni, si discute in generale l'equivalenza dei due schemi. Tale equivalenza è esplicitamente applicata alle funzioni vertice che sono dominate da mesoni vettori e sono dotate del comportamento asintotico proprio dell'algebra delle correnti del modello a quark. In tal modo si ottengono soddisfacenti predizioni per i decadimenti dei mesoni di SU_3, inclusi quelli che sono in disaccordo con il semplice modello a quark, eccetto il decadimento $\rho \to \pi\gamma$. Applicato ai decadimenti radiativi dei nuovi mesoni questo schema evita le difficoltà dei calcoli non relativistici e dà larghezze di decadimento sensibilmente minori.

Новое рассмотрение радиационных распадов мезонов.

Резюме (*). — Обосновывая хорошее согласие при описании глубоко неупругих процессов и некоторых радиационных распадов мезонов с помощью модели кварков и метода бесконечного насыщения векторными мезонами, мы доказываем, что эти два подхода являются эквивалентными. Показывается природа этой эквивалентности посредством введения доминантных вершин с векторными мезонами, которые имеют асимптотики, вытекающие из алгебры токов кварков. Этот метод дает удовлетворительные предсказания для распадов SU_3 мезонов, включая такие распады, для которых модель кварков несправедлива, за исключением распада $\rho \to \pi\gamma$. Применение этой схемы к радиационным распадам новых мезонов позволяет избежать трудностей, связанных с нерелятивистскими вычислениями, и предсказывает значительно меньшие ширины.

(*) *Переведено редакцией.*