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M. Lusignoli and Y. Srivastava: PION EM STRUCTURE FUNCTIONS FROM PION EXCHANGE IN LEPTOPRODUCTION

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M. Lusignoli[x] and Y. Srivastava[o]: PION EM STRUCTURE FUNCTIONS FROM PION EXCHANGE IN LEPTOPRODUCTION.

ABSTRACT:

The pion exchange contribution to e+p ⟶ e'+n+X is calculated using different models, both for pion exchange and for the pion EM structure functions. Numerical estimates of this contribution to deep inelastic scattering off protons are given.

1. - INTRODUCTION.

It is well known that the inclusive process p+p ⟶ n+X is dominated, in the triple Regge region, by one pion exchange[1,2]. This is due to the strength of the pion-nucleon coupling as compared to the couplings of other isovector Regge trajectories. For the same reason we expect[2] that an experimental study of the process $e(\mu)+$ $+p \longrightarrow e'(\mu')+n+X$ in the deep inelastic region and, for slow neutrons, can give us informations about the pion electromagnetic (EM) structure functions.

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In this work we present formulae for the calculation of one pion exchange to the above mentioned process. Some years ago, Sullivan (see ref. (3)) did a similar calculation using the simplifying assumption $F_2^{(\pi)} \simeq \frac{2}{3} F_2^{(p)}$ and limiting himself to a very small $|t|$ region, in order to safely neglect any t dependence in the vertices. We will instead extend the $|t|$ region using suitable t dependence for the pion exchange and use more detailed models for $F_2^{(\pi)}$ based on different extrapolations to the space-like region of the measured[4] pion structure functions in the time-like region.

Two interesting results emerge from our analysis. First, pion exchange contribution is found to be large enough so that a measurement of the above process appears to provide a viable method for obtaining the pionic structure functions. Secondly, at least $5 \div 10\%$ of the deep inelastic events ($e+p \longrightarrow e'+X$) should contain a slow neutron in the final state.

The paper is organized as follows: In section 2 after defining the kinematics, we give the Born term formulae for the cross-section. Section 3 deals with the models for pion exchange and Section 4 contains a discussion of different models for the pion structure functions. The results of numerical calculations are presented in Section 5 and the paper finishes with some concluding remarks in Section 6.

2. KINEMATICS AND BORN CROSS-SECTIONS.

The calculations are done, as usual, in the target proton rest frame (laboratory frame). We choose the lepton momenta as defining the xz plane, the z-axis being parallel to the direction of the virtual photon momentum. Neglecting the lepton masses and denoting by m and μ the nucleon and pion masses respectively, we define the 4-momentum of the particles as follows:

$k = E_L(1, \text{Sin}\,\psi\,, 0, \text{Cos}\,\psi\,)$ for the incoming lepton

$k' = E'_L(1, \text{Sin}(\psi+\vartheta_L), 0, \text{Cos}(\psi+\vartheta_L))$ for the outgoing lepton

$P_p = m(1,0,0,0)$ for the target proton

$P_n = (E, p\,\text{Sin}\,\alpha\,\text{Cos}\,\beta, p\,\text{Sin}\,\alpha\,\text{Sin}\,\beta, p\,\text{Cos}\,\alpha)$ for the neutron.

Here, ϑ_L is the angle between the leptons and $\tan\psi = \dfrac{E'_L\,\text{Sin}\,\vartheta_L}{E_L - E'_L\,\text{Cos}\,\vartheta_L}$, such that the internal photon momentum $q \equiv k-k'$, has no x or y component. We define, as usual, $\nu = q\cdot P_p/m = E_L - E'_L$ and $Q^2 = -q^2 = 4E_L E'_L \text{Sin}^2\,\vartheta_L/2$, to have

$$q = (\,\nu\,, 0, 0, \sqrt{\nu^2 + Q^2}\,).$$

We also define the Bjorken scaling variables $\omega = \dfrac{2m\nu}{Q^2}$ for the proton and an analogous quantity $\Omega = \dfrac{2\mu\nu_\pi}{Q^2}$ for the pion, where $\nu_\pi = \dfrac{q\cdot P_\pi}{\mu} \equiv \dfrac{q(P_p - P_n)}{\mu}$. Lastly, the momentum transfer between the nucleons is given by $t = p_\pi^2 \equiv (P_n - P_p)^2 = -2m(E-m)$.

The Feynman amplitude for the one pion exchange contribution (Fig. 1) to the process $e(k)+P(P_p) \longrightarrow e'(k')+n(P_n)+X$, can be written as follows:

$$M_n = \alpha\left[\bar{u}(k')\,\gamma^\varrho\,u(k)\right] \frac{4\pi}{q^2} \langle Xn|\; j_\varrho\,(o)\,|''\pi''\rangle \frac{\sqrt{2}\,g}{(t-\mu^2)} \cdot \left[\bar{u}(P_n)\,\gamma_5\,u(P_p)\right], \qquad (1)$$

with $\alpha \simeq \dfrac{1}{137}$ and $g^2/4\pi \simeq 14.7$. The inclusive cross-section is given by

$$d\sigma = \frac{1}{4mE_L} \sum_{\substack{(X_n)\\ \text{spins}}} \left|M_n\right|^2 (2\pi)^4\,\delta^4(p_\pi + q - P_{X_n}) \cdot \left[\frac{d^3k'}{(2\pi)^3\,2E'_L}\right]\left[\frac{d^3P_n}{(2\pi)^3\,(2E)}\right] \qquad (2)$$

The structure functions for the pion are defined as:

$$(2\pi)^4 \sum_{(X_n)} \delta^4(p_\pi + q - P_{X_n}) \langle \pi | j_\nu(o) | X_n \rangle \langle X_n | j_\varrho(o) | \pi \rangle =$$

$$= 4\pi\mu \left[\left(-g_{\nu\varrho} + \frac{q_\nu \; q_\varrho}{q^2}\right) W_1^{(\pi)} + \left(p_{\pi\nu} - \frac{p_\pi \cdot q}{q^2} q_\nu\right) \cdot \left(p_{\pi\varrho} - \frac{p_\pi \cdot q}{q^2} q_\varrho\right) \frac{W_2^{(\pi)}}{\mu^2} \right] \qquad (3)$$

and performing the spin sums we obtain:

$$\frac{d\sigma}{dE'_L \, d\,\mathrm{Cos}\,\vartheta_L} = \left(\frac{8\pi\alpha^2}{Q^4}\right)\left(\frac{\mu}{m}\right) g^2 \left[\frac{-t}{(\mu^2-t)^2}\right]\left(\frac{E'_L}{E_L}\right) \left\{ 2(k\cdot k')W_1 + \left[2(p_\pi \cdot k')(p_\pi \cdot k) - (k\cdot k')t\right] \frac{W_2}{\mu^2} \right\} \left[\frac{d^3 P_n}{(2\pi)^3 \, 2E}\right], \qquad (4)$$

where W_1 and W_2 are the structure functions for an off-shell pion of $(\mathrm{mass})^2 = t$. We introduce the functions $F_1 = \mu W_1$ and $F_2 = \nu_\pi W_2$, which for $t = \mu^2$ and in the Bjorken limit[(5)], should become functions of the variable Ω only, apart from corrections due to scaling violations. The result can now be written in the following form:

$$\frac{d\sigma}{dE'_L \, d\,\mathrm{Cos}\,\vartheta_L \, d^3 p_n} = \left(\frac{4\alpha^2}{\pi Q^4}\right)\left(\frac{g^2}{4\pi}\right)\left(\frac{E'^2_L}{mE}\right)\left(\frac{-t}{(\mu^2-t)^2}\right) \left\{ 2\,\mathrm{Sin}^2 \,\vartheta_L/2 \cdot F_1(\Omega, Q^2, t) + \left[\frac{(p_\pi \cdot k)(p_\pi \cdot k')}{E_L E'_L} - t\,\mathrm{Sin}^2 \frac{\vartheta_L}{2}\right] \frac{F_2(\Omega, Q^2, t)}{\mu\nu_\pi} \right\}, \qquad (5)$$

where, according to the previously given definitions,

$$\frac{(p_\pi\cdot k)(p_\pi\cdot k')}{E_L E'_L} = (m-E+p\,\mathrm{Sin}\,\alpha\,\mathrm{Cos}\,\beta\,\mathrm{Sin}\,\psi+p\,\mathrm{Cos}\,\alpha\,\mathrm{Cos}\,\psi)\Big[m-E+ +p\,\mathrm{Sin}\,\alpha\,\mathrm{Cos}\,\beta\,\mathrm{Sin}(\psi+\vartheta_L)+p\,\mathrm{Cos}\,\alpha\,\mathrm{Cos}(\psi+\vartheta_L)\Big], \quad (6)$$

and

$$\mathrm{Sin}\,\psi = \frac{E'_L\,\mathrm{Sin}\,\vartheta_L}{\sqrt{\nu^2+Q^2}}, \qquad \mathrm{Cos}\,\psi = \frac{E_L - E'_L\,\mathrm{Cos}\,\vartheta_L}{\sqrt{\nu^2+Q^2}}$$

$$\mathrm{Sin}(\psi+\vartheta_L) = \frac{E_L\,\mathrm{Sin}\,\vartheta_L}{\sqrt{\nu^2+Q^2}}, \qquad \mathrm{Cos}(\psi+\vartheta_L) = \frac{E_L\,\mathrm{Cos}\,\vartheta_L - E'_L}{\sqrt{\nu^2+Q^2}} \quad (7)$$

In the next section, we shall modify the cross section formula given by Eq. (5) to include the form factors.

3. MODELS FOR PION EXCHANGE.

Our formula for the cross section Eq. (5) has been derived from the Born term, i. e. an elementary pion with no cut off. Such an assumption is clearly unrealistic and we have to modify it, even for the limited t region which will be used throughout our analysis viz $-t \lesssim 0.5$ GeV^2. Outside this region, OPE is presumably quite small and other diagrams become important.

We have two different possibilities: either we assume the pion to lie on Regge trajectory $\alpha_\pi(t) = \alpha'(t-\mu^2)$ with $\alpha' \simeq 1\ \mathrm{GeV}^{-2}$, or we can use a model of elementary pion with a transverse momentum cut off which we proposed some years ago[2,6]. Both these models have been tested with the experimental data for the process $p+p \longrightarrow n+X$[7], in the range $-t \lesssim 0.5\ \mathrm{GeV}^2$. It should be mentioned that these data were contradictory, ISR data favoring the elementary pion model, while NAL data were in agreement with the Regge model. In the following results for both models shall be presented.

For the elementary pion with transverse momentum cutoff, the modification of Eq. (5) is quite simple. It requires to multiply g^2 by the factor exp $\left[-B(p_L^2+\mu^2)\right]$ with $B \simeq 14\ \mathrm{GeV}^{-2}$[7].

In the Reggeized pion model, for fixed, small t and at high energy, we can use the triple Regge formula[8]. This brings an extra power-law factor $(M_X^2/S)^{-2\alpha_\pi(t)}$, where $S=(q+P_p)^2$ and $M_x^2=(q+p_\pi)^2$. In terms of the variables defined earlier, we have $M_X^2/S=\left(\frac{\Omega-1+t/Q^2}{\omega-1+m^2/Q^2}\right)$. Also, the Regge propagators and residue factors can be introduced in terms of the parameters already determined, by making the substitution

$$\frac{1}{(\mu^2-t)^2} \longrightarrow \frac{\pi^2\alpha'^2}{2}\ \frac{1+\mathrm{Cos}\left[\pi\alpha_\pi(t)\right]}{\mathrm{Sin}^2\left[\pi\alpha_\pi(t)\right]}\ e^{b(t-\mu^2)},$$

where the fit to $p+p \longrightarrow n+X$[7] suggest $b \simeq 0,56\ \mathrm{GeV}^{-2}$.

4. PION STRUCTURE FUNCTIONS.

Here we describe some models for the pion structure functions $F_i(\Omega, Q^2, t=\mu^2)$, $i=1,2$.

From e^+e^- annihilation we have data for F_1 and F_2 in the time-like region ($\Omega < 1$) which show that in the region $0.5 \lesssim \Omega < 1$ scaling is obeyed and the cross-section is essentially transverse[4b]. In order to take advantage of these data we will assume that an analytic continuation from the annihilation to the scattering region is valid. Some experimental evidence supporting this hypothesis has been presented for the proton structure function near $\Omega = 1$[9].

One possible way to make the continuation is to utilize a reciprocity relation, first proposed by Gribov and Lipatov[10] which states

$F_2(\Omega) = \frac{1}{\Omega^3} F_2(\frac{1}{\Omega})$. However, there are other known constraints. For example, in the parton model the mean square charge of the partons is equal to $\int_1^\infty \frac{F_2(\Omega)}{\Omega^2} d\,\Omega$ [11]. Assuming fractionally charged quarks, one has the upper bound $\int_1^\infty \frac{F_2(\Omega)}{\Omega^2} d\Omega \lesssim 4/9$, allowing for a large part of the momentum to be carried by the sea quarks. Using the Gribov-Lipatov relation, one has $\int_1^\infty \frac{F_2(\Omega)d\Omega}{\Omega^2} = \int_0^1 x^3 F_2(x)dx$. Inserting in the last integral the scaling part of the SPEAR data[4] alone, one obtains $\int_{1/2}^1 x^3 F_2(x)dx \simeq 0.45$. Moreover, for the region $x < 1/2$ the present data do not scale and in fact show an increase in F_2 with Q^2. A rough estimate at the highest energies show that the contribution to the sum rule is $\simeq 1$. Thus, the sum rule seems to be inconsistent with the reciprocity relation.

Model I

In the fractionally charged quark-parton model, assuming π^+ as a bound state of u and $\bar{d}$ quarks, one would get for the sum rule the exact value (valence quarks only)

$$\int_1^\infty \frac{F_2(\Omega)}{\Omega^2} d\Omega = \frac{5}{18} \qquad (8)$$

We may use this as a constraint in building a phenomenological formula for $F_2(\Omega)$. We have chosen

$$F_2(\Omega) = \frac{(\Omega-1)^2}{\Omega^3} \left[C_1 e^{-K\Omega} + C_2 \Omega \right] , \qquad (9)$$

because SPEAR data suggest a quadratic dependence near $\Omega = 1$. The parameter C_2 is fixed to be $\simeq 0.2$ through the asymptotic behavior given by the quark model, $F_2^{(\pi)}(\infty) \simeq (2/3)\, F_2^{(p)}(\infty)$. If Eq. (8) is used as a constraint, only one parameter remains to be determined by a best fit to the scaling data[4c] as shown in Table I.

TABLE I

K	0.65	0.7	0.75	0.8	0.85	0.9
C_1	13.17	14.59	16.86	17.86	19.74	21.71
χ^2/NDF	1.11	0.81	0.59	0.49	0.51	0.67

In what follows, the values K=0.8 and C_1=17.86 will be used, even though the χ^2 dependence is not sharp enough to uniquely fix the best value. Luckily, in the region of our interest $1 < \Omega \lesssim 10$, the values of $F_2(\Omega)$ do not change appreciably ($\sim 10 \div 20\%$) by varying the parameters K and C_1 in the acceptable range. Thus, the actual choice is not very crucial.

As an independent check, using the above values for the parameters and Eq. (9), we have calculated the mean pion multiplicity pretending as if scaling were valid for all Ω and cross-section were purely transverse. Under such a hypothesis

$$\langle n_\pi \rangle = \frac{1}{R} \int_{2\mu/\sqrt{S}}^{1} x^2 F_2(x) dx,$$

where

$$R = \frac{\sigma_{had}(S)}{\sigma_{\mu\bar{\mu}}(S)} .$$

At $\sqrt{S}$ =4.8 GeV, assuming $R \simeq 5$ we get $\langle n_\pi \rangle \simeq 4.5$ and at $\sqrt{S}$ = 7.4 GeV with $R \simeq 5.5$ we get $\langle n_\pi \rangle \simeq 5.3$, in good agreement with experiment. Thus, not with-standing the fact for $\Omega < 1/2$ the data do not scale and have a large longitudinal component, the charge multiplicity is correctly given by our parametrization. This leads us to hope that our formula Eq. (9) can be used (atleast in the mean) even in region of Ω different from ($0.5 \div 1$) which were used to fit the parameters.

Model II

A different (and simpler) parametrization may be achieved without imposing the constraint given by Eq. (8) if one used Eq. (9) with $K=0$. A fit to the data gives $C_1=11.2$ ($C_2=0.2$ as before). In this case, the value of the integral in sum rule, Eq. (8), comes out to be 1, which is the limiting value in any parton model[11]. The corresponding pion multiplicities are $\langle n_\pi \rangle \simeq 3.3$ (3.8) for $\sqrt{S}=4.8$ (7.4) Gev.

Model III

Lastly, we have tried a formula for F_2 which satisfies the Gribov-Lipatov reciprocity relation, viz.

$$F_2(\Omega) = \frac{(\Omega-1)^2}{\Omega^3} \frac{A(\Omega^2+1)+A'\Omega}{(\Omega+1)}, \tag{10}$$

with $A=0.2$ (from the asymptotic value) and $A'=27$ (from a fit to the SPEAR data for $1/2 < \Omega < 1$). This formula gives for the sum rule integral in Eq. (8) the value 1.7 in complete contradiction with any parton model. On the other hand the pion multiplicity comes out too small. Conversely, if one modified Eq. (10) in order to obtain the right value for the multiplicity one would obtain a still larger value for the sum rule integral in Eq. (8), due to the connection imposed by the reciprocity relation between $\Omega < 1$ and $\Omega > 1$.

The formulae previously given for $F_2(\Omega)$ and the corresponding ones for $F_1(\Omega) = \frac{\Omega}{2} F_2(\Omega)$ will be used in the calculations to follow, ignoring any possible t dependence in the structure functions. This approximation is expected to be reasonable, since we limit ourselves to the small $|t|$ region.

5. NUMERICAL ANALYSIS

In this section, we present a numerical estimate of the contribution to the deep inelastic cross-section on protons from our diagram (Fig. 1) when the momentum transfer between the baryons is limited to $|t| \lesssim 0.5\ \mathrm{GeV}^2$.

As explained in Section 3, we take the modified Eq. (5) and integrate over the neutron momentum with the constraint $y = \frac{E-m}{m} \lesssim 0.3$ (corresponding to $-t \lesssim 0.53\ \mathrm{GeV}^2$) to obtain

$$\frac{d\sigma}{dE' d\cos\vartheta_L} = \left(\frac{4\alpha^2}{Q^4}\right)\left(\frac{E_L'^2}{m}\right)\left(\frac{g^2}{4\pi}\right)\frac{1}{\sqrt{1+\gamma}}\int_{z_1}^{z_2} dz \int_{y_1(z)}^{y_2(z)} dy\, y\, h(z,y) \cdot \tag{11}$$

$$\cdot \left\{ 2\sin^2(\vartheta_L/2) H_1(z,y) + \cos^2(\vartheta_L/2)\left(\frac{m}{\nu}\right) H_2(z,y) \right\},$$

with $\gamma = \frac{4m^2}{Q^2\omega^2} = \frac{Q^2}{\nu^2}$ and $z = \frac{\Omega}{\omega}$. The functions H (z,y) are defined as

$$H_1(z,y) = F_1^{\pi}(\omega z) + \left(\frac{m}{\nu}\right)\frac{1}{z}\left[\frac{\gamma}{2(1+\gamma)} y^2 + \left(1-\frac{z}{1+\gamma}\right) y - \frac{z^2}{2(1+\gamma)}\right] F_2^{\pi}(\omega z)$$

$$H_2(z,y) = \frac{1}{z}\left[\frac{3\gamma^2}{2(1+\gamma)^2} y^2 + \frac{\gamma}{(1+\gamma)}\left(1-\frac{3z}{1+\gamma}\right) y + \frac{(2-\gamma)}{2(1+\gamma)^2} z^2\right] F_2^{\pi}(\omega z) \tag{12}$$

Following Section 3, we have two alternative forms for the function h(z,y). For the Reggeized pion ($\alpha' = 1\ \mathrm{GeV}^{-2}$)

$$h(z,y) = (2\pi^2 m^4 \alpha'^2)\frac{1+\cos\left[\pi 2m^2\alpha'\left(y+\frac{\mu^2}{2m^2}\right)\right]}{\sin^2\left[\pi 2m^2\alpha'\left(y+\frac{\mu^2}{2m^2}\right)\right]} \exp\left\{-2m^2\left(y+\frac{\mu^2}{2m^2}\right)\cdot\right.$$

$$\left.\left[b + 2\alpha' \ln\left(\frac{1-\frac{1}{\omega}+\frac{m^2}{\omega Q^2}}{z-\frac{1}{\omega}-\frac{2m^2 y}{\omega Q^2}}\right)\right]\right\}, \tag{13}$$

while for the elementary pion with transverse momentum cut off

$$h(z,y)=\frac{1}{(y+\frac{\mu^2}{2m^2})^2}\exp\left\{-2m^2B\left[\frac{\gamma}{(2(1+\gamma)}y^2+(1-\frac{z}{1+\gamma})y-\frac{z^2}{2(1+\gamma)}+\frac{\mu^2}{2m^2}\right]\right\} \qquad (13')$$

The limits of integration in Eq. (11) are given by

$$z_1=1-\frac{1}{2}\;\frac{(1-\frac{2}{\omega})(\omega-1+\frac{2m^2-\mu^2}{Q^2})+\sqrt{1+\gamma}\sqrt{(\omega-1+\frac{2m^2-\mu^2}{Q^2})^2-\frac{4m^2}{Q^2}(\omega-1+\frac{m^2}{Q^2})}}{(\omega-1+\frac{m^2}{Q^2})},$$

$$\xrightarrow[\text{Bj}]{}\ \frac{1}{\omega}$$

$$z_2=-0.3+\sqrt{0.69(1+\gamma)}\ \xrightarrow[\text{Bj}]{}\ 0.53$$

$$y_1(z)=-1+\frac{1}{\gamma}\left[z-1+\sqrt{1+\gamma}\sqrt{(z-1)^2+\gamma}\right]\ \xrightarrow[\text{Bj}]{}\ \frac{z^2}{2(1-z)}$$

$$y_2(z)=\min\left\{0.3,\ \frac{Q^2}{2m^2}(\omega z-1)-\frac{\mu^2}{2m^2}\right\}\ \xrightarrow[\text{Bj}]{}\ 0.3\ .$$

In the above formulae we have kept the exact Q^2 dependence in the kinematic factors but have omitted it in $F^{\pi}_{1,2}$, since the calculations will be done for $Q^2 \gtrsim 2\ \mathrm{GeV}^2$, i.e. in the expected scaling domain.

The contributions to the proton structure functions are given by

$$F_i^{(p)}(\omega,Q^2)=\frac{1}{2\pi}\left(\frac{g^2}{4\pi}\right)\frac{1}{\sqrt{1+\gamma}}\int_{z_1}^{z_2}dz\int_{y_1(z)}^{y_2(z)}dy\,y\,h(z,y)H_i(z,y) \qquad (14)$$

Assuming the relation $F_1^{(\pi)}(\Omega)=(\Omega/2)\,F_2^{(\pi)}(\Omega)$ (spin 1/2 parton model) we find that in the Bjorken limit $F_1^{(p)}(\omega) = \frac{\omega}{2} F_2^{(p)}(\omega)$, since in this limit

$$H_1(z_1 y) \longrightarrow F_1^{(\pi)}(\omega z) = \frac{\omega}{2} z F_2^{(\pi)}(\omega z)$$

$$H_2(z_1 y) \longrightarrow z F_2^{(\pi)}(\omega z).$$

It is pleasing to note that the resulting proton structure functions are consistent with the spin 1/2 parton model. Actually the result is more general: the output proton structure functions scale the same way the pion structure functions do.

We now present results for the integrals appearing in Eq. (14) using the models described in Section 4 for $F_2^{(\pi)}(\Omega)$ and $F_1^{(\pi)}(\Omega) = \Omega/2$ $F_2^{(\pi)}(\Omega)$. The results will be multiplied by 3/2 to take account of π^0 exchange as well, namely a diagram similar to Fig. 1 where a proton appears in the final state.

The results for Model I and Reggeized pion, EQ. (13) are presented in Fig. 2 for $F_2^{(p)}$. They show that scaling is rather good, since the variation is everywhere less than $\sim$ 12% between Q^2=2GeV^2 and Q^2=25 GeV^2. The contribution of π exchange diagrams to $F_2^{(p)}$ is seen to be $\sim$ 15% of the total for values of $\omega \simeq 10$ in this case. This amounts to saying that about 10% of the deep inelastic events should contain a slow neutron in the final state. We remind the reader that our analysis includes values of $|t| \leq 0.53$ GeV^2 and that any t dependence of $F_2^{(\pi)}$ has been neglected. If $F_2^{(\pi)}$ were weakly dependent on t even at larger values (of t), the total pion exchange contribution to $F_2^{(p)}$ would be considerably larger than that shown in Fig. 2 especially for large ω.

In Fig. 3 we present the corresponding results for $F_1^{(p)}$ using the exact formula for Q^2=2 GeV^2 as well as from the relation $F_1^{(p)} = (\omega/2)$ $F_2^{(p)}$ and $F_2^{(p)}$ taken from Fig. 2 for Q^2=2GeV^2. As we can see, the agreement between the two results is very good. For this reason the function $F_1^{(p)}$ will not be presented for other models.

Fig. 4 contains results for Model I and an elementary pion with p_1 cutoff. (Eq. (13')). For this case, the strong cutoff reduces the pion exchange contribution to $\sim$ 7% of the total proton structure function. For the same reason this result should not change appreciably even if we extend the range of integration in t.

Model II of Section 4 gives the results presented in Fig. 5 and 6 for Reggeized and elementary pion cases respectively. As is clear the pion contribution is considerably larger in this case and in fact eventually saturates the true $F_2^{(p)}$ for the Reggeized pion.

Model III seems to be essentially ruled out, at least for the Reggeized pion, since the contribution from π exchange turns out to be much larger than the measured $F_2^{(p)}$. The results for the elementay pion, presented in Fig. 7, are also quite large and hardly acceptable. This shows that the Gribov-Lipatov reciprocity relation for the pion structure functions is invalid, unless accompanied by a rather strong t-suppression of $F_2^{(\pi)}$.

6. SUMMARY AND CONCLUSIONS.

In this work we have presented the slow neutron spectra in deep inelastic lepton scattering as given by one pion exchange, using both a Reggeized pion and an elementary pion with transverse momentum cutoff.

Since experimental data for the neutron spectra are not available at present, we have integrated to obtain the π exchange contribution to the proton structure functions. This analysis requires a parametrization of the EM structure functions for the pion, for which we have constructed three different models. One of the models, based on the Gribov-Lipatov reciprocity relation, seems to be essentially ruled out. (See Section 5 for discussion). From Model I, which has been constrained to fit a sum rule given by the quark parton model, we obtained the result that at least 5 ÷ 10% of the deep inelastic events for $\omega \gtrsim 7$ should contain a slow (kinetic energy $\lesssim$ 300 MeV) neutron in the final state.

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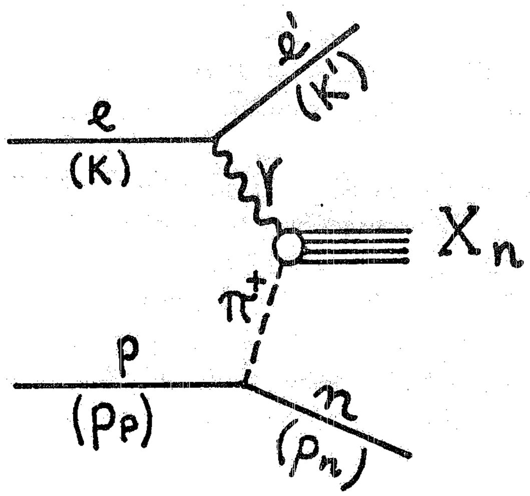

FIG. 1 - The one-pion exchange (OPE) diagram.

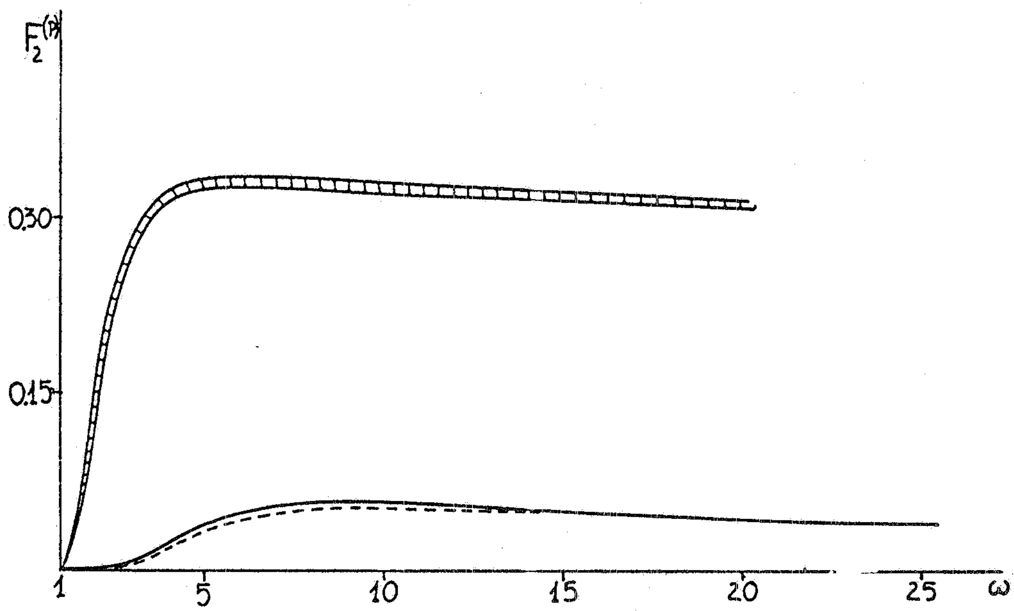

FIG. 2 - OPE contribution to $F_2^p(\omega)$ for Q^2 = 25 GeV^2 (full curve) and Q^2 = 2 GeV^2 (dashed curve) compared with experiment (shaded curve) for Model I and a reggeized pion.

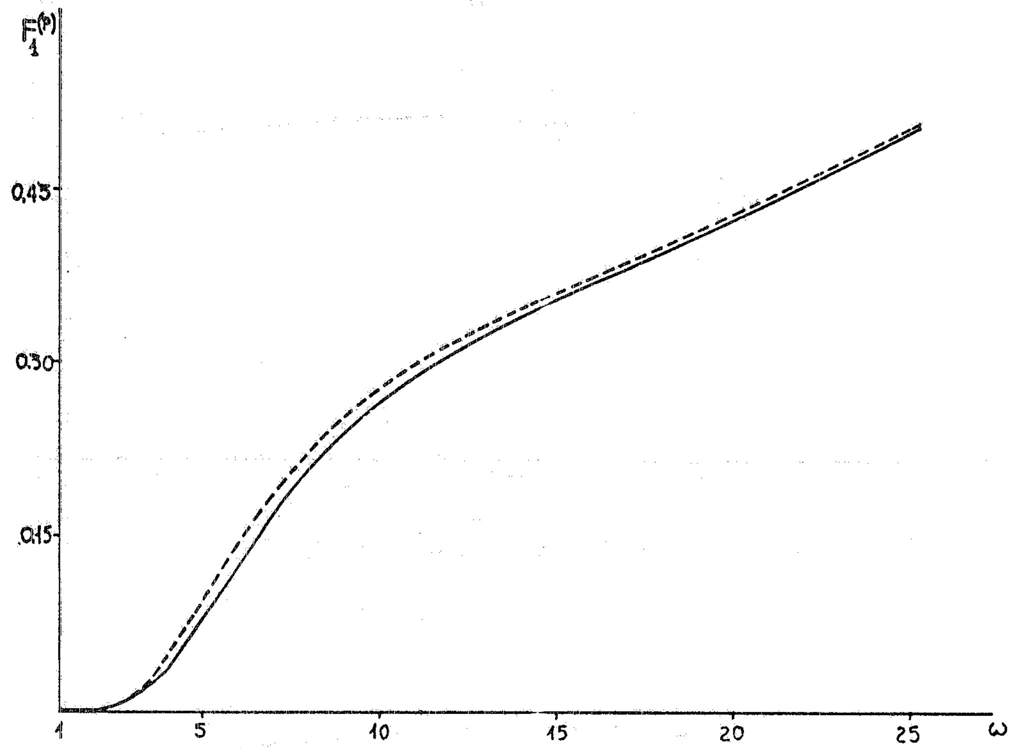

FIG. 3 - OPE contribution to $F_1^p(\omega)$ for $Q^2 = 2\ \mathrm{GeV}^2$ as calculated with the exact formula (dashed curve) and using the relation $F_1^p(\omega) = \frac{\omega}{2} F_2^p(\omega)$ (full curve) for Model I and a reggeized pion.

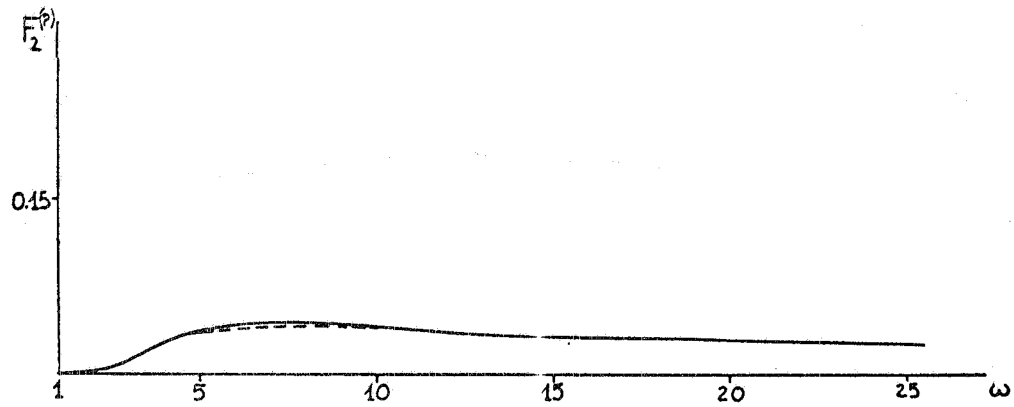

FIG. 4 - OPE contribution to $F_2^p(\omega)$ for $Q^2 = 25\,\mathrm{GeV}^2$ (full curve) and $Q^2 = 2\ \mathrm{GeV}^2$ (dashed curve) for Model I and a elementary pion with $p_\perp$ cut off.

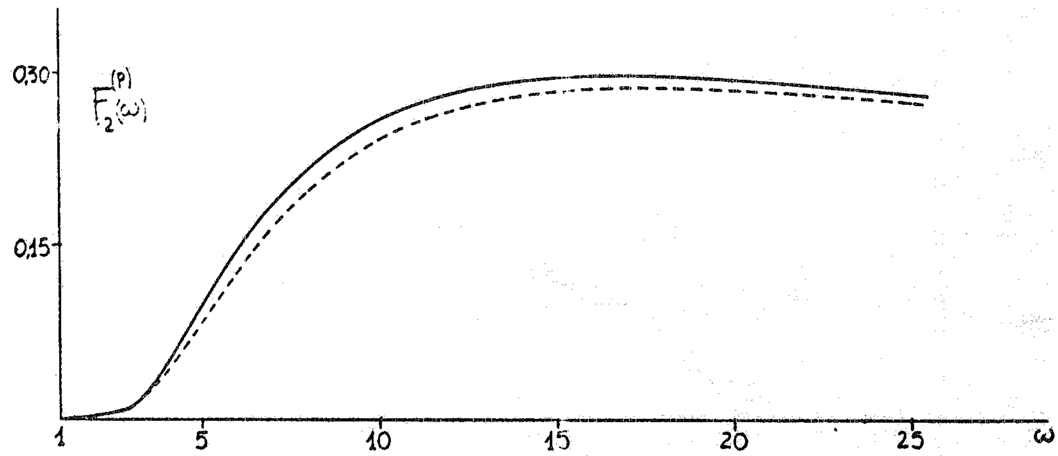

FIG. 5 - Same as Fig. 4 for Model II and a reggeized pion.

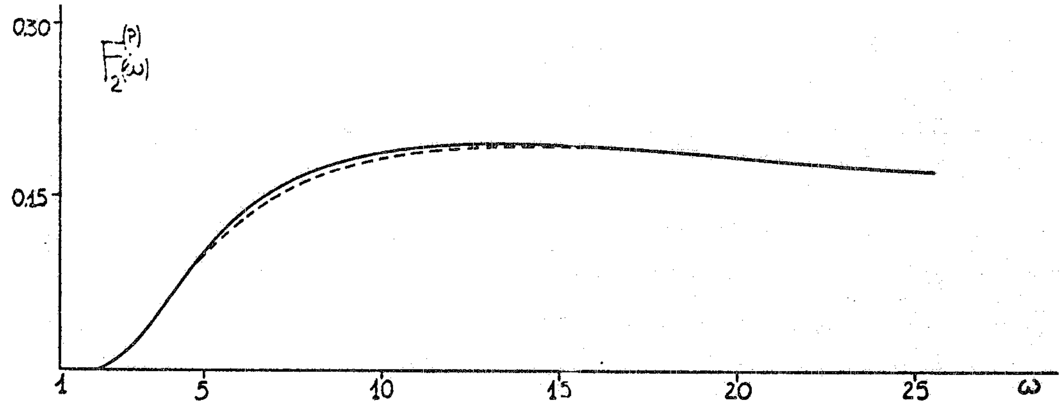

FIG. 6 - Same as Fig. 4 for Model II and an elementary pion.

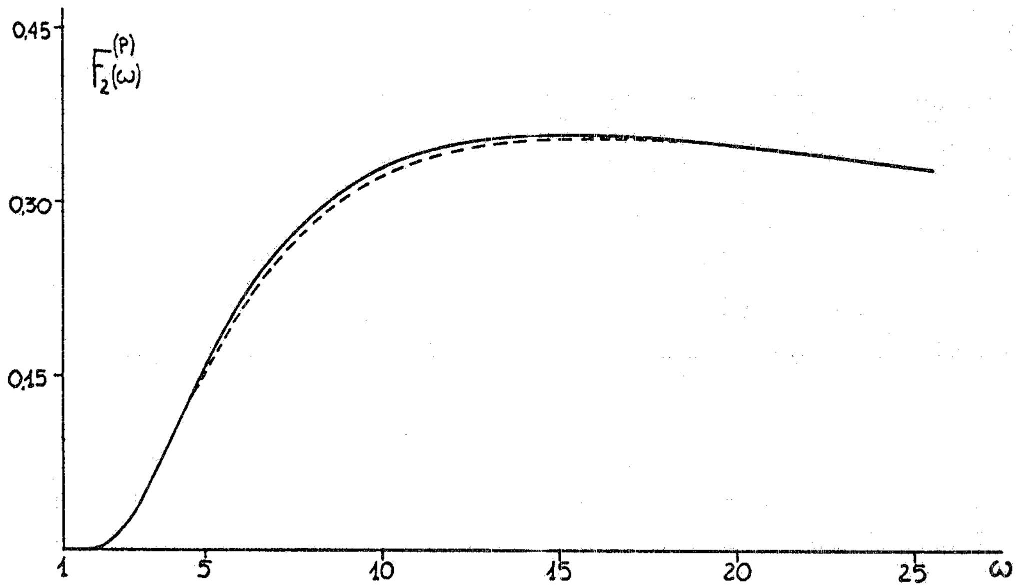

FIG. 7 - Same as Fig. 4 for Model III and an elementary pion.